APPLICATION OF MATHEMATICAL PROGRAMMING APPROACH TO WELDING FLUX DEVELOPMENT

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1.0 INTRODUCTION

1.1 Background of the Study

The objectives a welding flux is expected to achieve are many and often mutually incompatible. Such objectives or quality characteristics include weld-metal quality requirements which are determined by the chemical composition, mechanical properties and metallurgical features. Other quality characteristics are the operational requirements, which include arc stability, penetration, spatter, etc, while environmental requirements consist of minimum fume, minimum toxic content of fume, and minimum noxious odour (Adeyeye and Oyawale, 2010b). Studies have shown that these characteristics are influenced by the welding flux formulation; therefore, it is important to select the right type of welding flux ingredients and choose the appropriate proportions of the various flux ingredients to attain a good weld-metal quality (Pandey et al, 1994; De Resone et al, 2001 and 2002; Paniagua-Mercado et al, 2005; Kanjilal et al, 2004, 2005, 2006, 2007a and 2007b and Paniagua-Mercado et al, 2009). The incompatibility of the quality characteristics arises because improvement in one can only be made to the detriment of one or more of the other quality characteristics. Compromises and balances are often provided and designed into the flux by the Welding Flux Designer (WFD) such that as many as possible of the quality characteristics or specifications are met. The welding flux formulation problem is that of selecting the right types of flux ingredients in their right proportions such that the resulting flux gives the best balance among the competing flux quality requirements. The need for extensive experiments and lack of information on optimising algorithm for identification of optimal flux have been reported as the major challenges of welding flux formulation (Quintana et al 2006; Adeyeye and Oyawale 2008, 2009, 2010a & 2010b). The development of multi-criteria model to consider multiple flux quality requirements simultaneously is the subject of this study.

The traditional method employed by WFDs to achieve compromises and balances among the specifications is by lengthy experiments involving much iteration (Quintana et al 2006 and Adeyeye and Oyawale 2008). Drawing upon the principles of physics, chemistry and metallurgy tempered with accumulated experience, the WFD formulates an initial flux and performs welding to determine its operational characteristics. Next, the weld deposit is tested to determine its conformity to quality specifications. Based on the test results, the WFD makes guesses guided by the principles of science and accumulated experience to improve the flux. The cycle of 'formulate, weld, test and guess' continues until an acceptable flux is achieved (Fleming et al, 1996 and Pessoa et al, 2007). The 'formulate-weld-test-and-guess' approach to welding flux development has been described as 'try-and-test' method (Bhadesia 2004, Quintana et al, 2006 and Mostafa and Khajavi 2006).

The drawbacks of the 'try-and-test' welding flux design approach are many, namely (Quintana et al, 2006, Ren et al 2006 and Adeyeye and Oyawale 2008); (i) There is usually a very long lead-time because of the extensive experimental flux formulation, weld production and testing (ii) It is costly because of the labour requirements and consumption of considerable amount of resources including energy during the lengthy experiments (iii) The welding flux developed by the conventional method has a random character and it is difficult to guarantee optimal formulation (iv) The feasibility or otherwise of achieving the desired compromise formulation cannot be established until a lot of resources and efforts have been expended on try-and-test experiments (v) Trade-off exploration is difficult. These drawbacks have persisted in the state-of-the-art of welding flux design due to the paucity of optimising algorithms capable of simultaneously handling multiple competing flux quality requirements. To mitigate these limitations, a methodology that can simultaneously consider the multiple conflicting requirements and provide means of exploring available trade-off options so that the WFD can select the flux formulation that best meets his needs is required.

Advanced materials are being developed to improve energy efficiency, corrosion resistance, high temperature performance, cryogenic performance and mechanical properties of the many industries of the future (Caron, 2000; Tancret and Bhadeshia, 2003; Cala et al, 2008; de Carlan, 2004 and Wright et al, 2010). The uses of these materials as components in manufactured products require that they be welded; hence their effective deployment is highly dependent upon the development of welding flux technology (Tancret and Bhadeshia, 2003; Tancret et al, 2003a and Tancret et al, 2003b). Quintana et al (2006) observed that rapid deployment of these materials is hampered because arc welding technology has not been able to keep pace with the development of these new materials. The long lead-time due to the lengthy experiments has been a serious drawback in the deployment of new materials.

The need for the reduction of the number of experiments has been the concern of welding flux researchers and manufacturers. Although it is not possible to eliminate experiments completely, their reduction is very advantageous (Bhadesia, 2004 and Adeyeye and Oyawale, 2008). Quintana et al (2006) observed that a reduction in the number of experimental welds from 30 to 5 in gas metal arc welding (GMAW) led to about 80% energy savings. Obviously, the benefits of the reduction in the number of experimental welds would be more by the time the savings on materials, man-hours and time are considered. It is unlikely that the WFD will be able to rise to this challenge without better tools in the form of prediction and multi-criteria optimisation models with which he can determine the attainable set or the feasible criterion space (FCS). As a result of the absence of multi-criteria optimisation tools the WFD may not know whether or not it is possible to achieve the desired properties within the experimental domain until a lot of resources have been expended on experiments.

At times, a lot of resources is consumed searching for flux formulation that will achieve desired specifications in an experimental domain even when such formulation may not exist in that experimental region. On the other hand, the desired formulation may exist within the experimental domain but the WFD may not be able to identify it through the try-and-test experiments because of its random nature. The WFD often abandons the experimental domain to establish a new domain with the erroneous impression that the desired flux formulation can not be achieved or does not exist within the experimental space because he has not been able to identify it after a series of tedious experiments. Even in a situation where the WFD is able to get an acceptable flux through the formulate-weld-test-and-guess experimental approach, it cannot be guaranteed to be a noninferior formulation since it is not practical to explore all combinations of compositional variations due to time and cost limitations. There may exist flux formulation(s) that dominate(s) it in the flux design space but difficult to identify because of the absence of optimising algorithm. A methodology that can consider the multiple conflicting requirements simultaneously and provide trade-off options according to the preferences of the WFD is required.

In the last ten years, researchers have increased their efforts at reducing experiments and identifying optimal flux formulation by using experimental design (Kanjilal et al, 2004, 2005, 2006, 2007a, & 2007b and Ren et al, 2006). Ren et al, (2006) tried to overcome these problems by using a design of experiment method (DoE) known as uniform design (UD) to develop a new agglomerated flux for high speed and multi-arc SAW. The UD approach reduced the amount of experimental efforts. However, the best flux formulation from the UD experiment cannot be guaranteed to be optimal (Adeyeye and Oyawale, 2009). Even if par chance the best flux from the UD experiment coincides with an optimal flux formulation in the total experimental space, there is no quantitative means (optimality criteria) for its identification. Kanjilal et al, (2004, 2005, 2006, 2007a and 2007b) used another form of DoE technique known as the extreme vertices design proposed by McLean and Anderson (1966). They considered the simultaneous variation of flux ingredients and identified their direct and interaction effects on the responses. However, the application of multi-criteria optimisation on the responses has not been reported. The problems of simultaneous consideration of multiple flux quality requirements and determination of FCS set have remained unresolved. The resolution of these limitations constitutes the main interest of this study. This work attempts to bridge the gap and provide the industry with an easy to use multi-criteria optimisation tool for welding flux formulation.

1.2 Statement of the Problem

Welding fluxes are designed to meet the specifications of the users. The specifications are usually given as the desired numerical values for the various flux attributes. Some quality characteristics (QC) are very desirable, the larger their values the better, while some attributes are undesirable, the smaller their values the better. These specifications are described as Larger-The-Better (LTB) and Smaller-The-Better (STB), respectively. For some QCs, the achievement of a target numerical value is most desirable. Deviations from their respective target values are undesirable and as a result, the deviations are minimised (Adeyeye and Oyawale, 2010b). Such specifications are described as Nominal-The-Better (NTB). A user specification may involve LTB, STB and NTB simultaneously for various flux specifications. These specifications are often in conflict because improvement/achievement of one impairs the achievement of one or more of the other QCs. It is, therefore, difficult to get a utopia flux formulation. Instead, the WFD searches for a formulation that gives the best balance among the specifications (Adeyeye and Oyawale, 2010a & 2010b). Also the QCs are often of varying degrees of importance to the user. The specifications and the preferences of the user become the goals which the WFD pursues. The problem of welding flux formulation is that of determining the proportions of the various flux ingredients such that the welding flux gives the best balance among the QCs according to the preferences of the flux user.

The traditional lengthy and costly 'formulate-weld-test-guess' experimental approach to welding flux formulation may not guarantee an optimal flux (Adeyeye and Oyawale, 2008 & 2009). Optimisation modelling solution approach may have been made extremely difficult in welding flux technology because welding flux ingredients, welding wire constituents and welding parameters exhibit very complex reactions and interactions during the welding process. This appears to have precipitated difficulties of:

i. constructing objective and constraint functions that adequately express the multiple flux quality specifications;
ii. defining a feasible criterion space for systematic experimentation;
iii. resolving conflicts arising from simultaneously satisfying the desirable multiple flux attributes; and
iv. taking into consideration the preferences of the user.

The development of an approach which will effectively handle these situations is the problem addressed in this study.

1.3 Aim and Study Objectives

The main aim of this study is to provide the welding flux industry with an easy- to-use multi-criteria optimisation model for submerged arc welding flux formulation. Specifically, the following objectives will be pursued;

(i) To develop a computational procedure for the determination of a feasible criterion space.
(ii) To define and solve multiple response optimisation based welding flux formulation problem for the Nominal-The-Better situation.
(iii) To develop an approach for trade-off exploration for various flux formulation situations.

1.4 Scope of the Study

The scope of this research consists of the following elements:

i. This research is undertaken to develop multi-criteria optimisation model for SAW flux formulation.
ii. The research considers welding flux formulation situation in which all the attributes that define the quality of the flux depend on the same set of input variables.
iii. The modelling environment involves situation where a priori articulation of the preferences of the WFD is possible and the underlying response surfaces that describe the relationship between the flux attributes and the flux input variables are continuous and smooth over the domain of interest.

