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Effect of Porosity on Surface of Drawndown in an Unsteady State Drainage in Porous Material. I

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Abstract
This work considered the unsteady state drainage of fluid from a vertical column of porous material of varying porosities in an attempt to verify variation of drawndown surface with porosity and time using riverbed sand. Kerosene was used as the flowing fluid. Mathematical assumptions were made in connection with Darcy's law. The result showed that the experiment which was designed from the theoretical framework agreed with the theory to a large extent. However, this is not in perfect agreement with an earlier experiment which was performed using well rounded beads in which glycerine was used as the flowing fluid. A FORTRAN program was written to study the variation theoretically and this was compared with the experimental result. There exists, a degree of discrepancy between theory and experiment indicating that, the mathematical formulations did not perfectly agree with the complex earth system as compared with the bead model that was initially used. There is a need for modification of the mathematical formulations; nonetheless, an unsteady drawndown pattern was obtained with different porosity even in medium with very complex geometry.

Keywords: Darcy's law, drainage, drawndown, porosity and geometry.

Introduction
Ligon *et al* in 1962 [1,2] derived equations for the fluid surface location in a column of porous material and for the rate of discharge from this column after the beginning of drainage at complete saturation. He however applied this equation to simple homogeneous medium comprising of beads of 2mm in diameter and also a medium comprising of mixture of beads of 2mm and 0.5mm.

In this work, we want to see the workability of these sets of equations as applied to earth materials, basically porous sand, despite its complexities (from riverbeds) of porosities 0.33, 0.37, 0.38 and 0.39 and grain sizes ranging from 0.3mm – 0.7 . We want to examine the variability of drawndown surface with porosity and time in this medium. This research has application in agriculture in the use of irrigation in farm land. It gives information on how long water can be retained before been drained away in as much as the parameters of the soil are known.

A FORTRAN program was employed to generate values which were plotted to obtain curves and using the same expressions, an experiment was performed with each of the samples so as to make comparison between theory and experiment.

The column was initially saturated to the surface [3] and was allowed to begin at time $t=0$. ε is the fringe, δ is the hydraulic head at the lower end of the column, and was held constant throughout the drainage period. Basically, there are two equations for theoretical development for the rate of discharge of fluid per unit cross-sectional area at any time.

(a) Darcy's law applied to the column yields:

$$q_A = K\left(\frac{y-\varepsilon}{y+\delta}\right) \quad (1)$$

Where q_A is the rate of discharge per unit cross-sectional area of the column, K is the hydraulic conductivity of the porous medium, and y is the height of the surface of the fluid in the column.

(b) The rate of discharge per unit cross-sectional area is equal to the rate of fall of the surface of saturation multiplied by the porosity.

$$q_A = -\phi \frac{dy}{dt} \qquad (2)$$

Equating (1) and (2)

$$-\phi \frac{dy}{dt} = K\left(\frac{y-\varepsilon}{y+\delta}\right) \qquad (3)$$

