

APPLICATION OF MATHEMATICAL MODEL TO THE PRODUCTION CAPACITY OF A YAM FLOUR PRODUCING COMPANY

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Abstract : Yam flour is usually required for the preparation of pounded yam which is a daily nutritional food requirement for most Africans. It facilitates the preparation of pounded yam for both domestic and commercial consumption. The production process and the basic operations involved in the production of pounded yam flour have been investigated in order to estimate the production capacity of the company studied. A time study model was developed and applied to analyze the various basic operations involved in the production of pounded yam flour. These basic operations include; the selection of yam, weighing of yam, washing of yam, peeling of yam, slicing of yam with a slicing machine, parboiling of sliced yam with a parboiler, drying of parboiled yam with a dryer, milling of dried yam into yam flour with a hammer mill, packaging of the yam flour and sealing of the packages with a sealing machine.

In this paper therefore, a mathematical model was developed with the application of different techniques of differential calculus to the component elements of the production process of yam flour. The study reveals that the time it takes to produce some kilogram unit of pounded yam flour is directly proportional to the number of production stages involved and the time spent at each of these production stages. This time is being represented by some structural equations which are characteristics of the system being studied.

1.0 INTRODUCTION

Time study which is a branch of work study incorporates a range of concerns, including its definition and management. It involves a technique of work measurement which is aimed at assessing human effectiveness thereby making possible improved planning and control manning and as basis for sound incentive schemes, for higher productivity. It is a scientific technique which is used by most manufacturing organizations to aid dispute settlement between the employees' association of the company and management regarding issues of productivity (Oke, 2006). Time study is a technique of work measurement designed to establish the time for a qualified worker to carry out a specified job at a defined level of performance and research on time study incorporates a range of concerns, including its definition and management (Edo et al., 2001; Worrall and Smith, 1985; Watson, 1988; Aft, 2000).

Keywords : Mathematical Model, Yam Flour, Time Study Model, Production Capacity, Production Stages

The word yam comes from Portuguese name or Spanish name, which both ultimately derive from the Wolof word nyam, meaning "to sample" or "taste", in other African languages it can also mean "to eat" e.g. yamyam and nyama in Hausa (Mignouna, et al., 2003). Also yam is the common name for some species in the genus and they are perennial herbaceous vines cultivated for the consumption of their starchy tubers in Africa, Asia, Latin America and Oceania. They are used in a fashion similar to potatoes and sweet potatoes, (Brand Miller, et al., 2003). Yam products generally have a lower glycemic index than potato products (Kay, 1987), which means that they will provide a more sustained form of energy, and give better protection against obesity and diabetes (Walsh, 2003).

Yam which is one of the oldest known recipes to man is a tuber crop which belongs to the class of carbohydrate and has been a part of the African meal for centuries. Its starchy nature allows yam to form a bond when it is been beaten in a mortar which is then consumed as meal with a good choice of soup. Pounded yam is a staple food, which is consumed by almost every tribe in Nigeria. The indigenous process of production is very laborious. It requires physical pounding by two or more men or women, depending on the quantity, in mortars with pistons. The world production of yam was estimated at 28.1 million tonnes in 1993. Out of this production, 96% came from West Africa, the main producers being Nigeria with 71% of world production; Cote d'Ivoire 8.1%; Benin 4.3% and Ghana 3.5%. In the humid tropical countries of West Africa, yams are one of the most highly regarded food products and are closely integrated into the social, cultural, economic and religious aspects of life. Traditional ceremonies still accompany yam production, indicating the high status given to the plant. (food Information Net, 2008).

This paper is an attempt to present the methodology used in the production of pounded yam flour by the company which involves the basic operations of yam selection, weighing of yam, washing of yam, peeling of yam, slicing of yam with a slicing machine, parboiling of sliced yam with a parboiler, drying of parboiled yam with a dryer, milling of dried yam into yam flour with a hammer mill, packaging of the yam flour and sealing of the packages with a sealing machine. The time study technique was used to develop a model which was used to estimate the production capacity of each of these operational stages and the total productivity of the company.