2.0 LITERATURE REVIEW

Literature reveals two kinds of welding flux development activities in welding flux technology. The academic and national laboratory efforts focus on developing models by employing detailed scientific methodologies (kinetics, thermodynamics, slag chemistry, solution thermodynamics, arc plasma physics and chemistry, e.t.c…). These models are cumbersome and not easy to use. The models often consider effects of individual flux ingredients on a single response and not the effects of simultaneous variation of the ingredients on many responses. For instance, Terashina et al (1976) and Surian et al (1997) studied the effect of slag basicity on diffusible hydrogen, Du Plessis and Du Toit (2007) studied the effect of flux-oxidizing ingredients on diffusible hydrogen and Du Plessis et al (2007) and Baune et al (2000a & 2000b) investigated the effect of fluoride and calcite on diffusible hydrogen content. North et al (1978) and Eager (1978) studied the effects of FeO, MnO, metallic and ferro-metallic additions on weld-metal oxygen content individually. Farias et al (2004) considered the effects of wollastonite and quatz on fusion rate and short-circuit frequency. In the real world flux formulation situations, many competing quality requirements are simultaneously considered. For these reasons, models obtained based on physical science principles have limited applications in real-world industrial conditions where it is required that an optimal flux be developed at minimum costs and time. On the other hand, flux manufacturers have relied on extensive and expensive experimentations to drive their product development.

The quality characteristics a welding flux is expected to achieve are many and more often than not mutually incompatible. Compromises and balances are often provided and designed into the flux. To achieve the required balances the flux formulator first formulates an initial flux drawing upon the principle of physics, chemistry, metallurgy and accumulated experience. He then uses the flux to weld. During welding, he tests the flux to see if it meets operational and environmental specifications. After this, he tests to see if the weld deposit meets the chemical composition specifications as well as the mechanical and metallurgical requirements. Based on the results the WFD makes guesses to improve the flux dwelling on principles of metallurgy and experience. The cycle of formulate-weld-test-guess continues till an acceptable flux is achieved.

Fleming et al (1996) developed welding flux for SMAW of HSLA-100 grade steel. A sequential flux formulation methodology was used to study the effects of welding flux type on HSLA-100 steel weld-metal microstructure and mechanical properties. The flux compositions were systematically varied starting with an initial flux that can be classified as rutile-based and ending up with a more basic flux. The objective of the variations was to identify/develop a formulation of a SMAW flux that would exhibit the excellent welding behaviour found typically in a rutile electrode and balanced with the superior weld-metal properties deposited by a basic electrode. Nine separate series of electrodes were studied each with one substitution for a specific ingredient in the flux. The flux that produced the best results for a given series was used as the basis for the formulation of the next series. Although Fleming's et al (1996) flux produced acceptable weld deposit, it is not guaranteed to be a nondominated formulation. Another limitation of their methodology is that the interactions between the flux ingredients could not be identified and quantified. Prediction and optimisation of the measured responses cannot be easily determined.

A study on the effect of variation of the flux composition of covered rutile manual electrodes on both operational characteristics and deposited weld-metal properties was done by De Rissone et al (2001). Like the works of the previous investigators discussed above, De Rissone et al (2001) ignored possible interaction effects and their methodology lacks the ability to simultaneously consider multiple flux quality requirements. Hence, the flux so designed can not be guaranteed to be optimal.

De Rissone et al (2002) studied the effect of calcite on the operational characteristics and mechanical properties of weld-metal deposited by ANSI/AWS A5.1-91 E6013 rutile electrodes. Three fluxes were designed by varying calcite between 5% and 15% at the expense of cellulose and Si-bearing ingredients (quartz, kaolin, mica, and feldspar) in the dry mix. This replacement was undertaken to obtain an increased basicity of slag without varying TiO_2 content so that the operational characteristics of rutile electrode are maintained as far as possible. To be able to study the effect of calcite, Si-bearing ingredients were varied between 13% and 21% and cellulose between 0% and 6%. TiO_2 and Mn + Fe powder were varied over the narrow ranges of 52-55% and 15-16% respectively. Infact only K_2O remained constant throughout the experiment. In a simultaneous variation of ingredients as in this case, the observed responses cannot be attributed to only one ingredient or to the direct effects of the individual ingredients without first exploring the possible interactions of the ingredients.

Farias et al (2004) studied the effect of wollastonite on operational characteristics of AWS E6013 electrodes by replacing quartz with 0%, 8% and 16% of wollastonite in the flux. Three experimental AWS E6013 type electrodes were produced. The first one contained 16% quartz and 0% wollastonite; the second had 8% quartz and 8% wollastonite and the third flux 0% quartz and 16% wollastonite. They observed that the intermediate-wollastonite-content flux (8% quartz and 8% wollastonite) performed better in fusion rate analysis on direct current electrode positive (DCEP) and direct current electrode negative (DCEN). The intermediate-wollastonite-content electrode also tended to present higher short-circuit frequency on DC. They did not explain or give the reasons for the observed behaviour. One of the possible reasons for the better performance of the intermediate-wollastonite-content flux on these criteria might be due to the synergetic binary interaction effects of quartz and wollastonite. It may also be due to the ternary or even quaternary synergism of wollastonite, quartz, Mn powder and iron powder. Though the effects of Mn and Fe powders were not among the flux ingredients studied, they had to vary the amount of these powders as they varied the amount of quartz and wollastonite so that the chemical composition of the weld deposits for all the electrodes would be similar. The possible interactions were ignored. Assessment of ingredient interactions is being recognized as increasingly important in welding flux design, where it may be necessary to determine the combined synergetic and antagonistic effects of many flux ingredients (Kanjilal et al, 2004 and 2007). Knowledge of the individual and interaction effects can be very useful when developing new flux systems that will achieve optimum weld-metal properties.

The limitations of the formulate-weld-test-guess approach adopted by these investigators can be summarised as; (i) the lead time is usually long (ii) it is costly because of the consumption of considerable amount of materials, energy and man-hours during the extensive experiments (iii) the optimality of the flux can not be guaranteed (iv) interaction effects of flux ingredients can not be easily identified and quantified (v) trade-off exploration is difficult.

Since Lau et al (1986) reported the significant interactions effect of flux ingredients, little attention has been given to it by researchers until the last ten years. Applications of statistical design of experiments and computational techniques to welding flux design has been on the increase since 2004 (Kanjilal et al 2004, Paniagua-Mercado, et al 2005, Sui et al 2006, Zinigrad 2006 and Kanjilal et al 2006 and 2007, Achebo and Ibhadode, 2008 , 2009 and Achebo, 2009). Researchers seem to have realised that some of the information about flux behaviour may be lost without identifying and quantifying the interaction effects of the flux components as well as the need to reduce research time and labour by implementing designed experiment and using the experimental and or theoretical data to develop regression models which may be used to predict the responses. The use of predictive and optimisation models have been on the increase in other areas of welding research especially in the development of welding wire and rods but their application in welding flux formulation is scanty in the literature (Konjol and Koons 1978; Murugananth et al 2002; Bhadeshia 2004; Cho et al 2006; Gunaraj et al 2000a; Gunaraj et al 2000b and Adeyeye and Oyawale, 2008).

The studies of Kanjilal et al (2004, 2005, 2006, 2007a and 2007b) were prominent among researchers who used statistical design of mixture experiments to assist them to identify and quantify the direct and interactions effects of flux ingredients as well as be able to predict the values of flux attributes. They established mathematical relationship between the predictor variables (flux ingredients) and response variables (flux attributes). The models can only predict the values of individual welding flux attributes when the levels of flux ingredients are chosen. Optimisation of welding flux attributes either singly or jointly was not considered.

Ren et al, (2006) tried to achieve flux formulation with optimum properties by using a design of experiment method (DoE) known as uniform design (UD) to develop a new agglomerated flux for high speed and multi-arc SAW. Although the UD approach reduced the amount of experimental efforts, according to Adeyeye and Oyawale (2009), the best flux from the UD experiment cannot be guaranteed to be the optimal flux within the experimental space. The result at best may be near optimal. Even if par chance the best flux from the UD experiment coincides with an optimal flux in the total experimental space, there is no quantitative means (optimality criteria) for its identification.

Achebo and Ibhadode (2008, 2009) and Achebo (2009) used another DoE method known as Hadamard multivariate matrix design to develop welding fluxes for aluminium welding. The data from the experiment was used to develop regression model for shear strength in terms of the flux ingredients. Computer programmes based on non-linear multi-parameter regression were used by Paniagua-Mercado, et al, (2005) to predict the tensile properties and microstructure of submerged arc welded AISI 1025 steel using three flux compositions with low-carbon electrode. Sui et al (2006) developed multicomponent mixture regression model based on simplex algorithm of optimal design to investigate the physical properties of submerged arc welding flux. The model of Sui et al (2006) was for prediction purposes.

As far as we know, the works reported in the literature and available to us are limited to the development of regression models for prediction purposes and the identification and quantification of direct and interaction effects. In real world welding flux formulation situations, the WFD is interested in determining the flux ingredient levels that optimise all flux quality characteristics simultaneously and not individually. For instance, he may wish to achieve predetermined target values for some of the quality characteristics (nominal-the-better) while at the same time he may also want to maximise desirable attributes (larger-the-better) and minimise the values of undesirable attributes (smaller-the-better). The flux quality characteristics are often of differing degrees of importance to the WFD and needed to be taken into account during flux formulation. Apparently, none of the previous researches have addressed these situations. Studies that considered simultaneous optimisation of the various conflicting requirements as well as incorporate the preferences of the WFD are sparse in the open literature. The few that have appeared are the works of Adeyeye and Oyawale (2008, 2009, 2010a & 2010b). WFDs therefore need modelling tools that can handle multiple welding flux quality requirements as well as assist in exploring various trade-off options in order to be able to achieve optimal flux formulation.

A careful observation of the recent trends in activities and efforts of researchers in welding flux technology reveals areas where further research activities are required, namely,

- Using designed experiment instead of un-designed or trial-and-error experiments.
- Reduction of the amount of experimental efforts in order to reduce lead-time and costs associated with extensive experiments
- Identification of main and interaction effects of flux ingredients
- Development of optimisation tools for the simultaneous consideration of the conflicting quality requirements and the identification of the attainable set.
- Development of a means of answering 'what if' questions. That is a methodology that makes trade-off exploration possible.

The development of multi-criteria optimisation model for the determination of the best balance flux and the attainable set is the focus of this study.

3.0 METHODOLOGY

3.1 Welding Flux Design Problem

Welding flux formulation problem involves selecting the right types of flux ingredients in their right proportions such that the requirements of the welding flux designer (WFD) are achieved. The WFD is an individual or a team with experience and expertise in welding flux formulation. Typical welding flux requirements are weld-metal quality requirements, operational, environmental, manufacturability and storage requirements. Operational characteristics such as arc stability, deposition rate, slag control, etc…determine the productivity and cost of the welding process. Welding flux design therefore seeks to maximise the contribution of the welding flux to the society while minimising its cost to the manufacturer, user and the environment. Each lifecycle stage of the flux is taken into consideration during the design stage. Health and safety of the welder and other workers at the welding environment are also important. The flux is therefore expected to produce minimum fume, no or minimum noxious odours and minimum amount of toxic materials in the fume. Some of the commonly encountered requirements are presented in figure 3.1. Most of the requirements are bundles of other requirements and can be broken down to secondary and tertiary requirements. For instance, weld-metal quality depends on mechanical property, microstructure, bead morphology etc., all of which are also determined by other requirements (see Figure 3.1). The requirements presented in figure 3.1 are not exhaustive; depending on the situation more requirements may be added. The requirements the WFD selects for a particular flux depend on the welding method, the particular metal to be welded and the service requirement of the welded structure. These requirements are incompatible because it is not possible to improve one quality characteristic without decreasing the achievement or satisfaction of one or more of the other quality characteristics. The problem of flux design therefore, is that of determining the flux ingredients levels that will achieve the best balance among the various conflicting requirements. Multi-criteria optimisation tools that can handle multiple competing requirements such that the best compromise welding flux can be formulated will be considered.

