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STUDY OF PERMEABILITY OF SATURATED HOMOGENEOUS AND HETEROGENEOUS POROUS MEDIA

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ABSTRACT

Many practical seepage and drainage problems can be studied by constructing flow nets for section with a single permeability; however, many natural soil deposits are more or less stratified, often with horizontal bedding that make horizontal permeabilities much greater than the vertical. Three different types of heterogeneous media from five soil sample of different porosities were considered; with constant-head permeameter to determine the saturated hydraulic conductivity for each. The result shows that least permeable medium dominates in the permeability of heterogeneous medium and fluid flows faster in mixed heterogeneous medium than layered heterogeneous medium. However, fluid flow in homogeneous porous media is generally faster than that of heterogeneous medium of similar geometry and grains packing. Therefore, the effect of least permeable unit in heterogeneous medium must be considered in selecting a proper filter for seepage control.

Key words: Permeability, Heterogeneous media, Seepage control, Hydraulic gradient, Porosity.

INTRODUCTION

Water is the major component of soils that fluctuates with time and seasons; as it changes, the soil strength or volume may change correspondingly. Control of the water content, control of the movement of water, and prevention of the damaged caused by the movement of water in soils are vital aspects of soil engineering the study of seepage patterns in cross section with soils of more than one permeability is one of the most worthwhile and rewarding applications especially in selecting a protective filter or seepage control in man-made constructions. Control of seepage involves reducing the flow, reducing the water pressure, or increasing the load that resists the water pressure. Excessive seepages is caused by high permeability or short seepage path (Sower, 1970). Soil mass through which seepage occur is man-made, like septic tank or sewage disposal facility, the permeability can be reduced by the proper selecting materials, for example, mixing a small amount of clay with the sand (protective filter) used for construction can reduce the permeability greatly (Sower, 1970). A filter or protective filter is any porous materials whose opening are small enough to prevents movement of the soil into the drains and which is sufficiently pervious to offer little resistance to seepage. Frequently a soil is employed as a filter, and in preparing a good filter the knowledge of permeability of homogenous and heterogeneous media is very essential. A medium is homogenous if the permeability varies is constant from point to point over medium while it is heterogeneous if permeability from point to point in the medium. The permeability is the most important physical property of a porous medium, which is a measure of the ability of a material to transmit fluid through it. The application of Darcy's law enables hydraulic conductivity to be determined, from which permeability can be computed by using Hubert King relation. (Domenico and Schwartz, 2000). In this study riverbed sand samples were used as porous media for both homogeneous and heterogeneous media. The permeability properties of both media were determined in the laboratory using a vertical form of Darcy's equation. Five different homogenous media and three different heterogeneous media were used during the experiment. The homogeneous samples were obtained by the use of a sieve of a known grade while the heterogeneous samples were obtained by mixing sieved send of different grain sizes. The purpose of this study is to improve on the achievements made so far by the Darcy's law. The objectives are to investigate the difference in the flow rate of fluid in both homogenous and heterogeneous media with the respect to their varying permeabilities, and compare the permeability of different types of heterogeneous media. These are necessary for proper selection of material for seepage control in man-made constructions.

THEORY

When water flows across a boundary between dissimilar soil layers, the flow lines bend much in the way light rays are refracted in passing from air into water or from air into glass. Thus, when water flows from soil of high permeability into a material of lower permeability, the pattern develops in such a way that the flow remains in the more permeable material for the greatest possible distance. Conversely, it deflects as soon as possible into the material of higher permeability. To conserve energy, water seeks the easiest paths of travel (Cedergreen, 1976). The way flow lines deflect when they cross boundaries between soils of different permeabilities is shown in Fig.1.