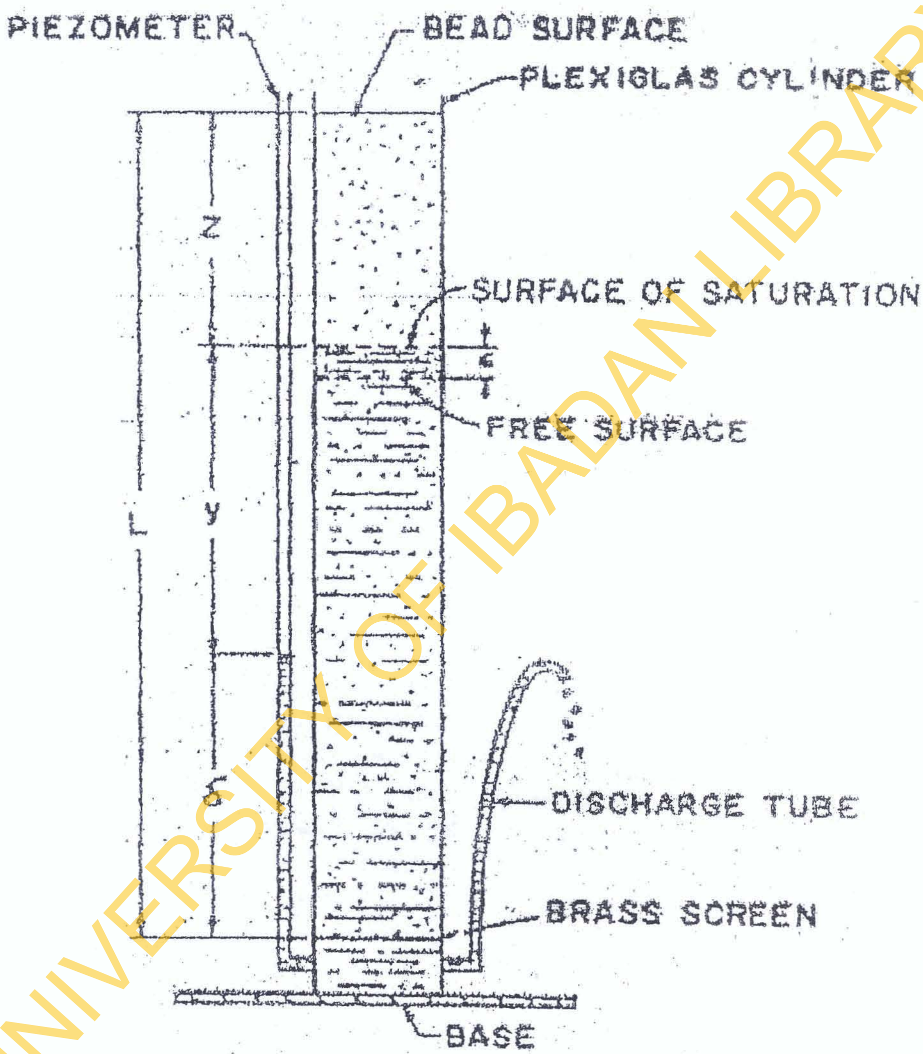

Fig. 1: The schematic diagram of the experimental setup

By separating variables, we have

$$\left(\frac{y+\delta}{y-\varepsilon}\right) dy = -\frac{K}{\phi} dt \qquad (4)$$

Integrating (4) and using the initial condition

i.e $y = L- \delta$ when $t= 0$, we obtain

$$\left(\frac{y-\varepsilon}{L-\delta-\varepsilon}\right)^{\delta+\varepsilon} exp\, y = exp\left(L-\delta-\frac{Kt}{\phi}\right) \qquad (5)$$

Note from the diagram that $y= L- \delta-z$ (6)

Substituting (6) into (5) and simplifying, we get

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$$\left(\frac{1-z}{(L-z-\delta)^{\delta+L}}\right)\exp(-z) = \exp(-Kt/\emptyset) \quad (7)$$

This can be converted into dimensionless form

$$\left(1-\frac{z-\delta}{\frac{L}{\delta}-\frac{z}{\delta}-1}\right)^{1+L/\delta} exp(-Kt/\delta\emptyset) \quad (8)$$

A FORTRAN program was written [4] to solve for equation (8) given the parameters as in Table 1.

z/δ is the dimensionless drawndown surface of saturation [5].

Experiment

Materials and Method

Sand, was obtained from streams within University of Ibadan and were sieved into different particle sizes using meshes with measured slit grades. Kerosene was used as the fluid, this is because it has better visibility than water during the flowing process. A saturated sample was prepared [6] by soaking sand in kerosene overnight. The transparent cylindrical glass column was oriented vertically by clamps attached to retort stand with discharge tube outlet focused over a measuring cylinder. A metre rule was attached to take measurement along the column. The saturated sand was poured into the tube which had a level of kerosene in it as quickly as possible to avoid compaction region [7, 8] between layers of settled sand. This process was stopped when a column of about 70-80cm of saturated sand was achieved.

The discharged tube was unlocked and drainage was allowed to begin. Also, the level of the saturation was drawndown simultaneously and measurements were taken at intervals for the discharge and drawndown of saturation surface from the measuring cylinder along the vertical column respectively. The piezometer reading was maintained at constant level throughout the period of the experiment and these fixed values were recorded accordingly. Measurements were stopped when drainage was observed to have ceased.