3.2 Model Assumptions

The following assumptions are set to construct the optimisation model of the welding flux formulation problem (Adeyeye and Oyawale 2010b).

I. All the response variables defining the quality/performance of the welding flux depend on the same set of predictor variables.
II. Response equations that describe the relationship between the response variables and the predictor variables can be estimated over the domain of interest.
III. The underlying response surface is continuous and smooth over the domain of interest.
IV. The welding wire type is fixed.
V. Welding parameters are fixed

3.3 The Concept of Feasible Criterion Space (FCS)

The need for the reduction of the number of experiments has been the concern of welding flux researchers and manufacturers. The extensive experiments are needed because it is difficult to know a priori, if the desired performance levels or targets for quality characteristics are achievable or not. The reactions and interactions of welding flux ingredients, welding wire constituents and welding parameters in the weld pool during welding to determine the numerous quality characteristics is complex. There is no theory that has the rigour or sophistication to simultaneously handle the large number of variables that control the welding process and weld-metal quality characteristics. The convectional way to approach such problem is to apply regression analysis in which experimental data are best fitted to some function. Such functions are expressed in terms of flux input variables. Once the response surfaces for all flux attributes or specifications are determined, the FCS can be defined to know the possibility of achieving a desired response target value with the same set of ingredients before further resources are expended on experiments.

3.3.1 General Multiple Criteria Welding Flux Formulation Problem

A general multiple criteria welding flux problem can be described as a multi-variable constrained problem. The problem is to optimise the function:

$$\begin{gathered} F(x) = [f_1(x), f_2(x), \dots f_m(x)] \\ \text{Subject to:} \\ x \in C_s \subset R^n \end{gathered} \tag{3.1}$$

where n is the number of design variables, m the number of design attributes/specifications and C_s denote set of constraints or design space. For a general welding flux formulation problem, $F(x)$ is non-linear and C_s may be defined by a set of linear, non-linear or both linear and non-linear constraints. The mapping of the flux design space into the design attribute space gives what we call the feasible criterion space or the attainable set (Marler and Arora, 2004). The obtainable criterion vectors, $\{F(x) \mid x \in C_s\}$ are denoted by Y, so $F: C_s \mapsto Y, that\ is\ C_s$ is mapped by F unto Y. In other words, the set $Y = F(x) \subset R^m$ is the mapping of the design space into the specification space. Each point within the design space, $\{x \in C_s \subset R^n\}$ maps to a point in Y (the FCS) as illustrated in Figure 3.2 below. The boundary of Y should be of particular interest to the rational WFD. If a desired specification value falls outside the boundary of Y, the WFD knows immediately without further experiments that it is not feasible to achieve that specification under the prevailing conditions.

- Flux requirement
  - Weld - metal requirement
    - Composition
      - *Chromium*
      - *Manganese*
      - *Carbon*
      - *Difussible – hydrogen*
    - Microstructure
      - Ferrite number
      - Acicular ferrite
      - Bainite e.t.c...
      - Inclusion volume fraction
      - Inclusion size
      - Inclusion number density
    - Mechanical property
      - Yield strength
      - Tensile strength
      - Charpy impact strength
      - Hardness
      - Elongation
    - Weld bead
      - Penetration
      - Reinforcement
      - Bead width
      - Dilution
  - Environmental requirement
    - Fume
      - Generation rate
      - Particle number
      - Mass distribution
    - Noxious odour
    - Toxic materials
      - Cr^{6+}
      - Mn
      - Ni, *etc..*
  - Manufacturability requirement
    - Extrudability
    - Good bonding
    - Uniform mixing
  - Storage requirement
    - Durability of coating
    - Moisture pick - up
    - Shelf - life
  - Operational requirement
    - Arc striking
    - Arc stability
    - Penetration control
    - Spatter
    - Slag control
    - Slag detachabilty
    - Deposition rate
    - Electrode overheating

Figure 3.1: Typical welding flux requirements (Source: Adeyeye and Oyawale, (2010a))

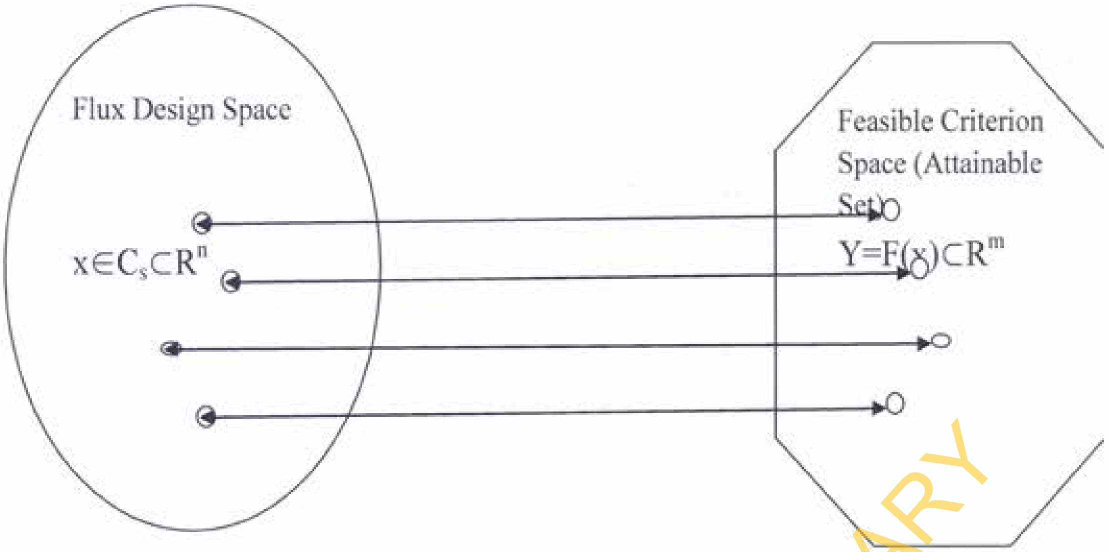

Figure 3.2: Mapping from Flux Design Space to Criterion Space

3.3.2 Determination of the Boundary of the Feasible Criterion Space

The steps the WFD may follow to determine the boundaries of the feasible criterion space are stated below (Adeyeye and Oyawale 2010b):

Step 1: The WFD determines the boundary of Y by first minimising and maximising each specification individually over the design space to obtain their minimum and maximum values possible, $f_i^{\min}(x)$ and $f_i^{\max}(x)$ respectively. This may be achieved by defining and solving Single Criterion Optimisation Problems (SCOPs) as in equations (3.2a and 3.2b) below.

$$Minimise, \eta_i = f_i(x)$$

Subject to: (3.2a)

$$x \in C_x$$

And,

$$Maximise, \eta_i = f_i(x)$$

Subject to: (3.2b)

$$x \in C_s$$

Step 2: The WFD writes the values of $f_i^{\min}(x)$ and $f_i^{\max}(x)$ for each $i \in I$. The criterion space or the attainable set is defined by $\{f_i^{\min}, f_i^{\max}\}$ and any point within or on the boundary is achievable. For a single criterion/specification case, the feasible criterion space is a straight line, for two quality specifications it is a plane while for m specifications it is R^m criterion space.

Step 3: The WFD writes the desired or target value, T_i for each $i \in I$ and compares them with their respective $\{f_i^{\min}, f_i^{\max}\}$ interval. If the T_i values for all $i \in I$ fall within or on the boundary of Y, then it is feasible to achieve the needed flux with the present flux ingredients. If one or more of the T_i values fall outside Y (the attainable set), then it is not feasible for the WFD to achieve the desired flux with the present flux ingredients without some changes.

Step 4: If it is feasible to achieve the desired flux, the WFD proceeds to the optimisation stage. Otherwise he goes back to experiments and makes necessary changes such as the addition of ferroalloys, addition or substitution of flux ingredients or any other actions based on accumulated experience and principles of metallurgy.

3.4 Optimisation Model

Once the boundary of Y is determined, and feasibility of achieving the desired flux quality characteristics has been established, the next stage is to identify or search for the point within the flux mixture space that maps to the point of best balance among the competing requirements in Y. This may be accomplished through the development and solution of the multiple criteria optimisation model.

3.4.1 Development and Solution of Multi-criteria Optimisation Models

The kind of model the WFD can use depends on the type of flux formulation problem, the goal of optimisation and his preferences. The commonly encountered flux formulation situation considered is the situation where many quality characteristics are important to the WFD and they are of roughly comparable importance. The WFD has target values he wants to achieve for each of the quality characteristics. Such situations are called Nominal-The-Better (NTB) flux design situations. Due to conflict between the competing QCs, the utopia point more often than not is not in Y. What the WFD searches for in Y, is the point of best balance or best compromise. Multi-criteria optimisation techniques developed for such situations is nonpre-emptive goal programming (NGP).

3.4.1.1 Nominal-The-Better (NTB) Flux Design Situations

Consider the flux formulation situation in which the WFD has some specific numeric values/ target values $(L_i, T_i \,\&\, U_i)$ he wants to achieve for each of the QCs. The problem is determining the flux ingredient proportions that minimise the weighted sum of the deviations of the quality characteristics from their respective target values. Non-pre-emptive goal programming (NGP) is a tested and validated multi-criteria optimisation procedure that can handle such flux formulation problems. In NGP, the various specifications are presumed to be of roughly comparable importance. Since it is not possible to achieve all the goals because of their conflicting nature, there will be deviations from their target values for all or some of the specifications. These deviations are unwanted and therefore, they should be minimised. The unwanted deviations are assigned weights according to their relative importance to the WFD and minimised as an Archimedian sum. The specific steps the WFD may follow are as follows (Adeyeye and Oyawale 2010a & 2010b):

Step 1: Establish the desired target levels $(T_i, L_i \,\&\, U_i)$ for each of the responses/quality characteristics, (e.g. acicular ferrite$\geq 50\%$, oxygen content = 240 ppm and diffusible hydrogen content ≤ 8 mL/100 g).

