A Biobjective Production Planning Model and Application of Three LP Procedures

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ABSTRACT

A biobjective model is proposed for production planning in a multi-stage, multi-facility production system. The decision situation considered was a case where the Decision Maker (DM) wants to determine the quantities of materials to be fed into each production facility at each stage of production that gives maximal realization of his objectives. A numerical example is solved using three Linear Programming procedures. The methods used are: Compromise Constraint Biobjective LP (CCBLP), Linear Combination of the Objective Functions (LCOF) and Goal Programming (GP).

The behaviour of the CCBLP model shows that it is superior to LCOF and non-preemptive GP in terms of its sensitivity to relaxations in the objectives. It also supports the result of an earlier research that the CCBLP gives the real compromise solution.

Key words: Biojective, Production Planning, Linear Programming, Goal Programming, Linear Combination of objective functions, Compromise Constraint Biojective LP.

1.0 INTRODUCTION

Some production planning problems, like many real world cases, may involve conflicting multiple objectives. Improvement in one objective may turn out to be detrimental to another if all objectives are not simultaneously considered.

A very common way of handling multiple objectives in LP is by Linear combination of the objective functions (LCOF) into one and then solving the single objective problem [1,3,5]. In some cases it may be rather difficult to include all conflicting objectives in a single criterion. A more flexible and practical methodology is goal programming (GP) developed by Charnes and others [6] for solving planning problems. This allows for simultaneous combination of multiple conflicting objectives.

A methodology known as Compromise Constraint Biobjective LP (CCBLP) for evaluating biobjective decision situations was first reported by Adulbhan et al [3,4]. However, the literature is sparse on wide scale application of his formulation and solution strategy. One possible reason may be that his approach has not been adequately tested in real life situation.

The biobjective production planning problem is the subject of this paper. In particular the Adulbhan's formulation will be re-examined and compared to the Linear Combination of Objective Functions (LCOF) and the GP formulation [1,2, 3, 5, 9, 10]. A numerical example is next solved to determine their relative advantages. First, the detailed description of the proposed model is presented.

2.0 MODEL FORMULATION

Consider a multi-stage, single product production planning situation in a process industry Given the final product material mix, daily production and installed plant capacity, the problem is that of determining the quantity of each raw and intermediate materials to be fed into each facility per stage such that the following criteria are satisfied: minimization of total production cost and maximization of capacity utilization of production facilities [4].

The kind of production system considered in this study is schematically depicted in fig. 1 with the following assumptions: deterministic operating environment and constant unit production cost (no economies or diseconomies of scale).

2.1 MODEL CONSTRUCTION

Three essential sets of constraints are considered in addition to the non-negativity constraint. More constraints may be added depending on the actual situation.

(i) Capacity Constraints:
The sum of the quantities of materials fed into a facility must not exceed the capacity of the facility

$$\sum_{i=1}^{I} x_{ijk} + y_{j,k} \leq d_{jk};\ j = 1, 2, \ldots, J_k;\ k = 1, 2, \ldots, K;\ y_{j,1} = 0\ \forall\ j \qquad (1)$$

x_{ijk}: Quantities of the ith process material fed into the jth facility of the kth stage of production.

d_{jk}: Available capacity of the jth facility of the kth stage of production.

y_{jk}: Quantity of intermediate product fed into the jth facility of the kth stage of production.

(ii) Material Mix Constraint:
Material proportion constraints depends on the actual production process. In this study we assume that the proportion of the quantity of each material fed into each

facility for each stage is measured as a ratio of a base material quantity (x_{rjk}) for that stage.

$$\frac{x_{ijk}}{x_{rjk}} = \gamma_{ijk};\ i = 1, 2, \ldots, I;\ j = 1, 2, \ldots, J_k;\ k = 1, 2, \ldots, K \qquad (2)$$

x_{rjk}: Base material quantity by which the proportion of the materials are determined for the jth facility at the kth stage of production.

γ_{ijk}: Proportion of the ith material fed into the jth facility of the kth stage of production.

