DETERMINATION OF DISCHARGE COEFFICIENT OF RECTANGULAR SIDE WEIRS

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ABSTRACT

The characteristics of flow over side weirs are taken into consideration to verify the discharge coefficient for subcritical flow conditions under the assumption of constant-specific energy. The main channel discharge, length of weir crest and sill height of the weir are treated as controlled variables. The discharge coefficient Cd is found to depend on the Upstream Froude No F_1 and also on the ratio of sill height to upstream flow depth S/Y_1. Simple linear regression analysis was conducted to establish the relationship of Cd with both F_1 and S/Y_1 taken separately while multiple regression analysis was conducted to establish the relationship of Cd with both F_1 and S/Y_1 taken together. Cd decreases with increase in F_1 while Cd increases with increase in S/Y_1 ratio when taken separately. When taken together, it was clearly evident that variation in Cd was largely due to S/Y_1 ratio while the effect of F_1 was relatively negligible.

INTRODUCTION

A side weir is an overflow weir set into the side of a channel and is used for water level control in canal systems, diverting excess water into relief channels during floods, as storm overflows from urban sewage systems and as head regulators of distributors. The ability to predict the flow diverted over side weirs, also known as lateral weirs, is also useful in the design of diversion structures and in flood alleviation works.

The attraction of weirs in use as a water-measuring device is their cheapness and the ease with which they may be made with simple workshop facilities from readily available sheet material. The flow over a side weir falls under the category of spatially varied flow. The concept of constant specific energy is often adopted for studying the flow characteristics of these weirs (Singh et al. 1994). However, their discharge coefficient is invariably considered as a function of the upstream Froude No only, though dimensional analysis also suggests it to be a function of the ratio of sill height to the upstream flow depth (Singh et al. 1994).

The objective of this study is to experimentally investigate and verify the effect of the upstream Froude No and sill height on the discharge coefficient of rectangular side weirs under subcritical flow conditions and the assumption of constant – specific energy in a prismatic rectangular main channel.

Prior to 1978, most of the studies on side flows in open channels were focused on the empirical derivation of discharge formulas (Cheong, 1991). Singh et al. (1994) related the discharge coefficient of a broad-crested rectangular side weir to the main channel upstream Froude No and S/Y_1 ratio and they came to a conclusion that by increasing the depth of flow at the upstream section, coefficient of discharge decreases. Multiple regression analysis were conducted by these researchers to establish the relationship of Cd with both F_1 and S/Y_1 taken together thus:

$$Cd = 0.99 - 1.26\,F_1 \quad (1)$$

$$Cd = 0.24 + 0.54\,(S/Y_1) \quad (2)$$

$$Cd = 0.33 - 0.18\,F_1 + 0.49\,(S/Y_1) \quad (3)$$

Furthermore, discussion by Jalili and Borghei (1996) on the discharge coefficient of rectangular side weirs shows that by increasing the flow depth at upstream section, coefficient of discharge increases – this is a contradiction from earlier experimental results. Therefore, they concluded that the relation between Cd and S/Y_1 is given thus:

$$Cd = 0.50 - 0.05\,(S/Y_1) - 0.45\,(S/Y_1)^{3\,3}. \qquad (4)$$

$$Cd = 0.71 - 0.41\,F_1 - 0.22\,(S/Y_1) \qquad (5)$$

DESIGN AND EXPERIMENTATION

In general, the design of the water channel, take into consideration the following parameters:

(i) Availability of construction materials
(ii) Durability
(iii) Capability
(iv) Ease of Construction;
(v) Cost of production
(vi) Channel Slope

The test facility consists of a main channel, 6.0m long, 0.25m wide, and 0.35m deep. The branch channel, set at 90° to the main channel was 2.0m long, 0.30m wide and 0.35m deep. Thin-plate weirs, baffles and V-notches were fabricated from mild steel plates and plasticine used as a modeling paste are employed in ensuring water tight edges of these components in place during the experimental test period. The experimental set up is shown in plate 1.