Step 2: Assign weights to each QC and their respective negative (n_i) and positive (p_i) deviations

Step 3: Construct the goal constraints of the problem. The goal constraint is usually given by;

$$f_i(x) + n_i - p_i = T_i, L_i, or U_i \quad \text{for each } i \in I \qquad (3.3)$$

Step 4: Construct the achievement function of each response as illustrated in Table 3.1 below.

Table 3.1: Construction of Achievement Function

Objective	**Description**	**Achievement Function**
$f_i(x) \geq L_i$	Under-achievement or negative (n_i) deviation (i.e. values below L_i) is unwanted and must be minimised.	Minimise n_i
$f_i(x) \leq U_i$	Over-achievement or positive deviation (p_i) (i.e. values above U_i) is unwanted and must be minimised.	Minimise p_i
$f_i(x) = T_i$	Both negative (n_i) and positive (p_i) deviations are unwanted and must be minimised	Minimise($n_i + p_i$)

Step 5: Construct the overall achievement function and add the goal constraints to the structural constraints of the problem. The complete NGP model of the problem may be stated as;

$$\text{minimize}, a = \sum_{i \in I} (u_i n_i + v_i p_i)$$

subject to;

$$f_i(x) + n_i - p_i = T_i, L_i, or\, U_i \text{ for each } i \in I \qquad (3.4)$$

$$x \in C_s$$

$$n_i \times p_i = 0 \text{ for each } i \in I$$

(It is not possible to have both p_i and n_i together for any response i). The weights u_i, v_i take the value zero if the minimisation of the corresponding deviational variable is not important to the WFD.

Step 6: Solve the model in step 5 to find the flux ingredient levels that minimises the weighted sum of the deviations.

Step 7: Use the values obtained to develop the needed welding flux. Trade-off exploration may be achieved by using different weight structures

3.5 Application of the Proposed Procedure

3.5.1 Data Collection

Data for the study were collected from the work of Kanjilal and co-investigators in the literature (Kanjilal et al, 2004, 2005, 2006, 2007a and 2007b). They used statistical design of mixture experiment to determine the treatment combinations. The flux ingredients used were the reagent-grade CaO, MgO, CaF_2 and Al_2O_3 with other ingredients which are of constant composition throughout the experiments. The fixed composition ingredients (FCI) are SiO_2, Fe-Mn, Fe-Si, Ni and bentonite (Table 3.3). The study was done in order to understand the effect of flux ingredients on weld-metal chemical composition, mechanical properties, microstructure and element transfer for submerged arc welding of C-Mn steel. The experiments were conducted with a low-carbon filler wire with a diameter of 3.15 mm at fixed welding parameters (current 400 A, voltage 26 V, speed 4.65 mm/s and electrode extension of 25 mm). The compositions of the base metal and filler wire are given in Table 3.2 while the flux formulations as per the mixture experiment design are given in Table 3.3. The corresponding response values from the experiments conducted with the formulations in Table 3.3 are presented in Table 3.4.

3.5.2 Development of Response Equations

Regression equations were fitted to the data in Tables 3.3 and 3.4 sequentially according to the procedure presented in Adeyeye and Oyawale (2008) with STATISTICA 8.0 software. Models with the highest R^2 values were chosen. Scheffe's special cubic polynomials have higher R^2 than Scheffe's quadratic canonical polynomial by Kanjilal et al. (2004, 2005, 2006, 2007a and 2007b) with the exception of the regression model for the nickel content of weld metal. The regression equations that relate the chemical composition of weld-metal, microstructure, mechanical properties and element transfer to the proportions of flux ingredients are presented in Tables 3.5-3.8. The adequacies of the response equations were tested using the analysis of variance (ANOVA) technique. According to Siva et al, (2009) if the calculated value of F-statistic of the response equation exceeds the standard tabulated value of the F-statistic for the desired level of confidence, the response equation can be considered adequate within the confidence limits. In this study the confidence limit was 95%. Details of the regression equations are presented in Appendix A.

Table 3.2 Base Metal and Filler Wire Composition

Element	Carbon (wt.%)	Manganese (wt.%)	Silicon (wt.%)	Sulphur (wt.%)	Phosphorus (wt.%)	Nickel (wt.%)	Oxygen (ppm)	Nitrogen (ppm)
Base metal	0.22	0.77	0.25	0.03	0.02	-	350	50
Filler wire	0.10	0.56	0.05	0.02	0.01	-	380	60

Source: Kanjilal et al. (2004, 2005, 2006, 2007a and 2007b

Table 3.3 Flux Formulations Determined by Mixture Design

Case No.	Mixture Variables/Flux composition (wt %)				Fixed Proportion (FP) Ingredients				
	CaO	MgO	CaF_2	Al_2O_3	SiO_2	Fe-Mn	Fe-Si	Ni	Bentonite
C1	15.00	15.00	10.00	40.00	10.0	4.0	3.0	1.0	2.0
C2	15.00	15.00	40.00	10.00	10.0	4.0	3.0	1.0	2.0
C3	15.00	32.40	10.00	22.60	10.0	4.0	3.0	1.0	2.0
C4	15.00	17.00	40.00	8.00	10.0	4.0	3.0	1.0	2.0
C5	15.00	32.40	24.60	8.00	10.0	4.0	3.0	1.0	2.0
C6	35.00	15.00	10.00	20.00	10.0	4.0	3.0	1.0	2.0
C7	17.00	15.00	40.00	8.00	10.0	4.0	3.0	1.0	2.0
C8	35.00	15.00	22.00	8.00	10.0	4.0	3.0	1.0	2.0
C9	29.60	32.40	10.00	8.00	10.0	4.0	3.0	1.0	2.0
C10	35.00	27.00	10.00	8.00	10.0	4.0	3.0	1.0	2.0
C11	24.43	23.14	24.43	8.00	10.0	4.0	3.0	1.0	2.0
C12	15.67	15.67	40.00	8.66	10.0	4.0	3.0	1.0	2.0
C13	25.92	24.36	10.00	19.72	10.0	4.0	3.0	1.0	2.0
C14	23.40	15.00	24.40	17.20	10.0	4.0	3.0	1.0	2.0
C15	19.87	32.40	14.86	12.87	10.0	4.0	3.0	1.0	2.0
C16	15.00	22.36	24.92	17.72	10.0	4.0	3.0	1.0	2.0
C17	35.00	19.00	14.00	12.00	10.0	4.0	3.0	1.0	2.0
C18	22.67	21.63	21.63	14.07	10.0	4.0	3.0	1.0	2.0

Source: Kanjilal et al. (2004, 2005, 2006, 2007a and 2007b)

Table 3.4: Experimental Values for the Various Responses

Case No.	Microstructure					Chemical Composition					Mechanical properties			Element Transfer			
	AF (wt%)	FAS (wt%)	SPF (wt%)	PF (wt%)	GBF (wt%)	Mn (wt%)	Ni (wt%)	O_2 (ppm)	Si (wt%)	S (wt%)	YS (MPa)	UTS MPa	CIT at - 20°C (J)	ΔMn in wt%	ΔO_2 in ppm	(ΔSi in wt%)	(ΔS in wt%)
C1	13	4	19	27	37	0.560	0.21	560	0.340	0.042	254	464	8.8	-0.08844	187.8	0.207	0.01472
C2	12	4	19	27	38	0.520	0.11	570	0.210	0.042	285	428	9.8	-0.13748	197.4	0.065	0.01499
C3	15	6	18	30	31	0.620	0.20	520	0.280	0.040	336	446	10.5	-0.03505	153.5	0.140	0.01415
C4	14	5	17	30	34	0.470	0.17	500	0.170	0.034	290	425	9.8	-0.18873	131.85	0.027	0.00899
C5	13	5	17	27	38	0.600	0.27	530	0.248	0.044	308	460	7.8	-0.05662	157.4	0.108	0.01931
C6	24	7	16	24	29	0.670	0.24	380	0.229	0.028	346	485	22.2	0.02138	8.4	0.092	0.00356
C7	16	5	20	25	34	0.488	0.32	490	0.270	0.040	298	431	13.7	-0.16789	117.8	0.126	0.01407
C8	19	5	16	29	31	0.580	0.29	480	0.200	0.028	305	458	14.4	-0.06937	113.7	0.064	0.00331
C9	28	6	17	20	29	0.690	0.23	330	0.260	0.027	346	455	16.7	0.04723	-31.3	0.132	0.00333
C10	16	5	19	29	31	0.540	0.31	480	0.193	0.034	316	456	14.7	-0.12466	109.5	0.043	0.00683
C11	35	7	14	20	24	0.700	0.50	300	0.120	0.021	382	474	26.0	0.04452	-60.4	-0.022	-0.00301
C12	26	6	15	24	29	0.601	0.34	350	0.150	0.037	326	455	15.8	-0.04369	-15.00	0.019	0.01022
C13	28	6	12	27	27	0.620	0.30	320	0.160	0.016	337	472	23.5	-0.03789	-41.5	0.017	-0.00768
C14	36	7	10	25	22	0.748	0.68	300	0.258	0.031	361	496	25.5	0.08902	-58.9	0.112	0.00474
C15	35	8	14	18	25	0.800	0.59	320	0.370	0.020	397	535	24.1	0.14669	-49.8	0.228	-0.0031
C16	10	4	21	31	34	0.507	0.50	600	0.200	0.024	286	385	9.1	-0.15139	226.8	0.060	-0.00125
C17	20	6	14	28	32	0.595	0.33	470	0.273	0.015	294	450	14.2	-0.05253	106.4	0.139	-0.00811
C18	16	4	19	28	33	0.517	0.29	540	0.160	0.023	285	435	11.6	-0.13447	168.2	0.026	-0.00219

Source: Kanjilal et al. (2004, 2005, 2006, 2007a and 2007b)