The above equation may be simplified as below:

$$x_{ijk} - x_{rjk}\gamma_{ijk} = 0 \qquad (3)$$

(iii) Material Balance Constraint:
Figure 2 gives the process flow diagram showing the quantities of material fed into each facility. The junctions are introduced for convenience and to help clarify the process flow diagram. The sum of the quantities of materials entering a junction is equal to the quantities of material leaving the junction.

$$\sum_{j=1}^{J} y_{j,k-1} + \sum_{j=1}^{J_{k-1}}\sum_{i=1}^{I} x_{ijk-1} = \sum_{j=1}^{J_k} y_{jk};\ k = 1, 2, \ldots, k;\ y_{j,0} = 0 \qquad (4)$$

(iv) Non-Negativity Constraint:
Negative quantity of materials makes no sense

$$x_{ijk}, y_{jk} \geq 0 \quad \forall\, i, j, k \qquad (5)$$

The two objectives considered are:

(i) Minimization of the total sum of production cost.
The total production cost is the sum of the products of the unit variable costs and the quantity of material processed by each facility. The objective is:

Minimize

$$Z_1 = \sum_{i=1}^{I} c_{jk}x_{ijk};\ j = 1, 2, \ldots, J_k;\ k = 1, 2, \ldots, K \qquad (6)$$

C_{jk} is the cost of processing one kilogramme of material on the jth facility of the kth stage of production.

(ii) Maximization of Capacity Utilization of Production Facilities:
Capacity utilization is the total sum of the individual utilization factor (i.e. load divided by maximum capacity). The objective is:

Maximize

$$Z_2 = \sum_{k=1}^{K}\sum_{j=1}^{J_k}\left(\frac{\sum_{i=1}^{I} x_{ijk}}{d_{jk}}\right) \qquad (7)$$

2.2 LINEAR COMBINATION OF OBJECTIVE FUNCTIONS (LCOF) APPROACH

For dimensional consistency, the objectives of the problem may be expressed as:

Maximize

$$\bar{Z}_1 = -\sum_{k=1}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I} C'_{jk}x_{ijk} \quad (8)$$

$$\bar{Z}_2 = \sum_{k=1}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I} d'_{jk}x_{ijk} \quad (9)$$

where $C'_{jk} = \frac{C_{jk}}{C_{3,1}}$ and $d'_{jk} = \frac{d_{3,2}}{d_{jk}}$ $j = 1, 2, \ldots, J_k;\ k = 1, 2, \ldots, k$

The objective functions are usually converted to their normal forms before they are combined into a single objective function because of the difference in the magnitude of their coefficients [3,4]. The biobjective LP problem is now transformed into a single objective problem as follows:

Maximize

$$Z_{12} = \frac{-W_1}{\sqrt{\sum_{k=1}^{K}\sum_{j=1}^{J_k} C'^2_{jk}}}\left(\sum_{k=1}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I} C'_{jk}x_{ijk}\right) + \frac{W_2}{\sqrt{\sum_{k=1}^{K}\sum_{j=1}^{J_k} d'^2_{jk}}}\left(\sum_{k=1}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I} d'^2_{jk}x_{ijk}\right) \quad (10)$$

subject to,

$$\sum_{i=1}^{I} x_{ijk} + y_{jk} \leq d_{jk};\ j = 1, 2, \ldots, J_k;\ k = 1, 2, \ldots, K \quad (11)$$

$$x_{ijk} - \gamma_{ijk}x_{rjk} = 0;\ i = 1, 2, \ldots, I;\ j = 1, 2, \ldots, J_k;\ k = 1, 2, \ldots, K$$

$$\sum_{j=1}^{J_{k-1}} y_{j,k-1} + \sum_{j=1}^{J_k}\sum_{i=1}^{I} x_{ij,k-1} = \sum_{j=1}^{J_k} x_{jk}^{k-1};\ \ k = 1, 2, \ldots, K \quad (12)$$

2.3 GOAL PROGRAMMING (GP) APPROACH

Two GP models are considered in this study, namely; Preemptive GP and non-preemptive GP. Included are the following goals (i) minimize the positive deviation from the desired goal level for the total sum of production cost, (ii) minimize the negative deviation from the desired goal level for the capacity utilization of production facilities. The GP model is given as;

Minimize

$$a = g_1(d_1^+), g_2(d_2^-) \tag{13}$$

subject to

$$\left.\begin{array}{l} \sum_{k=1}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I} C'_{jk}x_{ijk} - d_1^+ = Z_1^* \\ \sum_{k=1}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I} d'_{jk}x_{ijk} + d_2^- = Z_2^* \end{array}\right\} \text{Goal constraints} \tag{14}$$

plus the structural constraints.

Z_1^* and Z_2^* are the optimal values of Z_1 and Z_2 respectively when solved independently. d_1^+ is the positive deviation from the cost goal, while d_2^- is the under achievement of the capacity utilization goal.