Mathematical Design Analysis:
From constant-specific energy considerations, Singh *et al*, (1994) gave the following equation for a side weir located in a rectangular channel.

$$Cd = {}^{3}/_{2}\ {}^{B}/_{L}\,(\phi_2 - \phi_1) \qquad (6)$$

where Cd = discharge coefficient; B = Width of the main channel; L = Length of the weir crest; ϕ = Varied flow function; and suffixes 1 and 2 represent the upstream and downstream sections of the weir crest. The varied flow function can be expressed by the following equation.

$$\phi = \frac{2E-3S}{E-S}\sqrt{\frac{E-Y}{Y-S}} - Sin^{-1}\sqrt{\frac{E-Y}{Y-S}} \qquad (7)$$

Where E=-Specific energy; S = Weir sill height; and Y = Depth of flow.

Knowing the flow conditions at sections 1 and 2, and the geometrical configuration of the weir, the discharge coefficient Cd can be obtained. From dimensional analysis

$$Cd = f(F_1,\ S/Y_1,\ L/Y_1) \qquad (8)$$

For the constant – specific energy concept, Cd is assumed to be independent of L, hence (8) reduces to

$$Cd = f(F_1,\ S/Y_1) \qquad (9)$$

The specific energy is given as

$$E = Y + \frac{Q^2}{2gA^2} \qquad (10)$$

where A is cross-sectional area of flow in the rectangular channel given as

$$A = By \qquad (11)$$

Substituting (11) into (10) gives

$$E = Y + \frac{Q^2}{2g(By)^2} \qquad (12)$$

where Y = water depth; Q = Discharge, B = Width of main channel, y = Flow depth in the main channel; All measured at point x which is 3.5m upstream of the channel and
g = Acceleration due to gravity.

Experimentation: The range of variables studied is given in table 1. As part of the test procedure, water surface profiles in the vicinity of the side weir were recorded and head measuring locations were shown explicitly in a definition sketch as shown in figure 1.

RESULTS AND DISCUSSION

Water Surface Profile; Specific Energy; Upstream and Downstream Varied Flow function.

The water surface profile exhibited a rising trend along the length of the weir crest in all experimental runs, thus satisfying the constant specific energy assumption for sub-critical flow conditions in the main channel (Singh *et al.* 1994). Increase in sill height resulted in increase in specific energy and downstream varied flow function while there was a corresponding decrease in upstream varied flow function.

Cd; Ratio S/Y_1 and F_1.

From the computed values of Q_1; Q_2 and measured values of Y_1, Y_2, the discharge coefficient was calculated using equations (6) and (7). Cd values ranged from 0.23 to 1.21; however, the majority of the values were between 0.27 and 0.32. Similarly, ratio S/Y_1 ranged from 0.612 to 0.927; however, the majority of the S/Y_1, values were between 0.656 and 0.915. Also, F_1 values ranged from 0.11 to 1.53, however, the majority of the values were between 0.11 and 0.77.

Variation of Cd with F_1

Cd values were plotted against F_1 as shown in figure 2. From the figure it is evident that Cd decreases with increase in F_1 The regression line was given by the equation below.

$$Cd = 0.71 - 0.32\ F_1 \qquad (13)$$

Subramanya and Awasthy (1972), Ranga Raju et al. (1979) and Singh *et al.* (1994) reported that by increasing the Froude number, the coefficient of discharge decreases. The trend of this relationship is similar to the findings in this study.

Variation of Cd with S/Y_1

The values of Cd were plotted against S/Y_1 as shown in figure 3. The figure reveals that Cd increases with increase in S/Y_1 ratio. Hence, the regression line (or least square- fit relationship) was given by:

$$Cd = 2.14\ S/Y_1 - 1.13 \qquad (14)$$

The relationship of Cd with S/Y_1 ratio contradict the earlier findings by Subramanya and Awasthy (1972) that the sill height (S) has no effect on Cd. However, the trend of the relationship is similar to those reported by Singh et al. (1994) which states that by increasing the depth of flow at the upstream section, coefficient of discharge decreases.

Variation of Cd with F_1 and S/Y_1

Since Cd has been shown to depend both on F_1 and S/Y_1 taken together, the first order polynomial relating Cd with F_1 and S/Y_1 is of the following form:

$$Cd=0.8326+1.7536(S/Y_1)+0.0088F_1 \qquad (15)$$

It was clearly evident that variation in Cd was largely due to S/Y_1 while the effects of F_1 was relatively negligible. Hence, the result obtained from simple linear regression analysis was largely supported by the results obtained with multiple regression analysis.