Table 3.5 Response Equations for the Chemical Elements in the Weld Deposit

Element (wt%)	Response Equations
Manganese content	$f_{Mn}(x) = 0.028359x_{CaO} + 0.111599x_{MgO} + 0.014723x_{CaF_2} - 0.020999x_{Al_2O_3}$ $- 0.003890x_{CaO}x_{MgO} - 0.001485x_{CaO}x_{CaF_2} + 0.000662x_{CaO}x_{Al_2O_3} - 0.003692x_{MgO}x_{CaF_2}$ $- 0.001819x_{MgO}x_{Al_2O_3} + 0.004574x_{CaF_2}x_{Al_2O_3} + 0.000173x_{CaO}x_{MgO}x_{CaF_2}$ $+ 0.000059x_{CaO}x_{MgO}x_{Al_2O_3} - 0.000086x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.000123x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Silicon content	$f_{Si}(x) = 0.071547x_{CaO} + 0.117478x_{MgO} + 0.070621x_{CaF_2} + 0.087037x_{Al_2O_3}$ $- 0.004424x_{CaO}x_{MgO} - 0.003332x_{CaO}x_{CaF_2} - 0.003846x_{CaO}x_{Al_2O_3} - 0.004678x_{MgO}x_{CaF_2}$ $- 0.005156x_{MgO}x_{Al_2O_3} - 0.004499x_{CaF_2}x_{Al_2O_3} + 0.00009x_{CaO}x_{MgO}x_{CaF_2}$ $+ 0.000101x_{CaO}x_{MgO}x_{Al_2O_3} + 0.00013x_{CaO}x_{CaF_2}x_{Al_2O_3} + 0.000138x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Sulphur content	$f_S(x) = -0.002898x_{CaO} - 0.004501x_{MgO} - 0.003869x_{CaF_2} - 0.006079x_{Al_2O_3}$ $+ 0.000268x_{CaO}x_{MgO} + 0.000202x_{CaO}x_{CaF_2} + 0.000268x_{CaO}x_{Al_2O_3} + 0.000354x_{MgO}x_{CaF_2}$ $+ 0.000446x_{MgO}x_{Al_2O_3} + 0.000302x_{CaF_2}x_{Al_2O_3} - 0.000012x_{CaO}x_{MgO}x_{CaF_2}$ $- 0.000015x_{CaO}x_{MgO}x_{Al_2O_3} - 0.000002x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.000016x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Oxygen content	$f_{O_2}(x) = 60.3255x_{CaO} - 15.8052x_{MgO} + 23.7717x_{CaF_2} + 51.1951x_{Al_2O_3}$ $- 0.4345x_{CaO}x_{MgO} - 1.5757x_{CaO}x_{CaF_2} - 2.9029x_{CaO}x_{Al_2O_3} + 0.6963x_{MgO}x_{CaF_2}$ $- 0.2584x_{MgO}x_{Al_2O_3} - 3.2749x_{CaF_2}x_{Al_2O_3} - 0.0429x_{CaO}x_{MgO}x_{CaF_2}$ $+ 0.0003x_{CaO}x_{MgO}x_{Al_2O_3} + 0.0829x_{CaO}x_{CaF_2}x_{Al_2O_3} + 0.1065x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Nickel content	$f_{Ni}(x) = -0.0776x_{CaO} + 0.0556x_{MgO} - 0.0181x_{CaF_2} - 0.0058x_{Al_2O_3} + 0.0006x_{CaO}x_{MgO} +$ $0.0030x_{CaO}x_{CaF_2} + 0.0026x_{CaO}x_{Al_2O_3} - 0.0015x_{MgO}x_{CaF_2} - 0.0018x_{MgO}x_{Al_2O_3} +$ $0.0004x_{CaF_2}x_{Al_2O_3}$

Table 3.6 Response Equations for the Microstructure of the Weld Deposit

Microstructure (%)	Response Equation
Acicular Ferrite (AF)	$f_{AF}(x) = -1.63120x_{CaO} + 5.22221x_{MgO} + 0.47687x_{CaF_2} - 2.31415x_{Al_2O_3}$ $-0.13508x_{CaO}x_{MgO} - 0.01367x_{CaO}x_{CaF_2} + 0.13476x_{CaO}x_{Al_2O_3} - 0.22026x_{MgO}x_{CaF_2}$ $-0.08320x_{MgO}x_{Al_2O_3} + 0.26751x_{CaF_2}x_{Al_2O_3} + 0.01086x_{CaO}x_{MgO}x_{CaF_2}$ $+0.00328x_{CaO}x_{MgO}x_{Al_2O_3} - 0.0053x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.00794x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Grain Boundary Ferrite (GBF)	$f_{GBF}(x) = 3.033838x_{CaO} + 0.716844x_{MgO} + 0.857262x_{CaF_2} + 2.732242x_{Al_2O_3}$ $-0.066845x_{CaO}x_{MgO} - 0.059322x_{CaO}x_{CaF_2} - 0.144496x_{CaO}x_{Al_2O_3} + 0.025593x_{MgO}x_{CaF_2}$ $-0.078667x_{MgO}x_{Al_2O_3} - 0.070960x_{CaF_2}x_{Al_2O_3} - 0.001620x_{CaO}x_{MgO}x_{CaF_2}$ $+0.003143x_{CaO}x_{MgO}x_{Al_2O_3} + 0.001232x_{CaO}x_{CaF_2}x_{Al_2O_3} + 0.002198x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Polygonal Ferrite (PF)	$f_{PF}(x) = 0.40509x_{CaO} - 4.59375x_{MgO} - 1.45476x_{CaF_2} + 054702x_{Al_2O_3}$ $+0.14904x_{CaO}x_{MgO} + 0.06569x_{CaO}x_{CaF_2} - 0.07553x_{CaO}x_{Al_2O_3} + 0.24583x_{MgO}x_{CaF_2}$ $+0.14728x_{MgO}x_{Al_2O_3} - 0.07209x_{CaF_2}x_{Al_2O_3} - 0.00881x_{CaO}x_{MgO}x_{CaF_2}$ $-0.00143x_{CaO}x_{MgO}x_{Al_2O_3} + 0.0047x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.00100x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Ferrite with Aligned Second Phase (FAS)	$f_{FAS}(x) = 0.233493x_{CaO} + 0.985959x_{MgO} + 0.50953x_{CaF_2} - 0.181183x_{Al_2O_3}$ $-0.033559x_{CaO}x_{MgO} - 0.026052x_{CaO}x_{CaF_2} + 0.011146x_{CaO}x_{Al_2O_3} - 0.044696x_{MgO}x_{CaF_2}$ $-0.010883x_{MgO}x_{Al_2O_3} + 0.008816x_{CaF_2}x_{Al_2O_3} + 0.001915x_{CaO}x_{MgO}x_{CaF_2}$ $+0.000074x_{CaO}x_{MgO}x_{Al_2O_3} + 0.00007x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.0000507x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Side Plate Ferrite (SPF)	$f_{SPF}(x) = -0.791224x_{CaO} + 0.981259x_{MgO} + 0.861090x_{CaF_2} + 0.466069x_{Al_2O_3} + 0.0864$ $+0.033355x_{CaO}x_{CaF_2} + 0.074115x_{CaO}x_{Al_2O_3} - 0.006464x_{MgO}x_{CaF_2} + 0.025464x_{MgO}x_{Al_2O_3}$ $-0.133284x_{CaF_2}x_{Al_2O_3} - 0.002351x_{CaO}x_{MgO}x_{CaF_2} - 0.005066x_{CaO}x_{MgO}x_{Al_2O_3} - 0.00070$ $+0.007246_{MgO}x_{CaF_2}x_{Al_2O_3}$

Table 3.7 Response Equations for the Mechanical Properties of the Weld Deposit

Mechanical Property	Response Equations
Yield Strength (YS in MPa)	$f_{YS}(x) = -7.1772x_{CaO} + 22.7137x_{MgO} + 1.4682x_{CaF_2} - 22.8876x_{Al_2O_3}$ $- 0.4110x_{CaO}x_{MgO} + 0.0930x_{CaO}x_{CaF_2} + 1.3418x_{CaO}x_{Al_2O_3} - 0.8539x_{MgO}x_{CaF_2}$ $+ 0.3202x_{MgO}x_{Al_2O_3} + 1.76851x_{CaF_2}x_{Al_2O_3} + 0.0472x_{CaO}x_{MgO}x_{CaF_2}$ $- 0.0148x_{CaO}x_{MgO}x_{Al_2O_3} - 0.0491x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.0424x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Ultimate Tensile Strength (UTS in MPa)	$f_{UTS}(x) = 23.57549x_{CaO} + 46.46474x_{MgO} + 19.36357x_{CaF_2} + 18.49687x_{Al_2O_3}$ $- 1.71678x_{CaO}x_{MgO} - 1.02147x_{CaO}x_{CaF_2} - 0.84772x_{CaO}x_{Al_2O_3} - 1.46229x_{MgO}x_{CaF_2}$ $- 1.39665x_{MgO}x_{Al_2O_3} + 0.07021x_{CaF_2}x_{Al_2O_3} + 0.05751x_{CaO}x_{MgO}x_{CaF_2}$ $+ 0.04834x_{CaO}x_{MgO}x_{Al_2O_3} + 0.01672x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.01790x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Charpy Impact Strength (CIT in joules)	$f_{CIT}(x) = -2.16457x_{CaO} + 2.10089x_{MgO} - 0.85696x_{CaF_2} - 3.36851x_{Al_2O_3}$ $- 0.03716x_{CaO}x_{MgO} + 0.05330x_{CaO}x_{CaF_2} + 0.18269x_{CaO}x_{Al_2O_3} - 0.09231x_{MgO}x_{CaF_2}$ $+ 0.01431x_{MgO}x_{Al_2O_3} + 0.29284x_{CaF_2}x_{Al_2O_3} + 0.00638x_{CaO}x_{MgO}x_{CaF_2}$ $+ 0.00101x_{CaO}x_{MgO}x_{Al_2O_3} - 0.00800x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.00724x_{MgO}x_{CaF_2}x_{Al_2O_3}$