2.0 PRODUCTION PROCESS OF POUNDED YAM FLOUR

The production process of pounded yam flour consists of some basic operations as follows: Yam selection and weighing which involves the weighing of the selected yam and the thorough washing of the selected yam. Next is the peeling and slicing of the yam in which the washed tubers are carefully peeled manually while the peeled tubers are mechanically sliced to desirable thickness with a slicing machine made of stainless steel blades. The yam slices are parboiled for some minutes, depending on the thickness of the slices while the parboiled yam slices are dried in a dryer at a specified temperature. The dried yam slices are milled directly into flour of uniform particle size distribution, while the yam flour is packed into airtight, moisture-proof packaging materials. The basic production stages are shown in Figure-1. These basic operations include; the selection and weighing of yam, washing of yam, peeling of yam, slicing of yam with a slicing machine, parboiling of sliced yam, drying of parboiled yam, milling of dried yam into yam flour, packaging of the yam flour and sealing of the packages with a sealing machine.

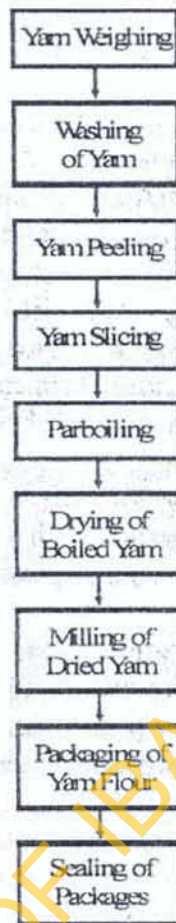


Fig-1 : Flow Diagram for the Production Stages of Pounded Yam Flour

3.0 THE MATHEMATICAL MODEL

The basic activities in yam flour production were studied in order to be able to develop mathematical model for the problem. The first mathematical expression for the time study model framework is as follows :

$$t = \sum_{i=1}^n t_i \quad (1)$$

where (t) represents the total time used in producing a unit of product. The variable (i) represents the various workstations of interests (i.e. selection and weighing of yam, washing of yam, peeling of yam, slicing of yam with a slicing machine, parboiling of sliced yam, drying of parboiled yam, milling of dried yam into yam flour, packaging of the yam flour and sealing of the packages with a sealing machine) With close observation of the various workstations, there are variations in the rate of working for both the individuals at the workstations and the machines doing the actual operation. Therefore, we introduce the rate of working for both the machines at the various workstations and the workers as differentials that are expressed mathematically. For instance, if machine i is represented as m_i , where m_i may be m_1 for the machine that does the work such as selection and weighing yam station, m_2 , is the machine that does the

work at washing of yam station, while m_3 is the machine that does the work at peeling of yam station, etc.).

If the time taken by the 'in-process' product is time t , then mathematical expression becomes;

$$\frac{dm_i}{dt} = \Delta m_i,$$

$$\text{where } \Delta = \frac{d}{dt}$$

Also, if (w_i) represents the human worker at workstation (i), and this worker works for a period of time t units, then we can express the rate of working of this worker as :

$$\frac{dw_i}{dt} = \Delta w_i$$

$$\text{where } \Delta = \frac{d}{dt}$$

Since in time study activities a provision of allowance is always very necessary, we now introduce a parameter ' T_a ' into the model.

Therefore, the general mathematical expression for the production time $g(t)_i$ at each workstation is given as :

$$T_i = \frac{\Delta}{m_i} \times \frac{\Delta}{w_i} \times f(y_i) + T_a \quad (2)$$

where $g(t)_i$ is a normalizing function which converts the expression into time units. Substituting Equation 2 into Equation 1 gives the following equation.

$$t = \sum_i^n \left(\frac{1}{\Delta m_i} \times \frac{1}{\Delta w_i} \times f(y_i) + t_a \right) \quad (3)$$

$$= \sum_i^n \left(\frac{1}{\Delta m_i} \times \frac{1}{\Delta w_i} \times f(y_i) + \sum_i^n t_a \right) \quad (4)$$

$$\text{but } = \sum_i^n t_a = nt_a$$

$$\therefore t = \sum_i^n \left(\frac{1}{\Delta m_i} \times \frac{1}{\Delta w_i} \times f(y_i) + nt_a \right) \quad (5)$$

It is assumed that the rate at which machines are producing and the working rate of workers is constant. Thus Equation 5 becomes;

$$t = \left(\frac{1}{\Delta m_i} \times \frac{1}{\Delta w_i} \right) \sum_i^n f(y_i) + nt_a$$

We generalize the model by taking $f(y_i)$ as $f(y)$, $\frac{1}{\Delta m_i}$ as $\frac{1}{\Delta m}$ and $\frac{1}{\Delta w_i}$ as $\frac{1}{\Delta w}$

$$\text{Thus } t = \frac{1}{\Delta m} \times \frac{1}{\Delta w} \int_1^n f(y) dy + nt_a \quad (6)$$

Assuming that the total number of products produced is denoted by symbol (X), while T is the total time spent for all the products, Equation 6 above becomes,

$$T = Xt = X \left(\frac{1}{\Delta m} \times \frac{1}{\Delta w} \int_1^n f(y) dy + nt_a \right) \quad (7)$$

Equation 7 is the general formula for the total time spent in producing x products.