The mean of Cd estimated by equations (6) and (7) is 0.56 with standard deviation error of the mean of 0.07, while the mean of Cd estimated by equation (15) is also 0.56 with standard deviation error of the mean of 0.05. Hence, student's t-distribution test shows that there is no significant difference between Cd values at 0.01 level of significance. Equation (15) therefore provides an easy means of estimating Cd.

CONCLUSIONS

- Cd was computed on the assumption of estimating the downstream depth on the basis of constant specific-energy. Cd evaluated using depth measurements of downstream section without recourse to constant specific-energy assumption gave standard error of 0.9239 (that is, S_n = 92.39%) from the analysis of the accuracy of the mean of flow depth measurement values of downstream section.
- A first-order polynomial from multiple regression analysis describing the relationship of Cd with both F_1 and S/Y_1 ratio taken together was developed and the result obtained validate the proposed expression for Cd by Singh *et al*, (1994).
- Student's t-distribution test used shows that there is no significant difference between Cd values from experimental data and from multiple regression analysis at 0.01 level of significance. Hence, equation (15) provides an easy means of estimating Cd of rectangular side wiers.

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APPENDIX : Notation

The following symbols are used:

A = Cross-sectional area of flow
B = Main channel width
Cd = Discharge coefficient
E = Specific energy
$f()$ = function
F_1 = Upstream Froude Number
g = Acceleration due to gravity
L = Length of Side Weir Crest
Q = Channel discharge
S = Weir Sill height
Y = Channel flow depth
ϕ = Varied flow function