Table 3.8 Response Equations for the Element Transferred into the Weld Deposit

Element Transferred	Response Equation
Manganese Transfer (ΔMn) in wt%	$f_{\Delta Mn}(x) = 0.021356x_{CaO} + 0.109620x_{MgO} + 0.010687x_{CaF_2} - 0.025374x_{Al_2O_3}$ $- 0.004065x_{CaO}x_{MgO} - 0.001581x_{CaO}x_{CaF_2} + 0.000632x_{CaO}x_{Al_2O_3} - 0.004021x_{MgO}x_{CaF_2}$ $- 0.002108x_{MgO}x_{Al_2O_3} + 0.004270x_{CaF_2}x_{Al_2O_3} + 0.000179x_{CaO}x_{MgO}x_{CaF_2}$ $+ 0.000061x_{CaO}x_{MgO}x_{Al_2O_3} - 0.000084x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.000110x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Oxygen Transfer (ΔO_2) in ppm	$f_{\Delta O_2}(x) = 53.6875x_{CaO} - 22.7569x_{MgO} + 16.2158x_{CaF_2} + 43.1139x_{Al_2O_3}$ $- 0.3489x_{CaO}x_{MgO} - 1.4801x_{CaO}x_{CaF_2} - 2.8283x_{CaO}x_{Al_2O_3} + 0.8400x_{MgO}x_{CaF_2}$ $- 0.0736x_{MgO}x_{Al_2O_3} - 2.9697x_{CaF_2}x_{Al_2O_3} - 0.0435x_{CaO}x_{MgO}x_{CaF_2}$ $+ 0.0001x_{CaO}x_{MgO}x_{Al_2O_3} + 0.0802x_{CaO}x_{CaF_2}x_{Al_2O_3} + 0.0926x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Silicon Transfer (ΔSi) in wt%	$f_{\Delta Si}(x) = 0.069361x_{CaO} + 0.118695x_{MgO} + 0.071856x_{CaF_2} + 0.089078x_{Al_2O_3}$ $- 0.004455x_{CaO}x_{MgO} - 0.003347x_{CaO}x_{CaF_2} - 0.003848x_{CaO}x_{Al_2O_3} - 0.004871x_{MgO}x_{CaF_2}$ $- 0.005363x_{MgO}x_{Al_2O_3} - 0.004876x_{CaF_2}x_{Al_2O_3} + 0.000090x_{CaO}x_{MgO}x_{CaF_2}$ $+ 0.000099x_{CaO}x_{MgO}x_{Al_2O_3} + 0.000134x_{CaO}x_{CaF_2}x_{Al_2O_3} + 0.000153x_{MgO}x_{CaF_2}x_{Al_2O_3}$
Sulphur Transfer (ΔS) in wt%	$f_{\Delta S}(x) = -0.002189x_{CaO} - 0.002910x_{MgO} - 0.003108x_{CaF_2} - 0.003501x_{Al_2O_3}$ $+ 0.000186x_{CaO}x_{MgO} + 000154x_{CaO}x_{CaF_2} + 0.000180x_{CaO}x_{Al_2O_3} + 0.000269x_{MgO}x_{CaF_2}$ $+ 0.000304x_{MgO}x_{Al_2O_3} + 0.000204x_{CaF_2}x_{Al_2O_3} - 0.000010x_{CaO}x_{MgO}x_{CaF_2}$ $- 0.000012x_{CaO}x_{MgO}x_{Al_2O_3} - 0.000001x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.000013x_{MgO}x_{CaF_2}x_{Al_2O_3}$

3.5.3 Optimisation Model

3.5.3.1 Variables:

$x_{CaO}, x_{MgO}, x_{CaF_2}$ and $x_{Al_2O_3}$ represent the respective weight percent of CaO, MgO, CaF_2 and Al_2O_3 in the flux mixture.

3.5.3.2 Constraints:

The constraints of the model are the lower and upper limits of the flux ingredients. These constraints define the flux mixture space. From Kanjilal et al (2004, 2005, 2006, 2007a and 2007b) the lower and upper limits on the flux ingredients are (Table 3.3):

$$15 \leq x_{CaO} \leq 35 \tag{3.5}$$

$$10 \leq x_{MgO} \leq 32.40 \tag{3.6}$$

$$10 \leq x_{CaF_2} \leq 40 \tag{3.7}$$

$$8 \leq x_{Al_2O_3} \leq 40 \tag{3.8}$$

There is an additional constraint that the proportions of these ingredients must sum up to 80%. The balance (20%) consists of SiO_2, Fe-Mn, Ni and bentonite all of them with fixed compositions throughout the experiments (Table 3.3).

$$x_{CaO} + x_{MgO} + x_{CaF_2} + x_{Al_2O_3} = 80 \tag{3.9}$$

3.5.4 Determination of Attainable set/ Feasible Criterion Space (FCS)

The FCS $\{f_i^{\min}(x), f_i^{\max}(x)\}$ for the flux formulation problem was determined according to procedure described in section 3.3.1. There were 17 responses/quality characteristics considered. For manganese content in the weld deposit, the $\{f_{Mn}^{\min}(x), f_{Mn}^{\max}(x)\}$ values were determined by solving equations (3.10 and 3.11) below;

$$\begin{aligned} Minimize\, f_{\mathrm{Mn}}(x) = {} & 0.028359x_{\mathrm{CaO}} + 0.111599x_{\mathrm{MgO}} + 0.014723x_{\mathrm{CaF_2}} - 0.020999x_{\mathrm{Al_2O_3}} \\ & - 0.003890x_{\mathrm{CaO}}x_{\mathrm{MgO}} - 0.001485x_{\mathrm{CaO}}x_{\mathrm{CaF_2}} + 0.000662x_{\mathrm{CaO}}x_{\mathrm{Al_2O_3}} - 0.003692x_{\mathrm{MgO}}x_{\mathrm{CaF_2}} \\ & - 0.001819x_{\mathrm{MgO}}x_{\mathrm{Al_2O_3}} + 0.004574x_{\mathrm{CaF_2}}x_{\mathrm{Al_2O_3}} + 0.000173x_{CaO}x_{MgO}x_{CaF_2} \\ & + 0.000059x_{CaO}x_{MgO}x_{Al_2O_3} - 0.000086x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.000123x_{MgO}x_{CaF_2}x_{Al_2O_3} \end{aligned}$$

subject to; (3.10)

Constraints (3.5)-(3.9)

And

$$\begin{aligned} Maximize\, f_{\mathrm{Mn}}(x) = {} & 0.028359x_{\mathrm{CaO}} + 0.111599x_{\mathrm{MgO}} + 0.014723x_{\mathrm{CaF_2}} - 0.020999x_{\mathrm{Al_2O_3}} \\ & - 0.003890x_{\mathrm{CaO}}x_{\mathrm{MgO}} - 0.001485x_{\mathrm{CaO}}x_{\mathrm{CaF_2}} + 0.000662x_{\mathrm{CaO}}x_{\mathrm{Al_2O_3}} - 0.003692x_{\mathrm{MgO}}x_{\mathrm{CaF_2}} \\ & - 0.001819x_{\mathrm{MgO}}x_{\mathrm{Al_2O_3}} + 0.004574x_{\mathrm{CaF_2}}x_{\mathrm{Al_2O_3}} + 0.000173x_{CaO}x_{MgO}x_{CaF_2} \\ & + 0.000059x_{CaO}x_{MgO}x_{Al_2O_3} - 0.000086x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.000123x_{MgO}x_{CaF_2}x_{Al_2O_3} \end{aligned}$$

subject to; (3.11)
Constraints (3.5-3.9)

Similarly, the $\{f_i^{\min}(x), f_i^{\max}(x)\}$ for the remaining 16 responses (Tables 3.5, 3.6, 3.7 and 3.8) were determined.

3.5.5 Nominal-The-Better Flux Design Situations

The WFD desired a welding flux that will deposit a weld-metal with the composition in Table 3.9 below (Adeyeye and Oyawale, 2010b).

Table 3.9: Desired Weld-metal Composition

Element	Mn (%)	Si (%)	S (%)	O_2 (ppm)	Ni (%)
Amount desired	$= 0.760$	≥ 0.200	≤ 0.035	$250 - 350$	$= 0.460$

Source: Adeyeye and Oyawale, 2010b

Two welding flux design situations were considered, namely;

(i) All deviations were of equal importance to the WFD
(ii) Some deviations were of greater concern to the WFD than others

Case 1: All deviations were of equal concern to the WFD hence all the deviations were assigned equal weights. The goal constraints of the problem may be as stated below using Tables 3.5 and 3.9 (Adeyeye and Oyawale, 2010b);

(Manganese content goal constraint)

$$\begin{aligned}
&0.028359x_{CaO} + 0.111599x_{MgO} + 0.014723x_{CaF_2} - 0.020999x_{Al_2O_3} \\
&- 0.003890x_{CaO}x_{MgO} - 0.001485x_{CaO}x_{CaF_2} + 0.000662x_{CaO}x_{Al_2O_3} - 0.003692x_{MgO}x_{CaF_2} \\
&- 0.001819x_{MgO}x_{Al_2O_3} + 0.004574x_{CaF_2}x_{Al_2O_3} + 0.000173x_{CaO}x_{MgO}x_{CaF_2} \\
&+ 0.000059x_{CaO}x_{MgO}x_{Al_2O_3} - 0.000086x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.000123x_{MgO}x_{CaF_2}x_{Al_2O_3} \\
&+ d_{mn}^- - d_{mn}^+ = 0.760
\end{aligned} \tag{3.12}$$

(Silicon content goal constraint)

$$\begin{aligned}
&f_{Si}(x) = 0.071547x_{CaO} + 0.117478x_{MgO} + 0.070621x_{CaF_2} + 0.087037x_{Al_2O_3} \\
&- 0.004424x_{CaO}x_{MgO} - 0.003332x_{CaO}x_{CaF_2} - 0.003846x_{CaO}x_{Al_2O_3} - 0.004678x_{MgO}x_{CaF_2} \\
&- 0.005156x_{MgO}x_{Al_2O_3} - 0.004499x_{CaF_2}x_{Al_2O_3} + 0.00009x_{CaO}x_{MgO}x_{CaF_2} \\
&+ 0.000101x_{CaO}x_{MgO}x_{Al_2O_3} + 0.00013x_{CaO}x_{CaF_2}x_{Al_2O_3} + 0.000138x_{MgO}x_{CaF_2}x_{Al_2O_3} \\
&+ d_{Si}^- - d_{Si}^+ = 0.200
\end{aligned} \tag{3.13}$$

(Sulphur content goal constraint)

$$\begin{aligned}
&f_S(x) = -0.002898x_{CaO} - 0.004501x_{MgO} - 0.003869x_{CaF_2} - 0.006079x_{Al_2O_3} \\
&+ 0.000268x_{CaO}x_{MgO} + 0.000202x_{CaO}x_{CaF_2} + 0.000268x_{CaO}x_{Al_2O_3} + 0.000354x_{MgO}x_{CaF_2} \\
&+ 0.000446x_{MgO}x_{Al_2O_3} + 0.000302x_{CaF_2}x_{Al_2O_3} - 0.000012x_{CaO}x_{MgO}x_{CaF_2} \\
&- 0.000015x_{CaO}x_{MgO}x_{Al_2O_3} - 0.000002x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.000016x_{MgO}x_{CaF_2}x_{Al_2O_3} \\
&+ d_S^- - d_S^+ = 0.035
\end{aligned} \tag{3.14}$$

(Oxygen content lower bound goal constraint)

$$\begin{aligned}
&60.3255x_{CaO} - 15.8052x_{MgO} + 23.7717x_{CaF_2} + 51.1951x_{Al_2O_3} \\
&- 0.4345x_{CaO}x_{MgO} - 1.5757x_{CaO}x_{CaF_2} - 2.9029x_{CaO}x_{Al_2O_3} + 0.6963x_{MgO}x_{CaF_2} \\
&- 0.2584x_{MgO}x_{Al_2O_3} - 3.2749x_{CaF_2}x_{Al_2O_3} - 0.0429x_{CaO}x_{MgO}x_{CaF_2} \\
&+ 0.0003x_{CaO}x_{MgO}x_{Al_2O_3} + 0.0829x_{CaO}x_{CaF_2}x_{Al_2O_3} + 0.1065x_{MgO}x_{CaF_2}x_{Al_2O_3} \\
&+ d_{LO_2}^{-} - d_{LO_2}^{+} = 250
\end{aligned} \tag{3.15}$$