2.4 COMPROMISE CONSTRAINT BIOBJECTIVE (CCBLP) MODEL FOR THE SYSTEMS PRODUCTION PLANNING

In [3,4], the biobjective LP problem is transformed into a single objective problem as follows.

Maximize any one of:

$$\bar{Z}_1 = -\sum_{k=1}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I} C'_{jk}x_{ijk} \tag{15}$$

$$\bar{Z}_2 = \sum_{k=1}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I} d'_{jk}x_{ijk} \tag{16}$$

$$Z_{12} = \frac{-W_1}{\sqrt{\sum_{k=1}^{K}\sum_{j=1}^{J_k} C'^2_{jk}}}\left(\sum_{k=1}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I} C'_{jk}x_{ijk}\right) + \frac{W_2}{\sqrt{\sum_{k=1}^{K}\sum_{j=1}^{J_k} d'^2_{jk}}}\left(\sum_{k=1}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I} d'^2_{jk}x_{ijk}\right) \tag{17}$$

subject to

compromise constraint,

$$\frac{-W_1}{\sqrt{\sum_{k=1}^{K}\sum_{j=1}^{J_k} C'^2_{jk}}}\left(\sum_{k=1}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I} C'_{jk}x_{ijk} - Z_1^*\right) + \frac{W_2}{\sqrt{\sum_{k=1}^{K}\sum_{j=1}^{J_k} d'^2_{jk}}}\left(\sum_{k=1}^{K}\sum_{j=1}^{J_k}\sum_{i=1}^{I} d'^2_{jk}x_{ijk} - Z_2^*\right) = 0 \tag{18}$$

plus the structural constraints.

3.0 MODEL APPLICATION

Tables 1 and 2 give the data of a toothpaste industry for the application of the model. Figure 1 shows the process flow diagram. We consider a case where the factory operates at full capacity.

4.0 RESULTS AND DISCUSSION

The summary of the results from the three LP methods are given in table III. The first and the last columns show the ideal solutions, that is, the optimum solutions considering each objective individually. In all the methods, the authors acted as Decision Maker. All the results were obtained by using Quantitative Systems for Business (QSB) package. The preemptive GP model was solved sequentially following the sequential Linear Goal Programming (SLGP) algorithm proposed by Ignizio et.al. [8].

All the modelling methods have been able to help the Decision Maker to determine the material mix for each facility at each stage of the production process. However, their sensitivity to relaxations in the objectives vary. Table III shows a concise comparison of the methods.

Since there is no 'optimal' solution to this example of biobjective decision making problem, it is very difficult to compare the solutions with one another. The comparison is based on the sensitivity of the various methods to the relaxation information provided by the Decision Maker. The 'best' compromise solutions were moulded according to the goals or relaxation information provided by the Decision Maker.

Two preemptive GP solutions were obtained using two goals and two priority structures. The solution of the preemptive GP model when cost was assigned the first priority was the same as the ideal solution of the cost objective. Similarly, the solution of the preemptive GP model when the priority structures were reversed is identical with the ideal solution of the capacity utilization objective.

That the solutions of the preemptive GP model are identical with the ideal solutions does not mean that it could be regarded as another way of solving single objective LP. For the case under study, the objectives do not have alternate optimal solutions. If there were, the preemptive GP method could have selected the solution that improves the achievement of the second goal without impairing the achievement of the first goal.

In this study, the ideal solutions were determined before the statement of goals. This is because a priori determination of goals could be too difficult or too arbitrary without a previous exploration of potentials provided by the two objectives. If the goals are set too low a suboptimal and even dominated solution might be computed.

In the case of non-preemptive GP, LCOF and CCBLP various weights of the objectives were employed to explore different trade-off options for informed decision making. CCBLP is the most sensitive to the relaxations in the objectives followed by non-preemptive GP while LCOF is the least sensitive (Table III). When $W_1 = 0.75$ and $W_2 = 0.25$ the LCOF and non-preemptive GP methods did not respond to the relaxation in the objectives. Their alleged 'best' compromise solution for this relaxation could be misleading. The CCBLP responded to this relaxation. The least utilized facility has a utilization of 20.324% as against 0.18% for LCOF and non-preemptive GP. The solution of the CCBLP is the real compromise solution.