Results and Discussion

For each of the samples, z/δ was plotted against $Kt/\emptyset\delta$ so as to be able to evaluate both theoretical and experimental behaviour under conditions underlying the theory of the work. Table 1 shows the parameters used for the experiment and which also served as boundary conditions for the theoretical simulation. The graphs as seen in Figures 2-5, explains the deviation or similarities for each sample considered.

For theoretical drawndown however, the nature of the curves have varying degrees of steepness. The sharpest gradient was observed for the coarsest particles while the most extended slope was observed for finest sand. This observation is also true for the experimental drawndown when the two (theoretical and experimental) were considered independently. Considering the two simultaneously, it is obvious that for each of the samples, the rate of drawndown is far more pronounced in experimental result than it is in the theoretical.

Physically, this means that the rate of drawndown of surface of saturation is sharpest in coarse than fine sand as expected. The graph of z/δ versus $Kt/\emptyset\delta$ for all the samples for both theoretical and experimental data revealed a marked degree of inadequacy of the theoretical assumption for the cases considered. Though these mathematical assumptions fitted well when Legon *et. al* applied them to a homogeneous medium with fairly defined pore connectivity, but for the earth material (sand) a noticeable deviation was observed which could have resulted due to the increased complexities of pore that it can perfectly describe the interaction of fluid with matrix in complex porous medium, which the earth medium represents. This is being looked into and will be presented in the part II of this presentation among other things.

Table1: Parameters used for the experiment and theoretical simulation

Porosity	Grain size(mm)	L (m)	Hydraulic Conductivity (cm/sec)
0.33	0.30	0.80	8.11E-3
0.37	0.90	0.80	2.99E-2
0.38	0.74	0.80	4.09E-2
0.39	0.90	0.80	5.14E-2