(Oxygen content upper bound goal constraint)

$$\begin{aligned}
&f_{O_2}(x) = 60.3255x_{CaO} - 15.8052x_{MgO} + 23.7717x_{CaF_2} + 51.1951x_{Al_2O_3} \\
&- 0.4345x_{CaO}x_{MgO} - 1.5757x_{CaO}x_{CaF_2} - 2.9029x_{CaO}x_{Al_2O_3} + 0.6963x_{MgO}x_{CaF_2} \\
&- 0.2584x_{MgO}x_{Al_2O_3} - 3.2749x_{CaF_2}x_{Al_2O_3} - 0.0429x_{CaO}x_{MgO}x_{CaF_2} \\
&+ 0.0003x_{CaO}x_{MgO}x_{Al_2O_3} + 0.0829x_{CaO}x_{CaF_2}x_{Al_2O_3} + 0.1065x_{MgO}x_{CaF_2}x_{Al_2O_3} \\
&+ d_{UO_2}^{-} - d_{UO_2}^{+} = 350
\end{aligned} \tag{3.16}$$

(Nickel content goal constraint)

$$\begin{aligned}
&- 0.0776x_{CaO} + 0.0556x_{MgO} - 0.0181x_{CaF_2} - 0.0058x_{Al_2O_3} + 0.0006x_{CaO}x_{MgO} + \\
&0.0030x_{CaO}x_{CaF_2} + 0.0026x_{CaO}x_{Al_2O_3} - 0.0015x_{MgO}x_{CaF_2} - 0.0018x_{MgO}x_{Al_2O_3} + \\
&0.0004x_{CaF_2}x_{Al_2O_3} + d_{Ni}^{-} - d_{Ni}^{+} = 0.460
\end{aligned}$$

From Table 3.9, it was required that the Mn content be 0.760% therefore deviations above and below the target were unwanted and must be minimised. Si content above 0.200% was acceptable to the WFD but values below it were not desirable and must be minimised. Sulphur content in the weld must not exceed 0.035%, therefore deviations above this value was minimised. For oxygen content deviations above the upper limit (350 ppm) and below the lower limit (250 ppm) were minimised while for Ni both the positive and negative deviations were minimised. Since all the weights associated with the deviation were equal and they must sum up to one, the weight assigned to each deviation variable was 0.125. The complete NGP model to minimise the Archimedian sum of the unwanted deviations was;

$$\begin{aligned}
&Minimise, asum = 0.125d_{Mn}^{-} + 0.125d_{Mn}^{+} + 0.125d_{Si}^{-} + 0.125d_{S}^{+} + 0.125d_{LO_2}^{-} + 0.125d_{UO_2}^{+} \\
&+ 0.125d_{Ni}^{-} + 0.125d_{Ni}^{+}
\end{aligned}$$

subject to;

Goal constraints (the constraints (3.12-3.16)) i.e.

(Manganese content goal constraint)

$$\begin{aligned}
&0.028359x_{CaO} + 0.111599x_{MgO} + 0.014723x_{CaF_2} - 0.020999x_{Al_2O_3} \\
&- 0.003890x_{CaO}x_{MgO} - 0.001485x_{CaO}x_{CaF_2} + 0.000662x_{CaO}x_{Al_2O_3} - 0.003692x_{MgO}x_{CaF_2} \\
&- 0.001819x_{MgO}x_{Al_2O_3} + 0.004574x_{CaF_2}x_{Al_2O_3} + 0.000173x_{CaO}x_{MgO}x_{CaF_2} \\
&+ 0.000059x_{CaO}x_{MgO}x_{Al_2O_3} - 0.000086x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.000123x_{MgO}x_{CaF_2}x_{Al_2O_3} \\
&+ d_{mn}^{-} - d_{mn}^{+} = 0.760
\end{aligned}$$

(Silicon content goal constraint)

$$f_{Si}(x) = 0.071547x_{CaO} + 0.117478x_{MgO} + 0.070621x_{CaF_2} + 0.087037x_{Al_2O_3}$$
$$-0.004424x_{CaO}x_{MgO} - 0.003332x_{CaO}x_{CaF_2} - 0.003846x_{CaO}x_{Al_2O_3} - 0.004678x_{MgO}x_{CaF_2}$$
$$-0.005156x_{MgO}x_{Al_2O_3} - 0.004499x_{CaF_2}x_{Al_2O_3} + 0.00009x_{CaO}x_{MgO}x_{CaF_2}$$
$$+0.000101x_{CaO}x_{MgO}x_{Al_2O_3} + 0.00013x_{CaO}x_{CaF_2}x_{Al_2O_3} + 0.000138x_{MgO}x_{CaF_2}x_{Al_2O_3}$$
$$+d_{Si}^{-} - d_{Si}^{+} = 0.200$$

(Sulphur content goal constraint)

$$f_S(x) = -0.002898x_{CaO} - 0.004501x_{MgO} - 0.003869x_{CaF_2} - 0.006079x_{Al_2O_3}$$
$$+0.000268x_{CaO}x_{MgO} + 0.000202x_{CaO}x_{CaF_2} + 0.000268x_{CaO}x_{Al_2O_3} + 0.000354x_{MgO}x_{CaF_2}$$
$$+0.000446x_{MgO}x_{Al_2O_3} + 0.000302x_{CaF_2}x_{Al_2O_3} - 0.000012x_{CaO}x_{MgO}x_{CaF_2}$$
$$-0.000015x_{CaO}x_{MgO}x_{Al_2O_3} - 0.000002x_{CaO}x_{CaF_2}x_{Al_2O_3} - 0.000016x_{MgO}x_{CaF_2}x_{Al_2O_3}$$
$$+d_S^{-} - d_S^{+} = 0.035$$

(Oxygen content lower bound goal constraint)

$$60.3255x_{CaO} - 15.8052x_{MgO} + 23.7717x_{CaF_2} + 51.1951x_{Al_2O_3}$$
$$-0.4345x_{CaO}x_{MgO} - 1.5757x_{CaO}x_{CaF_2} - 2.9029x_{CaO}x_{Al_2O_3} + 0.6963x_{MgO}x_{CaF_2}$$
$$-0.2584x_{MgO}x_{Al_2O_3} - 3.2749x_{CaF_2}x_{Al_2O_3} - 0.0429x_{CaO}x_{MgO}x_{CaF_2}$$
$$+0.0003x_{CaO}x_{MgO}x_{Al_2O_3} + 0.0829x_{CaO}x_{CaF_2}x_{Al_2O_3} + 0.1065x_{MgO}x_{CaF_2}x_{Al_2O_3}$$
$$+d_{LO_2}^{-} - d_{LO_2}^{+} = 250$$

(Oxygen content upper bound goal constraint)

$$f_{O_2}(x) = 60.3255x_{CaO} - 15.8052x_{MgO} + 23.7717x_{CaF_2} + 51.1951x_{Al_2O_3}$$
$$-0.4345x_{CaO}x_{MgO} - 1.5757x_{CaO}x_{CaF_2} - 2.9029x_{CaO}x_{Al_2O_3} + 0.6963x_{MgO}x_{CaF_2}$$
$$-0.2584x_{MgO}x_{Al_2O_3} - 3.2749x_{CaF_2}x_{Al_2O_3} - 0.0429x_{CaO}x_{MgO}x_{CaF_2}$$
$$+0.0003x_{CaO}x_{MgO}x_{Al_2O_3} + 0.0829x_{CaO}x_{CaF_2}x_{Al_2O_3} + 0.1065x_{MgO}x_{CaF_2}x_{Al_2O_3}$$
$$+d_{UO_2}^{-} - d_{UO_2}^{+} = 350$$

(Nickel content goal constraint)

$$-0.0776x_{CaO} + 0.0556x_{MgO} - 0.0181x_{CaF_2} - 0.0058x_{Al_2O_3} + 0.0006x_{CaO}x_{MgO} +$$
$$0.0030x_{CaO}x_{CaF_2} + 0.0026x_{CaO}x_{Al_2O_3} - 0.0015x_{MgO}x_{CaF_2} - 0.0018x_{MgO}x_{Al_2O_3} +$$
$$0.0004x_{CaF_2}x_{Al_2O_3} + d_{Ni}^{-} - d_{Ni}^{+} = 0.460$$

Technological/Structural constraints (the constraints (3.5-3.9)):

$$x_{CaO} + x_{MgO} + x_{CaF_2} + x_{Al_2O_3} = 80 \qquad (3.17)$$
$$x_{CaO} \geq 15$$
$$x_{CaO} \leq 35$$
$$x_{MgO} \geq 15$$
$$x_{MgO} \leq 32.40$$
$$x_{CaF_2} \geq 10$$

$x_{CaF_2} \leq 40$

$x_{Al_2O_3} \geq 8$

$x_{Al_2O_3} \leq 40$

Case 2: The WFD desired to achieve the same weld-metal content as in case 5 but the deviations were not of equal concern to him. Many methods exist by which the WFD may assign weights to the deviations to reflect his concern. In this study, the pairwise comparison method was used (see Table 3.10). The normalised scores/weights from Table 3.10 are; $u_{Mn} = 0.30$, $v_{Mn} = 0.06$, $u_{Si} = 0.17$, $v_S = 0.26$, $u_{LO_2} = v_{UO_2} = 0.20$, $u_{Ni} = 0.15$ and $v_{Ni} = 0.02$.