The flow lines bend to confirm the relationship given as

$$\frac{\tan\beta}{\tan\alpha}=\frac{k_1}{k_2} \quad (1)$$

where

k_1 = permeability of medium 1

k_2 = permeability of medium 2

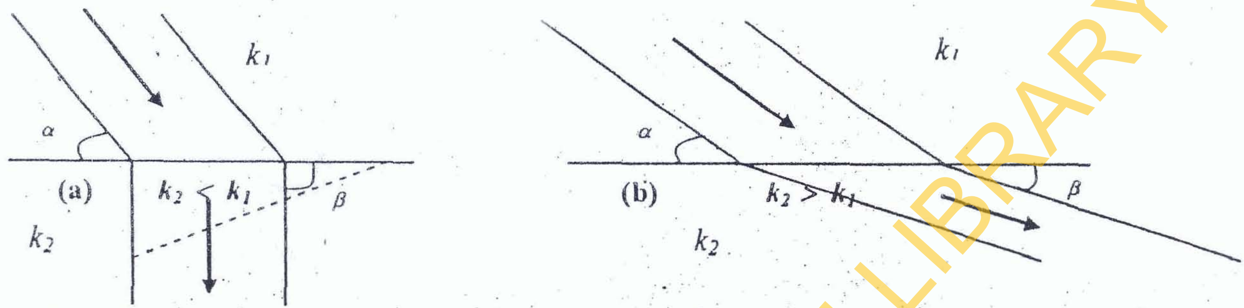

Fig. 1: Transfer conditions at boundaries between soil of different permeabilities (Casagrande, 1937).

When water flows from a soil of low permeability into a soil of higher permeability, less area is required to accommodate the same quantity of water and lower gradients are needed. If the flow is from high permeability into lower permeability, steeper (or higher) gradient are required and a relatively more area is needed to accommodate the flow. (Cedergreen, 1976). If layers of beds of porous media of different porosity is considered and it is assumed that each layer is homogenous and isotropic, then each layer is however characterized by a different hydraulic conductivity rendering the sequence as a whole heterogeneous. Theoretically, it has been found by Leonards (1962), that an equivalent horizontal hydraulic conductivity in the horizontal or x-direction is

$$K_x=\frac{\sum(m_i k_i)}{\sum m_i} \quad (2)$$

Where

K_x = the equivalent horizontal conductivity;

k_i = homogeneous conductivity of an individual layer and

m_i = the thickness of the layer

For vertical or direction at right angle to the stratification

$$K_z=\frac{\sum(m_i)}{\sum(m_i/k_i)} \quad (3)$$

where K_z = the equivalent vertical conductivity

It was found that for horizontal flow, the most permeable unit dominates the system. For vertical flow the least permeable unit dominates the system. Under the same hydraulic gradient, horizontal flow is of the order of six orders of magnitude faster than vertical flow. (Domenico and Schwartz, 2000)

MATERIALS AND METHOD

Sand samples were collected from the riverbed of two different rivers. The samples were washed, rinsed, sun dried and later placed in an oven. The samples were later sieved into five different grain sizes. Each of these samples was used as homogeneous medium. Three different types of heterogeneous media were formulated. They were mixed heterogeneous media, ascending layered heterogeneous medium and descending layered heterogeneous medium. The first medium is the mixture of the five homogeneous media in the same proportion. The second medium is layers of each of the five homogenous medium of the same thickness in ascending order of their porosity from the bottom. The third heterogeneous medium is the layers of each of the five homogenous medium of the same thickness in descending order of their porosity from the bottom. In the experiment, the porosity of the media were determined using volumetric method, while the volume fluxes were determined from vertical permeameter set up with constant head method. The transparent cylindrical tube of cross-sectional area $7.06 \times 10^{-4} m^2$ was used as a permeameter (Fig. 2). A continuous steady supply of water was fed through the sand samples of length L packed under gravity and at height h a hole was drilled, this enable the height h to be maintained. The volume of water discharge, Q through each sample for a period of 60 seconds was measured by measuring cylinder at different hydraulic gradient, i and the volume flux was determined from respective volume discharged using eqn (5). The hydraulic conductivity of each of the medium was obtained from Darcy's equation of the form.