3.1 Raw Materials and Electricity Supply.

Consider the issue of unavailability of raw materials which are the fresh palm fruit bunches and irregular electricity supply and assuming that $f(y)$ is a function of these two parameters of indices such that we have $f(y)$ and $f(y, z)$. Therefore Equation 7 can now be expressed as follows :

$$T = Xt = X \left(\frac{1}{\Delta m} \times \frac{1}{\Delta w} \int_1^n f(y, z) dydz + nt_a \right) \quad (8)$$

This equation gives the real general formula for the total time spent in producing y products.

4.0 APPLICATION OF THE TIME STUDY MODEL AND DISCUSSION.

This study which is a case study of a pounded yam flour processing company, is a real life situation of a production company in Edo state of Nigeria. The company specializes in producing production of pounded flour with 18 workers. The company has different types of machines and other facilities for its production. The company operates a nine hours daily production cycle. However, during festive periods such as Christmas, a large number of customers usually patronize the company's products resulting in huge spike sales. This sometimes, usually leads to increase in the daily working hours of operation.

This general formula for the total time spent in producing y products is given in the equation below (Oke, 2006).

$$T = Xt = X \left(\frac{1}{\Delta m} \times \frac{1}{\Delta w} \int_1^n f(y) dy + nt_a \right) \quad (9)$$

where T is the total time spent for all the products, X = number of products produced
n = number of workstations and t_a = the time allowance

Assuming that the electricity supply index (y) obeys a linear function such as $2y + 5$, then the expression is now $f(y) = 2y + 5$. From the above equations, we know that (n) is the number of workstations while (t_a) is the time allowance, from the actual production observation, the mathematical model that fit the time problem in terms of number of machines is :

$$t = my^3 + m^2y^2 + y \quad (10)$$

Differentiating Equation 10 gives :

$$\frac{dt}{dm} = y^3 + 2my^2 \quad (11)$$

Also, the mathematical expression that represents time with respect to the number of workers is :

$$t = wy^3 + w^2y^2 + y \quad (12)$$

Differentiating above gives :

$$\frac{dt}{dw} = y^3 + 2wy^2 \quad (13)$$

Note that (n) has been stated earlier as the number of workstations, and (t_a) is the time allowance.

If 87 gm of yam flour are produced by the company for 0.056 second per unit product, then we have allowance.

$$t_n = 87 \times 0.056 \text{ seconds.} = 4.872 \text{ seconds.}$$

Therefore $t_n = 4.872$ seconds,

$$\text{while } nt_n = 7 \times 4.872 = 34.104 \text{ seconds.}$$

$$t = t_j = \frac{1}{\Delta m} \times \frac{1}{\Delta w} \int_1^n f(y) dy + nt_n$$

But $\frac{1}{\Delta m} = x^3 + 2mx^2$ and $\frac{1}{\Delta w} = x^3 + 2wx^2$. There are 7 workstations for the yam flour production processes, hence $n = 7$. From equation 6, we can now estimate the values of t .

$$\text{We know that } t = t_j = \frac{1}{\Delta m} \times \frac{1}{\Delta w} \int_1^n f(y) dy + nt_n \text{ and the values of } \frac{1}{\Delta m} \text{ as } x^3 + 2mx^2$$

$$\text{and } \frac{1}{\Delta w} \text{ as } x^3 + 2wx^2, n = 7 \text{ and } t_n = 4.872 \text{ seconds.}$$

The average period electricity fails in a day is 65 minutes, while the average daily working time is 9 hours.

Note that x is the ratio of the period when electricity fails in a day to that of the working hours for that same day.

$$\text{Thus, } x = \frac{65 \text{ min utes}}{9 \times 60 \text{ min utes}} = \frac{65}{540} = 0.1204.$$

This gives an index value of 0.1204.