The NGP model is (Adeyeye and Oyawale 2010b);

$$Minimise, asum = 0.30d_{Mn}^{-} + 0.06d_{Mn}^{+} + 0.17d_{Si}^{-} + 0.26d_{S}^{+} + 0.02d_{LO_2}^{-} + 0.02d_{UO_2}^{+} + 0.15d_{Ni}^{-} + 0.02d_{Ni}^{+}$$

subject to; (3.18)

Same set of technological/structural and goal constraints as in (3.30) above

Table 3.10: Pairwise Comparison of Deviations for Weight Determination

Deviation	d_{Mn}^{-}	d_{Mn}^{+}	d_{Si}^{-}	d_{S}^{+}	$d_{LO_2}^{-}$	$d_{UO_2}^{+}$	d_{Ni}^{-}	d_{Ni}^{+}	Total Weight (t)	Normalised Weight $\left(\frac{t}{T}\right) = Weight$
d_{Mn}^{-}	–	2	2	2	2	2	2	2	14	0.30
d_{Mn}^{+}	0	–	0	0	1	1	0	1	3	0.06
d_{Si}^{-}	0	1	–	0	2	2	1	2	8	0.17
d_{S}^{+}	0	2	2	–	2	2	2	2	12	0.26
$d_{LO_2}^{-}$	0	0	0	0	–	0.5	0	0.5	1	0.02
$d_{UO_2}^{+}$	0	0	0	0	0.5	–	0	0.5	1	0.02
d_{Ni}^{-}	0	1	0	0	2	2	–	2	7	0.15
d_{Ni}^{+}	0	0	0	0	0.5	0.5	0	–	1	0.02
Grand Total (T)									47	1.00

KEY

WFD's Relative Concern	**Score**
High concern	2
Moderate Concern	1
Equal Concern	0.5
Less Concern	0

4.0 RESULTS AND DISCUSSION

4.1 Feasible Criterion Space

The boundaries of the attainable set/FCS, under the experimental domain are presented in Table 4.1. The models were solved with Lingo 12 software. Values of desired specifications that fall within the FCS are achievable under the prevailing conditions. The flux formulation that will achieve the desired response values can be determined by the development and solution of the appropriate single criterion optimisation models. The ability to establish feasibility is a great advantage of this methodology because of the cost savings in terms of time, labour, materials, and energy. In the case of the conventional welding flux design approach, the feasibility or otherwise of achieving the desired flux performance level is not easy to ascertain until a lot of time, labour and resources have been expended on try-and-test experiments. For the case under study, the attainable set was identified with only 18 experiments (Tables 3.3, 3.4 & 4.1). For instance the WFD need not waste time and resources on further experiments for him to know that YS above 410.58 MPa and sulphur content below 0.023% are not feasible under the prevailing conditions (Table 4.1). Once the required quality characteristics are identified and their desired values known, the WFD only needs to check to see if they fall within the FCS. If the desired quality specifications fall within the FCS, then the feasibility of achieving it has been established. The WFD can know how far the desired QC(s) deviate(s) from the FCS in situations where they do not fall within the FCS. This knowledge can serve as a useful guide for the WFD in determining the next line of action.

Table 4.1: The Boundary of the Feasible Criterion Space

S/N	Response/ Quality Characteristics	Lower Limit $\left[f_i^{\min}(x)\right]$	Upper Limit $\left[f_i^{\max}(x)\right]$
1	AF (%)	4.86	39.35
2	FAS (%)	3.56	7.90
3	SPF (%)	9.75	23.89
4	PF (%)	12.98	37.18
5	GBF (%)	23.98	37.25
6	Mn (wt%)	0.3968	0.8965
7	Ni (wt%)	0.1044	0.6150
8	O_2 (ppm)	249.39	655
9	Si (wt%)	0.0822	0.3771
10	S (wt%)	0.0230	0.0522
11	YS (MPa)	253.93	410.58
12	UTS (MPa)	381.02	526.32
13	CIT at -20^{0}C (J)	4.62	28.28
14	ΔMn (wt%)	-0.2662	0.2287
15	ΔO_2 (ppm)	-117.96	335.41
16	ΔSi (wt%)	-0.0700	0.2349
17	ΔS (wt%)	-0.00942	0.01853

4.2 Results of NTB Flux Design Situations

The NGP model solutions are presented in Table 4.2. Lingo software was used to solve the problems. Without any further experiments the flux levels that will give the best balance among the various chemical elements were established. In case 1, where all deviations were equally weighted, the flux formulation that gave the best balance among the goals was 21.44% CaO, 32.40% MgO, 10.00% CaF_2 and 15.1% Al_2O_3 (Table 4.2). Mn was underachieved by 0.007 which is 0.92% of the 0.760 target while Ni content was achieved in strict equality sense as desired. The Si and S contents were also at the acceptable levels. For case 2 where the deviations are of different concerns to the WFD, the flux formulation that achieved the best balance was 22.47% CaO, 32.40% MgO, 10.00% CaF_2 and 16.16% Al_2O_3

(Table 4.2). Mn and Ni content targets were achieved without deviations. In case 6 the concern of the WFD for underachievement (negative deviation) for Mn was higher hence the weight of 0.3 was assigned compared to 0.125 in case 1. With the higher weight the target value was achieved unlike case 1 where there was an underachievement. Si and S contents were at acceptable levels.

The NGP approach provides flexibility to the WFD, who is able to use different weight structures for the deviations from the targets to explore various trade-off options before choosing the one that best meets his needs. Apart from weld-metal chemical composition optimisation, NGP method may also be useful in other multiple design attributes welding-flux design situations. The determination of welding-flux ingredient levels that will achieve the desired values of acicular ferrite, polygonal ferrite, bainite and grain boundary ferrite contents in the weld-metal microstructure, or give the desired balance among mechanical properties such as yield strength, tensile strength, Charpy impact strength, hardness and elongation are such examples.

Table 4.2: Results of NTB Flux Design Situations **(Cases 1 and 2)**

Elements	Target Values	NGP Model Values		Deviations						Best Compromise Welding Flux Formulation (%) from NGP Model	
		Case 5 (equal weights)	Case 6 (different weights)	Case 5			Case 6			Case 5	Case 6
				(-ve)	(+ve)	(%)	(-ve)	(+ve)	(%)		
Mn(%)	0.760	0.753	0.760	0.007	0.0	0.92	0.0	0.0	0.0	CaO (21.44%) MgO (32.40%) CaF_2 (10.00%) Al_2O_3 (16.16%)	CaO (22.47%) MgO (32.40%) CaF_2 (10.00%) Al_2O_3 (15.13%)
Si(%)	≥ 0.200	0.322	0.322	0.0		0.0	0.0		0.0		
S (%)	≤ 0.035	0.032	0.031		0.0			0.0			
O_2(ppm)	250-350	312	297								
Ni (%)	0.460	0.460	0.460	0.0	0.0	0.0	0.0	0.0	0.0		

5.0 Conclusions

The NGP approach for multi-response optimisation of weld-metal chemical composition from welding-flux ingredients is proposed. The major conclusions are:

- It is feasible for the WFD to simultaneously consider many mutually incompatible responses or objectives with the NGP method.
- If all the responses depend on the same set of predictor variables and the models that capture the relationship between the response and predictor variables can be assumed over the experimental domain, then the proposed methodology can be used to determine the best balance between the responses.

- The proposed methodology can be used to establish the feasible solution space and the feasibility or otherwise of achieving the desired performance level of the welding flux before a lot of resources are expended on experiments.
- The random character of the welding flux developed by traditional approach is eliminated because the NGP model ensures that the flux that gives the best balance between the objectives of the WFD is formulated.
- The WFD can use different weight structures to explore trade-off options before choosing the formulation that best suits his needs.

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Appendix A: Analysis of Variance (ANOVA) for Testing Adequacy of Response Equations

Response	Degree of Freedom			R^2	F statistic (Tabulated)	F-statistic (Calculated)	P significance	Remark
	Response	Residual	Total Adjusted					
$f_{Mn}(x)$	13	5	18	0.9955	4.66	85.40	0.0001	adequate
$f_{Si}(x)$	13	5	18	0.9838	4.66	23.36	0.0013	adequate
$f_{S}(x)$	13	5	18	0.9915	4.66	44.89	0.0003	adequate
$f_{O_2}(x)$	13	5	18	0.9851	4.66	25.45	0.0011	adequate
$f_{Ni}(x)$	10	8	18	0.9032	3.35	7.46	0.0045	adequate
$f_{AF}(x)$	13	5	18	0.9631	4.66	10.03	0.0095	adequate
$f_{GBF}(x)$	13	5	18	0.9934	4.66	58.14	0.0001	adequate
$f_{PF}(x)$	13	5	18	0.9979	4.66	179.62	0.0000	adequate
$f_{FAS}(x)$	13	5	18	0.9854	4.66	25.93	0.0010	adequate
$f_{SPF}(x)$	13	5	18	0.9920	4.66	47.41	0.0002	adequate
$f_{YS}(x)$	13	5	18	0.9963	4.66	104.63	0.0000	adequate
$f_{UTS}(x)$	13	5	18	0.9989	4.66	338.81	0.0000	adequate
$f_{CIT}(x)$	13	5	18	0.9647	4.66	10.53	0.0086	adequate
$f_{Mn}(x)$	13	5	18	0.8474	4.66	12.70	0.0021	Adequate
$f_{O_2}(x)$	13	5	18	0.8150	4.66	15.11	0.0029	adequate
$f_{ASi}(x)$	13	5	18	0.9321	4.66	5.28	0.0387	adequate
$f_{AS}(x)$	13	5	18	0.9402	4.66	6.04	0.0291	adequate

Appendix B: Paired-sample Sign Test

Appendix B1: Paired-Sample Sign Test: A Maximising Case

S/N	Quality Characteristics	Optimisation model Maximum (OPT)	Experimental Maximum (EXP)	Sign of the Difference
1	AF (%)	39.35	36.0	+
2	FAS (%)	7.90	8.0	-
3	SPF (%)	23.89	21.0	+
4	PF (%)	37.18	31.0	+
5	GBF (%)	37.25	38.0	-
6	Mn (wt%)	0.8965	0.80	+
7	Ni (wt%)	0.6150	0.68	-
8	O_2 (ppm)	655	600	+
9	Si (wt%)	0.3771	0.370	+
10	S (wt%)	0.0522	0.044	+
11	YS (MPa)	410.58	397	+
12	UTS (MPa)	526.32	525	equal
13	CIT at -20^0C (J)	28.28	26.0	+
14	ΔMn (wt%)	0.2287	0.1467	+
15	ΔO_2 (ppm)	335.41	227	+
16	ΔSi (wt%)	0.2349	0.228	+
17	ΔS (wt%)	0.0185	0.190	equal

Sample size (SS) =17, Pairs of equal rating (PER) =2, number of plus signs (N_+) = 12

Let the two independent groups (results of Optimisation and Experiments) be represented by OPT and EXP respectively.
Null hypothesis, H_0: OPT = EXP
Alternative hypothesis, H_1: OPT > EXP
Level of significance, α=0.05
Reduced sample size (RSS) = SS-PER=15

$$Z_{0.05} = \frac{N_+ - 0.5 - RSS(P_o)}{\sqrt{RSS(P_o)(1-P_o)}} = \frac{12 - 0.5 - 15(0.5)}{\sqrt{15(0.5)(1-0.5)}} = 2.066$$

Decision: since the calculated $Z_{0.05}(2.066)$ exceeds tabulated $Z_{0.05}(1.645)$, the null hypothesis must be rejected. In other words, the data support the claim that the optimisation models achieve higher values for the responses when maximised than the values from experiments.