$$q = k\left(\frac{h}{L} + 1\right) \quad \text{(Frick and Taylor, 1978)} \qquad (4)$$

where

q = volume flux (ms^{-1})
K = hydraulic conductivity (ms^{-1})
h = head constant (m); and
L = length of the sample in the permeameter (m)
$i = \left(\frac{h}{L} + 1\right)$ = hydraulic gradient

$$q = Q/A \quad \text{(Jacob and Arnold, 1990)} \qquad (5)$$

where

$A = \pi(d/2)^2$
d = diameter of the cylindrical tube used as the permeameter.
q = volume flux and
Q = volumetric flow rate

The slope of the graph of volume flux against hydraulic gradient indicates the hydraulic conductivity. The hydraulic conductivities obtained were later converted to permeabilities by using the relation.

$$k = \frac{\mu}{\rho g} K \quad = (1.02 \times 10^{-7} \text{ ms})\, K \quad \text{(Hubert, 1940)} \qquad (6)$$

where

k = permeability (m^2)
μ = viscosity of water = $0.001 Nsm^{-2}$
ρ = density of water = $1000 kgm^{-3}$
g = acceleration due to gravity = $9.81 ms^{-2}$
K = hydraulic conductivity (ms^{-1})
μ/ρ = kinematic viscosity of water = $0.000001 m^2s^{-1}$

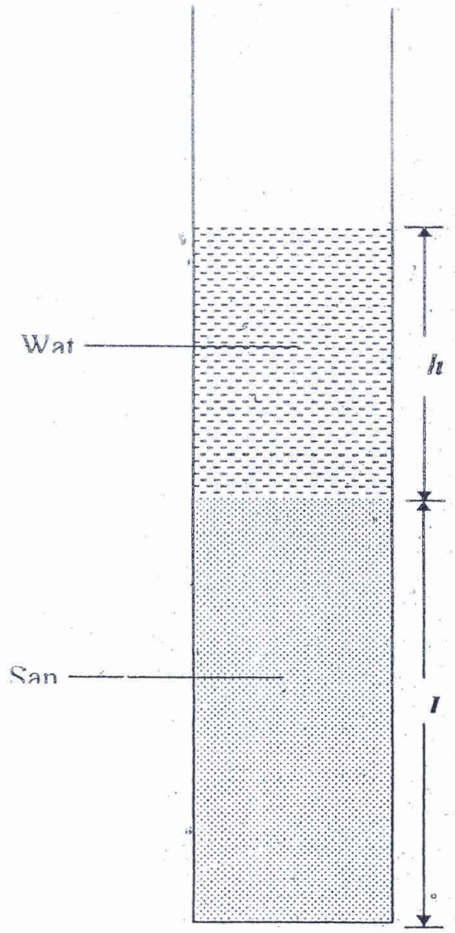

Fig. 2: Sand Model for vertical flow under head *h* (Jacob, 2001)

DISCUSSION

Table *1* is the values of volume flux for five different homogenous medium. They are tagged *A,B, C, D and E* with porosities *0.250, 0.374, 0.391, 0.510 and 0.620* respectively. The slopes of the plot of volume flux, q against hydraulic gradient i for each of the medium indicate the hydraulic conductivities for each medium (Fig. 3-7). The permeabilities were computed from hydraulic conductivities using Hubert King relation and are presented in table *2*. Table *3* shows the results of volume flux (specific discharge) for the three heterogeneous media and the slope of the line of volume flux q against hydraulic gradients i indicates the hydraulic conductivity for each medium (Fig. 8-10), and from which permeability was computed and presented in table *4*. The results show that the permeabilities of the three heterogeneous media are lower than the permabilities of all the homogeneous media except in sample *A*. Among the three heterogeneous media, the mixed heterogeneous medium has the highest permeability. This indicates that fluid flows faster in mixed heterogeneous medium than layered heterogeneous medium because the mixture of samples of different porosity provides a well-sorted (or non-uniform) grains distribution with large pore size interconnectivity, which enhanced high permeability. Also, it was observed that the least permeable sample or unit dominates in all the three types of heterogeneous media considered. This is true because the permeability of sample *A* which is 0.62×10^{-11} m^2, is relatively more closer to that of the three heterogeneous media than other homogeneous media (Table 4). Re-arrangement of grains distribution is one the factors that responsible for the reduction in fluid flow in descending layered heterogeneous media. This is true because the finest grains of the least permeable sample/unit at the topmost of the layers, migrate downward under the influence of water weight and there by fill in the pores in the subsequence more permeable and leads to reduction in permeability of the entire medium. Thus, the hydraulic connectivity of fluid through the medium becomes low. However, this could not occur easily in ascending layered heterogeneous medium because the grains at the top most are bigger in size relatively to the subsequence layer.