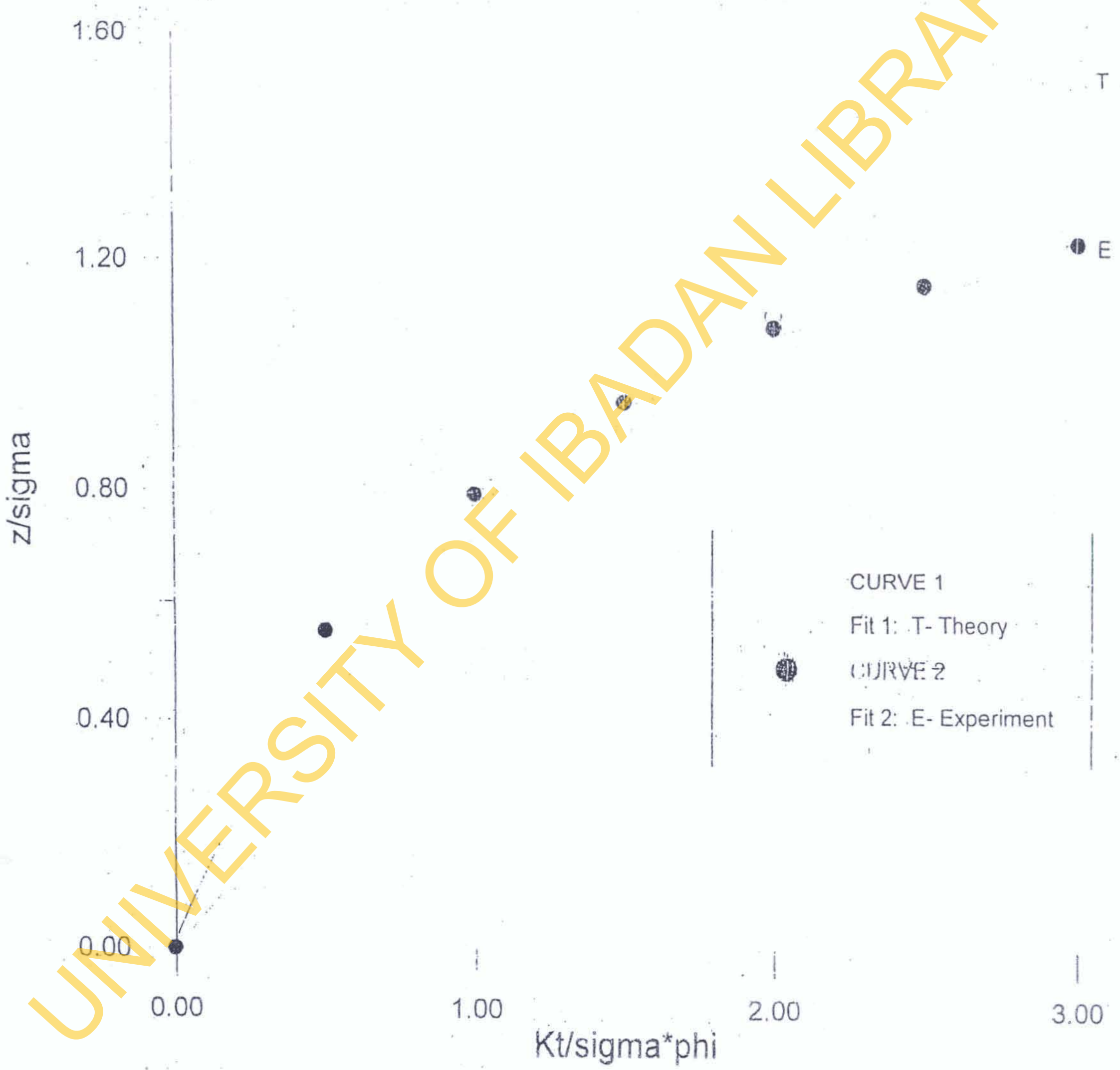

Fig. 1: Graph of z/δ versus $Kt/\phi\delta$ for both theoretical and experimental result.

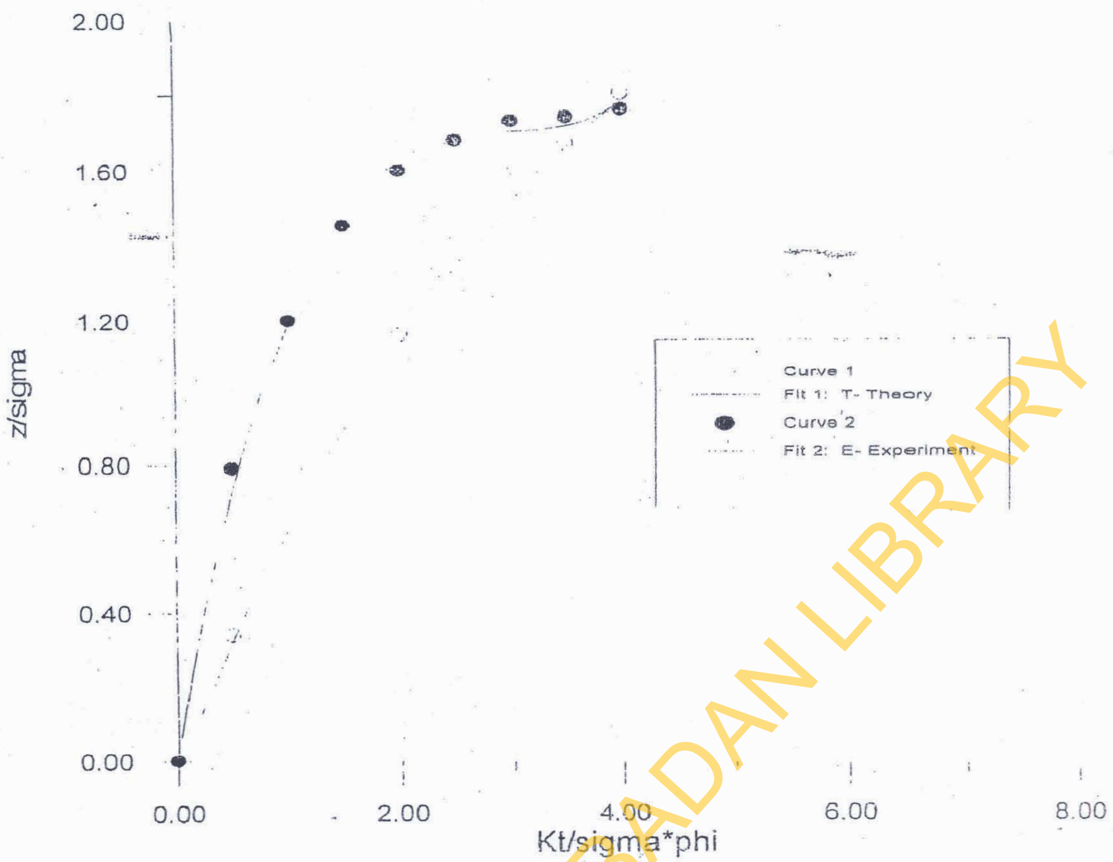

Fig. 2: Graph of *z/ δ* versus *Kt/øδ* for both theoretical and experimental result.