Subscripts

1 = Upstream section; and
2 = Downstream section

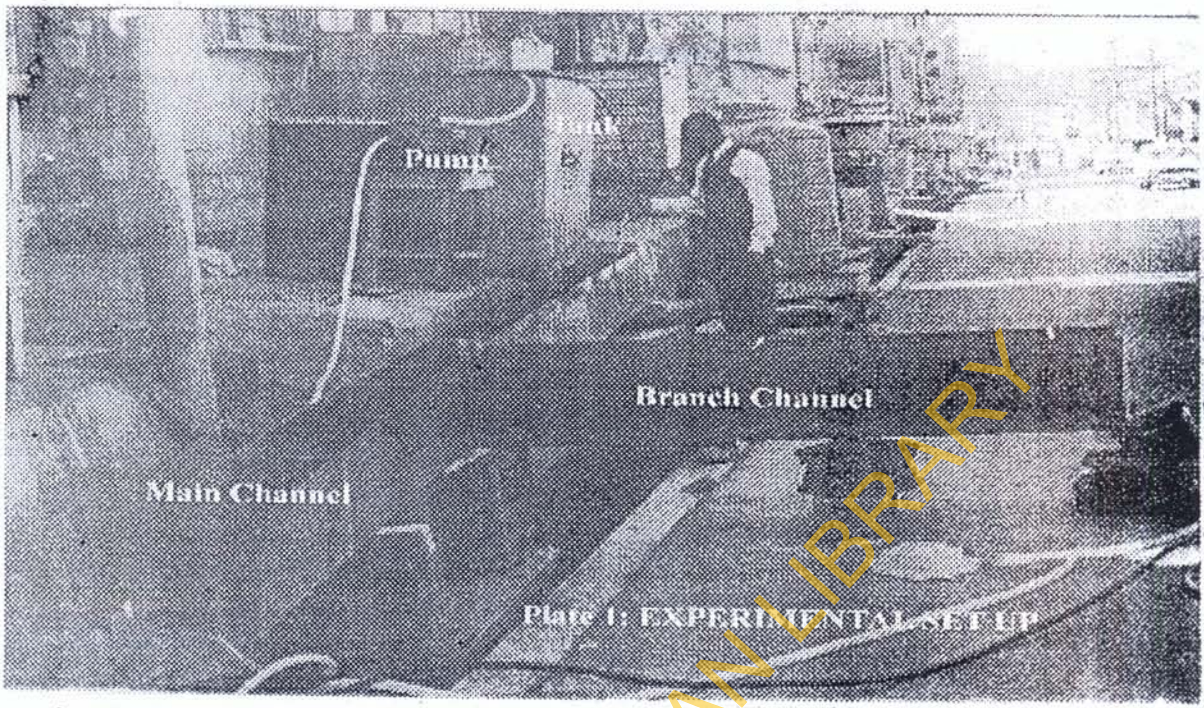

Plate 1: EXPERIMENTAL SET UP

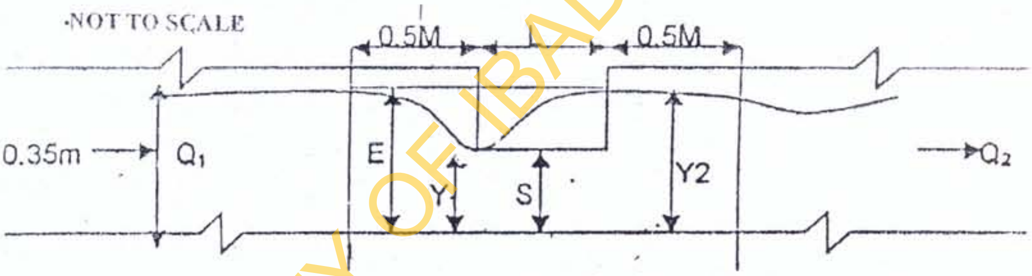

Fig. 1: Sketch of Water Surface Profile

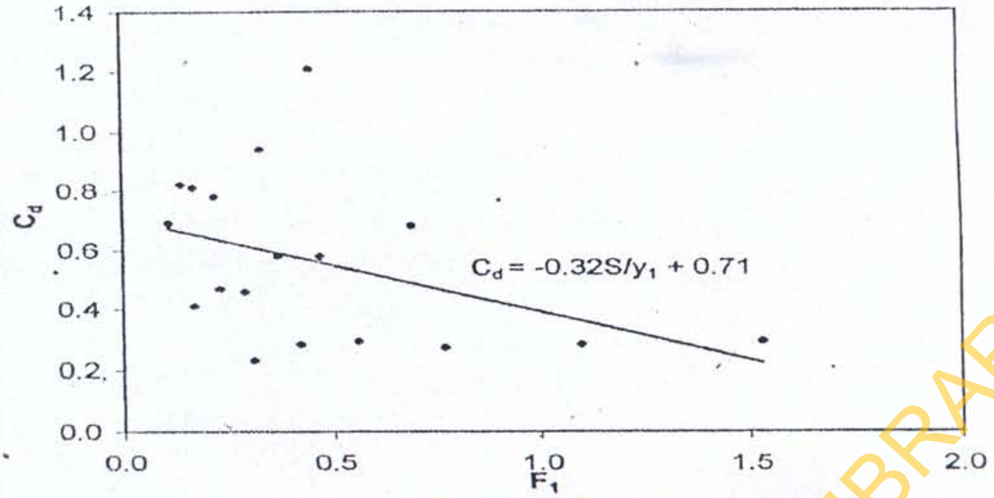

Figure 2: Relationship between C_d and F_1

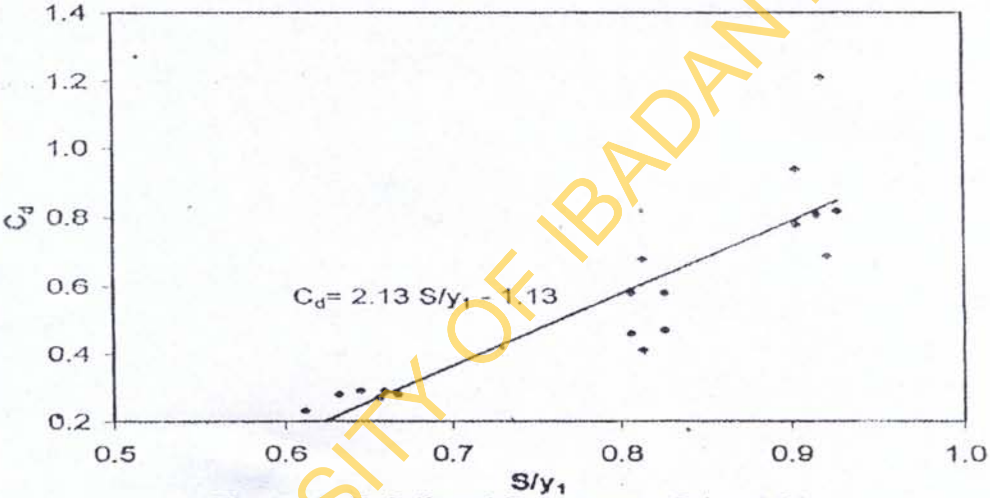

Figure 3: Relationship between C_d and S/y_1.