RESULTS AND DISCUSSION

Table 1: Experimental determined values of volume flux for homogeneous media at various hydraulic gradients

i	A $q \times 10^{-4}$ (m/s)	B $q \times 10^{-4}$ (m/s)	C $q \times 10^{-4}$ (m/s)	D $q \times 10^{-4}$ (m/s)	E $q \times 10^{-4}$ (m/s)
2.64	1.465	5.961	11.823	18.904	36.882
2.90	1.512	6.448	12.849	20.563	39.467
3.22	1.758	7.081	14.408	23.300	42.450
3.63	1.952	7.767	16.559	25.545	48.359
4.14	2.052	9.818	19.197	29.308	55.440
4.83	2.348	11.184	20.616	33.752	63.113
5.80	2.978	13.529	27.063	36.782	75.369
7.25	3.710	16.606	33.019	4.019	101.114
9.67	4.590	25.059	42.691	62.280	127.685
14.50	9.039	28.916	59.496	103.359	168.623

Table 2: Value of porosity, Hydraulic conductivity and Permeability for homogeneous medium

Samples	Porosity	Hydraulic conductivity (m/s)	Permeability (m^2)
A	0.250 ± 0.010	0.61 x 10^{-4}	0.62 x 10^{-11}
B	0.374 ± 0.003	2.10 x 10^{-4}	2.14 x 10^{-11}
C	0.391 ± 0.001	4.09 x 10^{-4}	4.17 x 10^{-11}
D	0.510 ± 0.001	6.41 x 10^{-4}	7.04 x 10^{-11}
E	0.620 ± 0.010	11.61 x 10^{-4}	11.90 x 10^{-11}

Table 3: Experimental determined volume flux for different heterogeneous medium at different hydraulic gradient

i	(MHT) volume flux $q \times 10^{-4}$ (m/s)	(ALHT) volume flux $q \times 10^{-4}$ (m/s)	(DLHT) volume flux $q \times 10^{-4}$ (m/s)
1.93	3.224	4.004	5.668
2.90	4.717	4.736	6.841
5.80	7.527	7.767	8.693

Note: ** *MHT*: Mixed heterogeneous medium
ALHT: Acceding order layered heterogeneous medium
DLHT: Descending order layered heterogeneous medium

Table 4: Hydraulic conductivities Permeabilities of heterogeneous media

Heterogeneous medium	Hydraulic conductivity $K \times 10^{-4}$ (m/s)	Permeability $k \times 10^{-11}$ (m/s)
MHT	1.08	1.10
ALHT	0.99	1.01
DLHT	0.75	0.77

Note $k = \dfrac{\mu}{\rho g} K = 1.02 \times 10^{-7} K$ (King Hubert Relation)

Where k = permeability (m^2)
K = hydraulic conductivity (ms^{-1})
μ = viscosity of water
ρ = density of water and
g = acceleration due to gravity = 9.81ms^{-2}
μ/ρ = kinematic viscosity of water = 0.000001m^{2}s^{-1}