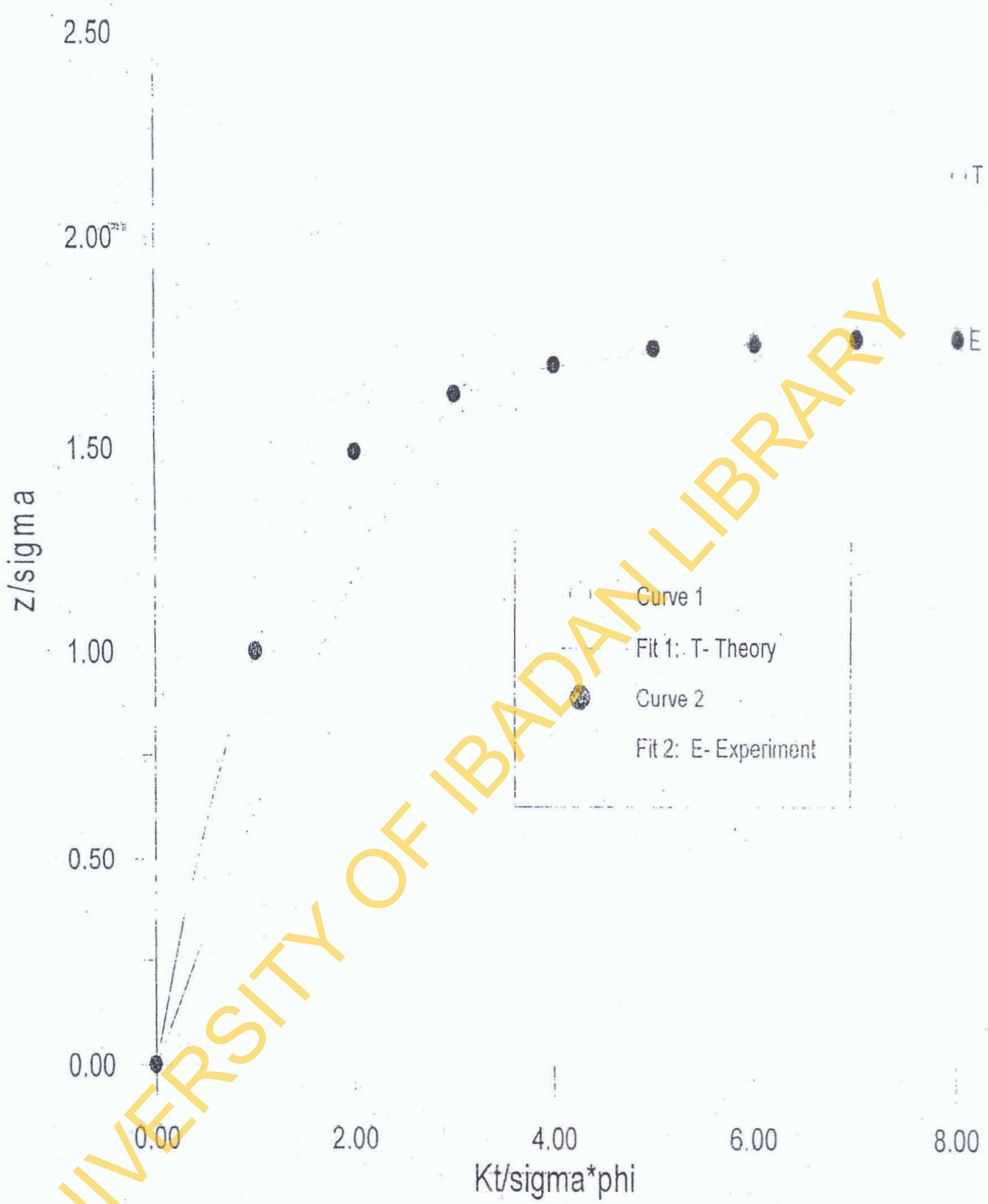

Fig. 3: Graph of z/δ versus $Kt/\o\delta$ for both theoretical and experimental result.