The differences in the sensitivity of the methods to relaxations in the objectives is due to their constraint sets. In the case of LCOF, the resulting single objective was solved subject to the original set of constraints. The solution merely takes one of the extreme points of the feasible region. This could be misleading because the point of the real best compromise may not coincide with any extreme point of the convex set.

Usually, with LCOF models in cases where there is no extreme point in-between the two optima (taking each objective independently) the resulting compromise solution simply takes either one of the previous optima partly depending upon which objective is given the higher weight. Even in cases where there are extreme points in-between the two optima the compromise solution may still take one of the previous optima as in the case under study (Table III).

In the case of GP, the addition of goal constraints to the original constraint set changes the feasible region and creates new vertices. The solution of the GP takes any one of the vertices of the new feasible region. The vertices of the new feasible region consists of some of the vertices of the original feasible region and the new vertices introduced by the goal constraints. This appears to be the reason for the identical solutions of LCOF and non-preemptive GP models when W_1 takes the two different values: 0.75, 0.5. The difference in the solutions of LCOF and non-preemptive GP models for $W_1 = 0.25$ might be that the non-preemptive GP solution takes one of the new vertices introduced by the goal constraints. The solution of the non-preemptive GP, like LCOF may not give the real compromise solution. This is because the real compromise may not coincide with any of the vertices.

For CCBLP, the assigned weights together with the ideal solutions [3,4] were used to derive the compromise constraint which is added to the original constraint set. The compromise constraint passes through the original feasible region and forces both objectives to settle on a common point in the boundary of the original feaisble region, even other than the extreme points of the convex sets. The CCBLP gives the real compromise solution, since each relaxation of the objectives requires that the compromise constraint be derived anew. Different relaxation of the objectives can not give identical solutions unlike the LCOF and non-preemptive GP.

5.0 CONCLUSION

A biojective model is proposed for the single product, multi-stage, multi-facility process material mix problem. The application of the model shows that it is feasible to formulate the problem as LCOF, GP and CCBLP problem. All the methods were found useful for making informed trade-off decision.

The pre-emptive GP method is useful for biobjective decision situations in which one objective is of overriding importance. LCOF, non-preemptive GP and CCBLP are good for decision situations in which the two objectives are of comparable importance. However, CCBLP is superior to LCOF and non-preemptive GP in terms of the sensitivity to relaxations in the objectives.

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Table I: Major Production Facilities

Stage	Facility None	Capacity Per Month (kg)	Normalized Cost Coefficient/kg
Premix	Premix Vessel 1(PM1)	9600	2.00
	Premix Vessel 2(PM2)	14400	1.00
	Premix Vessel 3(PM3)	24000	1.00
Processing	Processing Plant 1(PP1)	25000	2.00
	Processing Plant 2(PP2)	25000	1.80
	Processing Plant 3(PP3)	40000	1.40
	Processing Plant 4(PP4)	30000	1.60
Storage	Storage Tank 1(ST1)	80000	0.30
	Storage Tank 2(ST2)	45000	0.45
	Storage Tank 3(ST3)	20000	0.20

Table II: Process Materials

Stage	Process Materials Required	Proportion of Process Materials
Premix	(i) Glycerin	
	(ii) Carboxymethylcellulose (CMC)	10% by wt. of glycerin
	(iii) Water (H_20)	130% by wt. of glycerin
Processing	(i) Intermediate product from stage 1 (Int. Prod.)	
	(ii) Abrassives (Abr.)	96% by wt. of Int. Prod.
	(iii) Preservatives (Pre)	1.042% by wt. of Int. Prod.
	(iv) Flavour (Fla.)	5.21% by wrt. of Int. Prod.
	(v) Moisturizing Agent (MA)	6.25% by wt. of Int. Prod.
Storage	Finished product.	