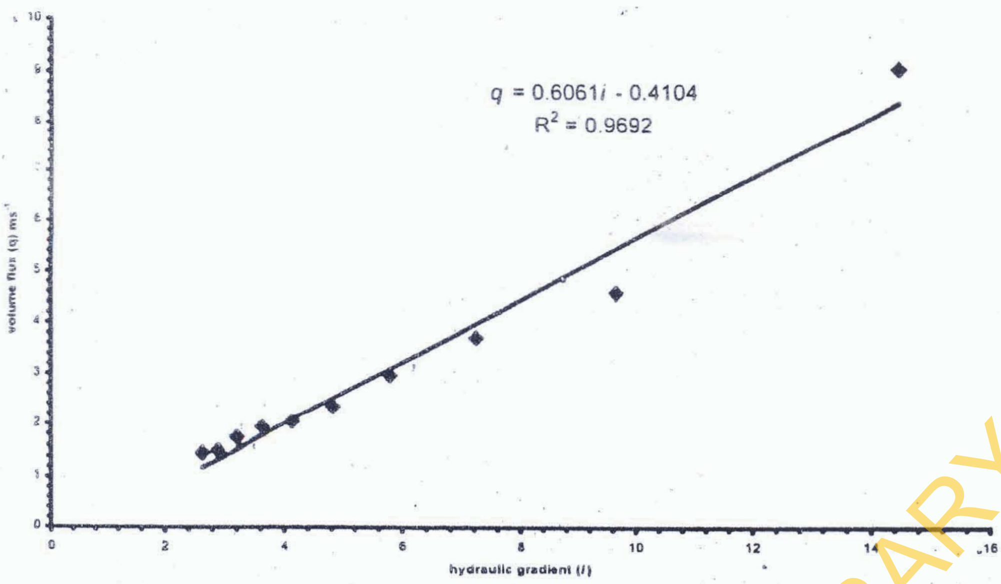

Fig 3: Plot of Volume flux against hydraulic gradient for sample A

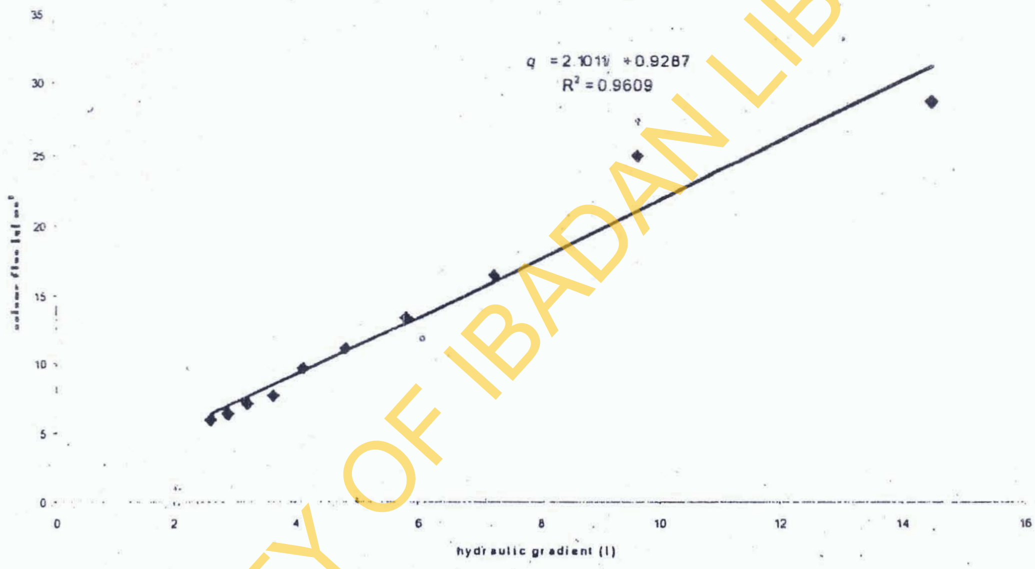

Fig 4: Plot of Volume flux against hydraulic gradient for sample B

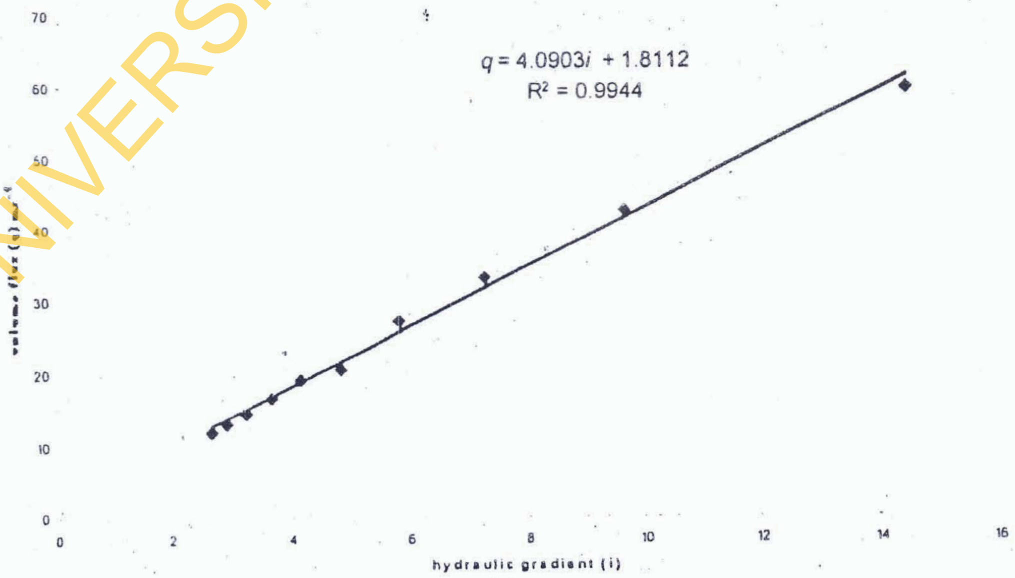

Fig 5: Plot of Volume flux against hydraulic gradient for sample C

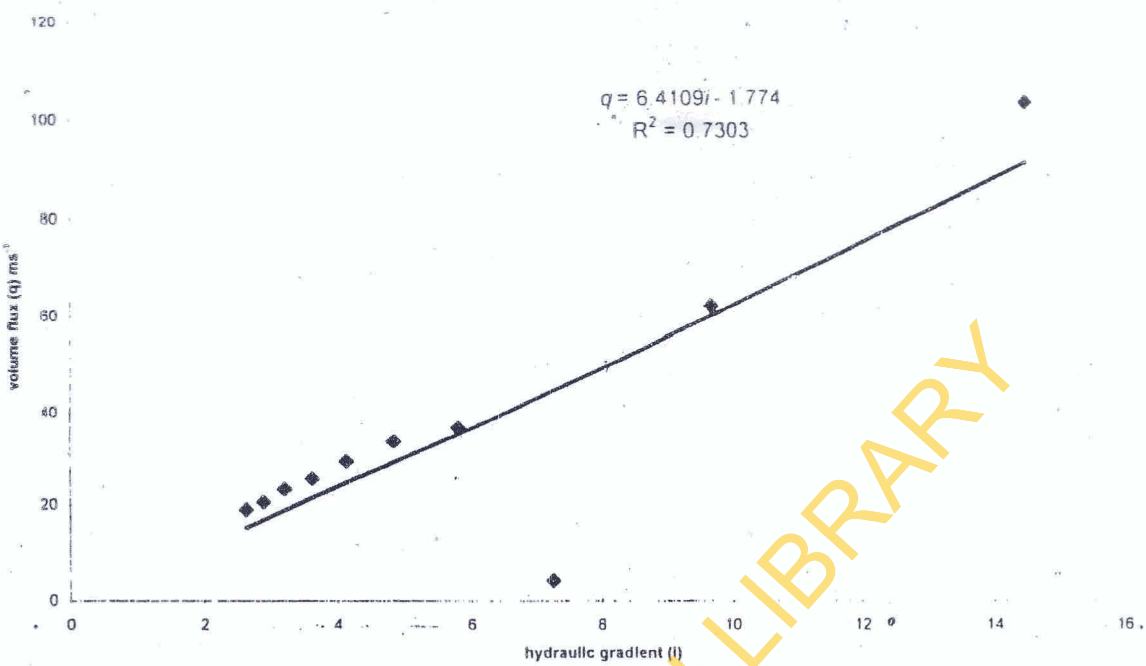