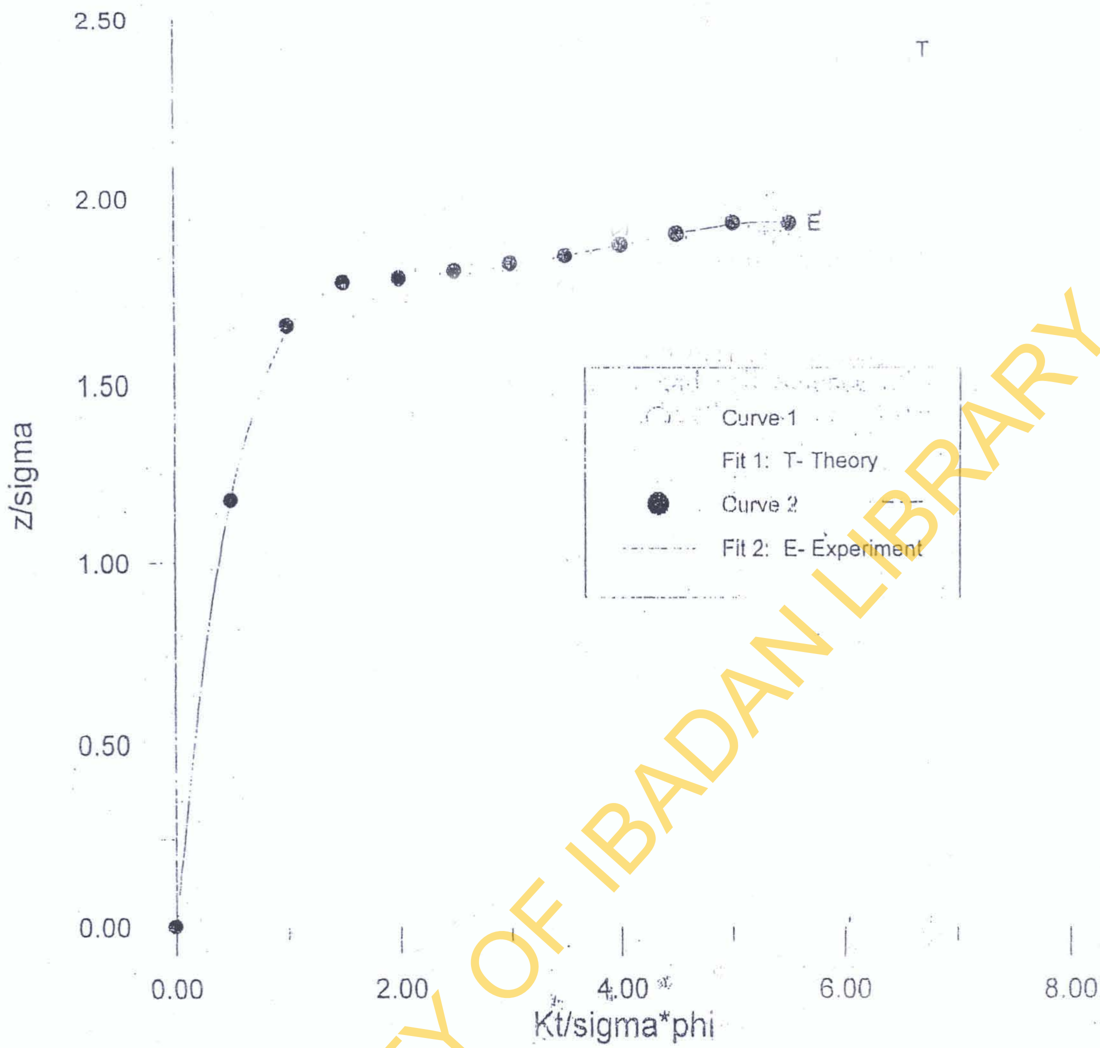

Fig. 4: Graph of z/δ versus $Kt/\phi\delta$ for both theoretical and experimental result.

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