Note that the number of machines $m = 5$, number of workers $w = 18$. Then since $f(y) = 2y + 5$, we now evaluate the function by substituting into equation 6 as follows;

$$t = \frac{1}{\Delta m} \times \frac{1}{\Delta w} \int_1^n f(2y + 5) dy + nt_n$$

$$\text{so } t = \frac{1}{\Delta m} \times \frac{1}{\Delta w} (y^2 + 5y + c) + nt_n$$

Note that at the start of production process, all the factors are zero since no product has been produced. This gives the production constant c to be zero.

$$\therefore t = \frac{1}{\Delta m} \times \frac{1}{\Delta w} (y^2 + 5y) + nt_n \quad (14)$$

Now substituting the required values into the equation gives :

$$t = t_j = (y^3 + 2my^2)(y^3 + 2wy^2)(y^2 + 5y) + nt_n \quad (15)$$

$$\therefore t = (0.1204^3 + 2 \times 5 \times 0.1204^2)(0.1204^3 + 2 \times 18 \times 0.1204^2)(0.1204^2 + 5 \times 0.1204) + (7 \times 4.872) \text{ seconds}$$

$$\therefore t = (0.146706937)(0.523607097)(0.61649616) + 34.104 = 34.151357 \text{ seconds.}$$

$$\therefore t = 34.151357 \text{ seconds.}$$

$$\text{or } t = 34.2 \text{ seconds.}$$

Note that $t_1 = 0.075$ second per unit product, therefore the total products produced in 34.2 seconds;

$$= \frac{34.2 \text{ sec onds}}{0.075 \text{ sec ond per unit product}} = \frac{34.2}{0.075} \text{ unit product}$$

= 456 units of product in 34.2 seconds.

Therefore the total units of product produce in average daily working period of 9 hours

$$\text{is } = \frac{456 \times 9 \times 3600 \text{ sec onds}}{34.2 \text{ sec onds}} \text{ grams of yam flour} = 432,000 \text{ grams of yam flour.}$$

$$= 432 \text{ kilograms of yam flour.}$$

That is 432 kilogram units of yam flour would be produced in an average daily working period of 9 hours.

In conclusion, we have therefore be able to apply a time study mathematical model in computing the time required for operational activities in the production processes for yam flour which is used for making pounded yam and it is seen that 432 kilogram units of yam flour could be produced in nine hours.

5.0 CONCLUSION

The production of yam flour by the company studied has been thoroughly examined. The nine basic operational activities were thoroughly studied and these operational activities include; the selection of yam, weighing of yam, washing of yam, peeling of yam, slicing of yam with a slicing machine, parboiling of sliced yam, drying of parboiled yam, milling of dried yam into yam flour, packaging of the yam flour and sealing of the packages with a sealing machine.

Our studied reviewed that the setting of standards for achieving production targets for the yam flour company is very important and one of the techniques for achieving this aim is the application of a mathematically developed time study models in the monitoring and control of employees on the company production floor. In this paper therefore, a time study mathematical model was developed with the application of different techniques of differential calculus to the component elements of the production process of yam flour in other to analyze the various production activities of the company.

REFERENCES

- Aft, L.S. (2000) : Work Measurement and Methods Improvement., John Wiley and Sons. ISBN: 0471370894.
- Brand-Miller, J., Burani, J., Foster-Powell, K.. (2003) : The New Glucose Revolution - Pocket Guide to the Top 100 Low GI Foods. ISBN 1-56924-500-2.
- Edo, M., Evans, T.D., and Viengkham, O.V. (2001) : "Study on Structure and Time of Assembly Motion from a Viewpoint of the Motion Velocity". Bulletin of the College of Engineering, Forest Ecology and Management. New York.
- Food Information Network, (2008) : Wageningen University, The Netherlands.
- Kay, D.E. (1987) : Root Crops. Tropical Development and Research Institute : London
- Mignouna, H.D., Abang, M.M. and Asiedu, R. (2003) : Harnessing Modern Biotechnology for Tropical Tuber Crop Improvement: Yam (*Dioscorea spp.*) Molecular Breeding.
- Oke, S.A. (2006).: "A Case Study Application of Time Study Model in an Aluminum Company". Pacific Journal of Science and Technology. 7(2): 153-162.
- Walsh, S. (2003) : Plant Based Nutrition and Health. ISBN 0-907337-26-0. New York.. 47pp.
- Watson, I.J. (1988) : Electronic Time Study. Colliery Guardian. 236(5): 144.
- Worrall, B.M. and Smith, M.D. (1985) : Application of Computerized Time Study to Establish Time Standards. Springer-Verlag: Berlin, pp. 745-750.