Fig 6: Plot of Volume flux against hydraulic gradient for sample D

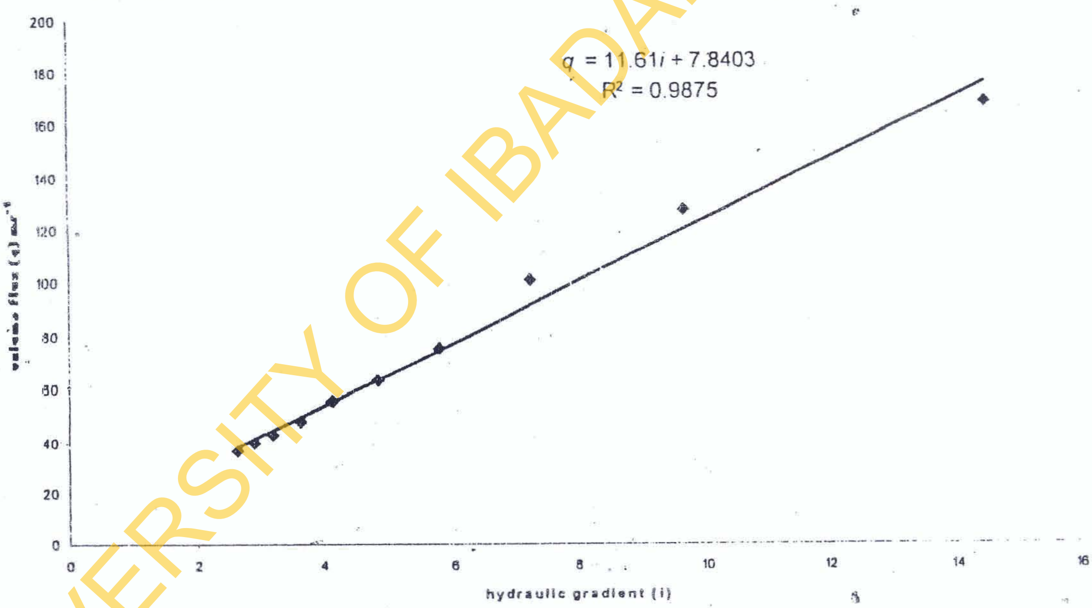

Fig 7: Plot of Volume flux against hydraulic gradient for sample E

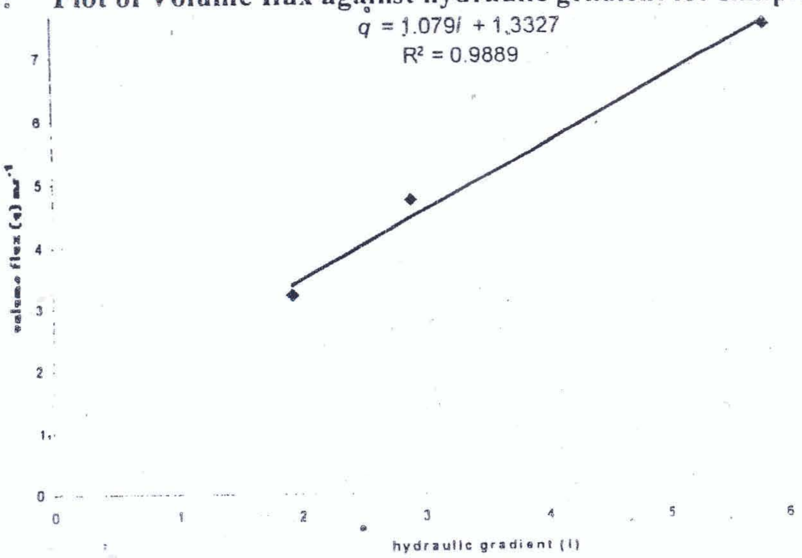

Fig 8: Plot of Volume flux against hydraulic gradient for MHT

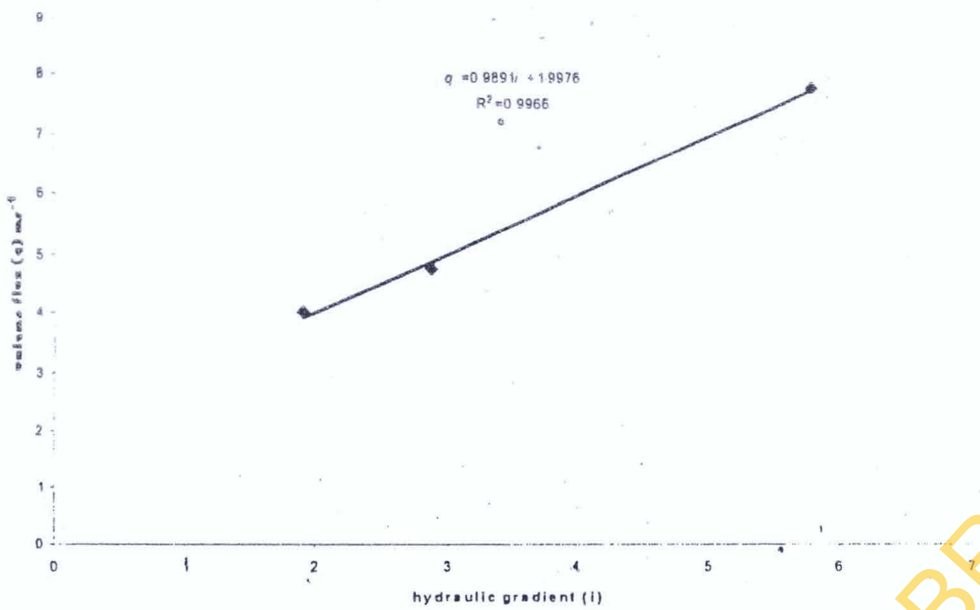

Fig 9: **Plot of Volume flux against hydraulic gradient for ALHT**