[1] Table III: Summary of the % Utilization of Production Facilities

	$W_1 = 1$, $W_2 = 0$	$W_1 = 0.75$, $W_2 = 0.25$			$W_1 = 0.5$			$W_1 = 0.25$, $W_2 = 0.75$			
Facility Name	Ideal Soln of cost obj.	LCOF	NGP*	CCBLP	LCOF	NGP*	CCBLP	LCOF	NGP*	CCBLP	Ideal Solution of Cap. Utl.obj.
Premix Vessel 1(PM1)	100	100	100	100	100	100	100	100	100	100	100
Premix Vessel 2(PM2)	100	100	100	100	100	100	100	100	100	100	100
Premix Vessel 3(PM3)	100	100	100	100	100	100	100	100	100	100	100
Pro.Plant 1(PP1)	20.324	20.324	20.324	20.324	20.324	20.324	25.196	100	100	58.424	100
Pro.Plant 2(PP2)	100	100	100	100	100	100	100	100	100	100	100
Pro.Plant 3(PP3)	100	100	100	100	100	100	96.955	50.203	100	76.188	50.203
Pro.Plant 4(PP4)	100	100	100	100	100	100	100	100	33.6	100	100
Sto.Tank 1(ST1)	100	100	100	66.845	43.851	43.851	43.851	43.851	43.851	43.851	43.851
Sto.Tank 2(ST2)	0.18	0.18	0.18	59.123	100	100	100	100	100	100	100
Sto.Tank 3(ST3)	100	100	100	100	100	100	100	100	100	100	100

[1]* NGP: non-preemptive GP.

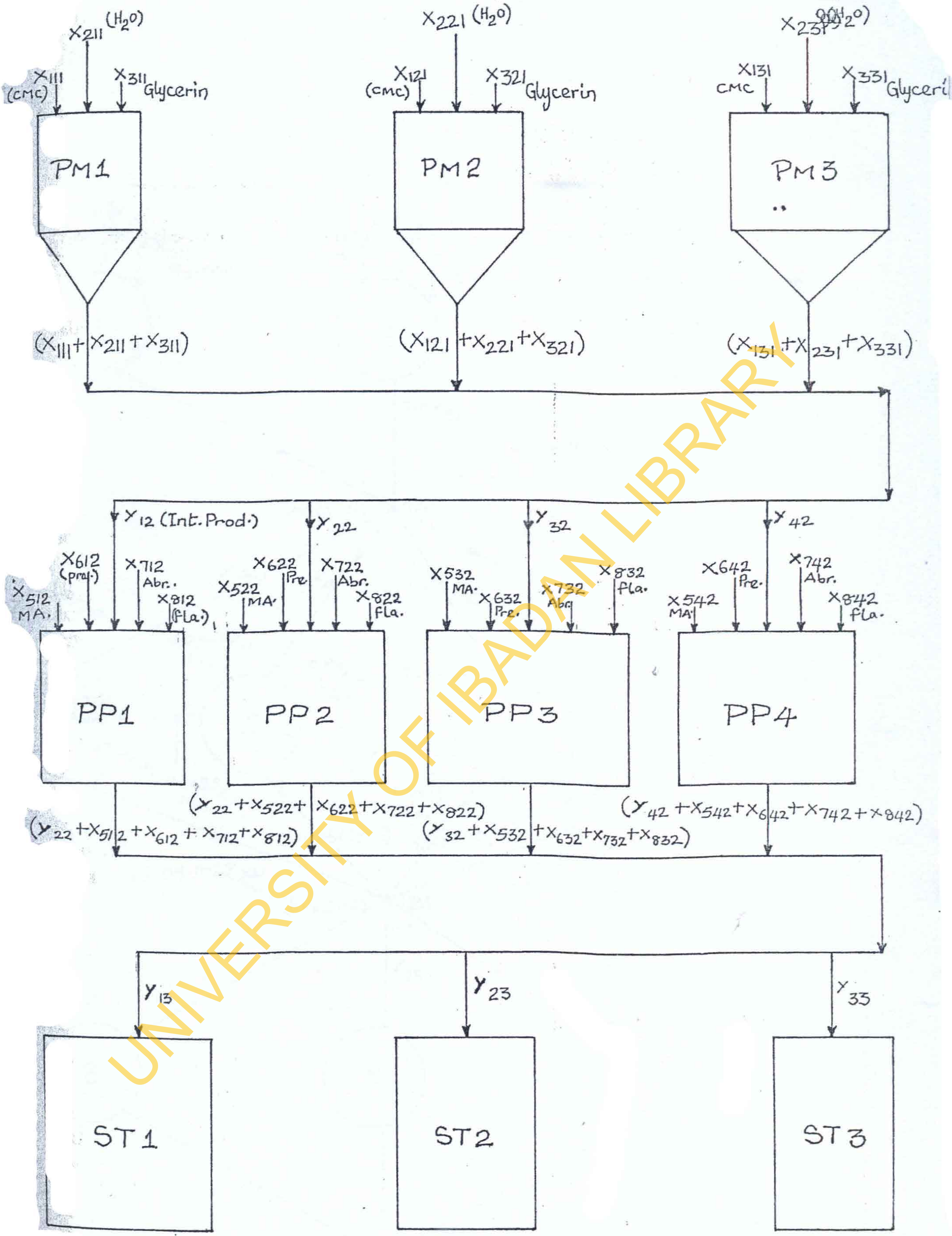

Fig. 1 Process Flow Diagram of The Multi-Stage, Multi-Facility Production System.