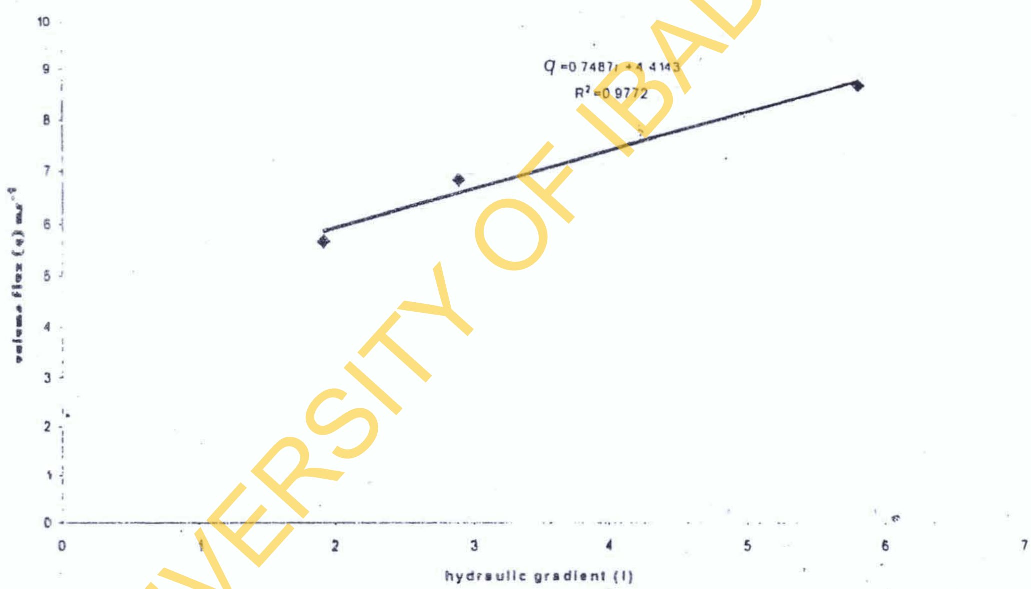

Fig 10: Plot of Volume flux against hydraulic gradient for DLHT

CONCLUSION

At the end of the study, it was found that: the least permeable medium dominates in the permeability of both bed layers and mixed heterogeneous media; under the same hydraulic gradient, the vertical fluid is faster in a mixed heterogeneous porous medium then layered heterogeneous porous media; and fluid flow faster in homogeneous porous media than in heterogeneous porous medium. Thus, in selecting a suitable material of lower permeability for seepage control, a medium of layered heterogeneous which is made up of sand samples arranged in descending order of porosity will serves better.

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