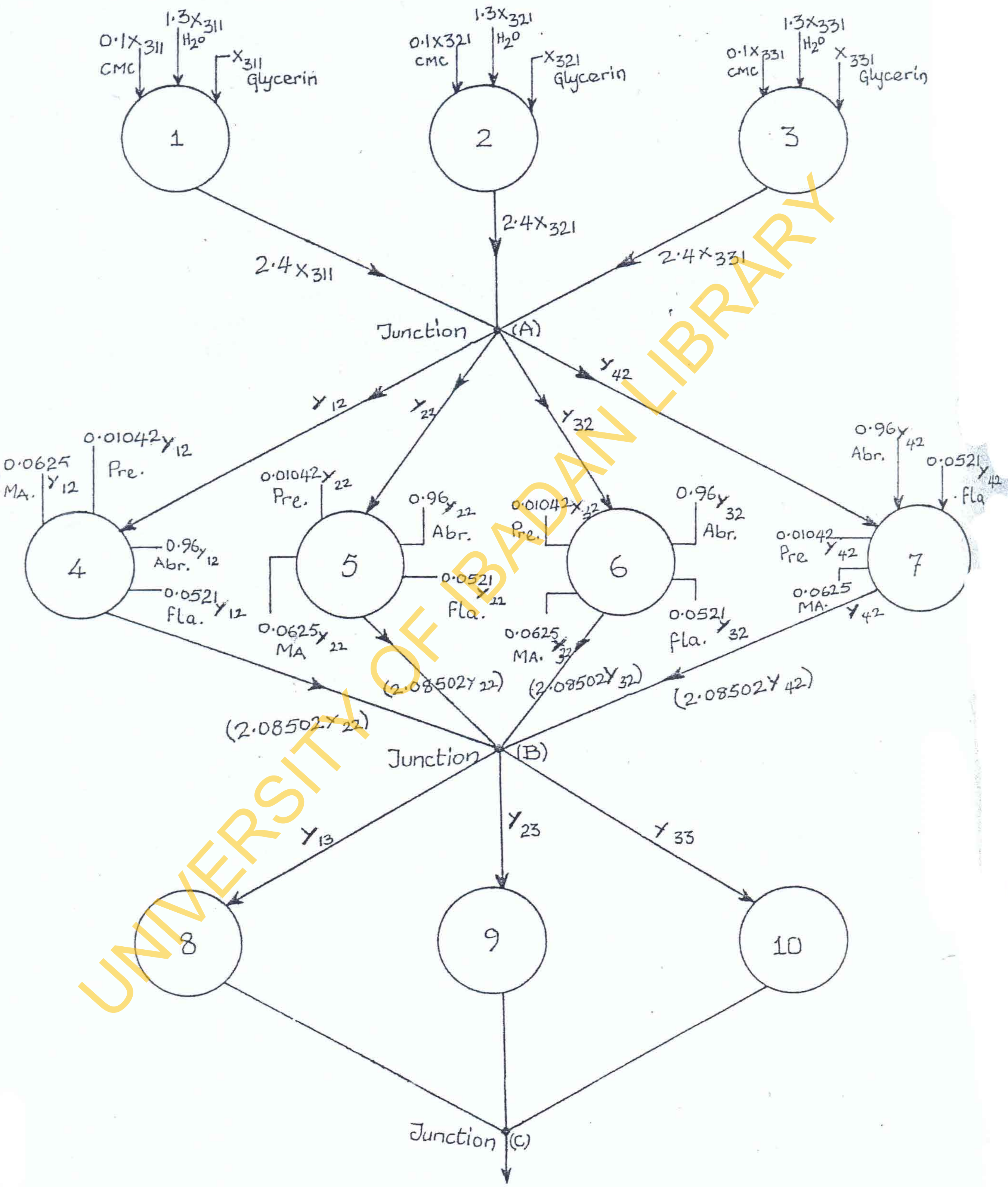

Fig 2 Process Flow Diagram Indicating Proportion of Materials.