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Analysis of Water Cresting in Horizontal Wells

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Abstract

Horizontal well application has sometimes been employed as a way of minimizing excessive water production arising from coning commonly encountered during oil production in vertical wells. Lots of efforts on water coning in vertical wells have been published. Available predictive models in horizontal wells vary from rather simplistic to complex models. This studt investigated the development of practical models that combine ease of use with accuracy.

Conformal mapping was used to combine steady state flow, volumetric voidage and pressure drop due to gravity effects in horizontal wells to obtain models that predict critical rates and breakthrough times. The results were compared with some existing correlations under varied reservoir fluid and rock properties. The models were also applied to vertical wells.

It was also observed that critical rates and breakthrough times in horizontal wells are affected directly by effective permeability, well length, oil column height, density contrast between water and oil, the height of the water crest. There is however, an inverse relationship with oil viscosity and production rate.

It is concluded that simple and accurate correlations that can be applied to coning problems in both horizontal and vertical wells have been developed. They provide a means of comparing the performance of horizontal and vertical wells.

Introduction and Review of literature

Excessive water production as result of coning and cresting has been a serious problem in the oil and gas industry. Coning or cresting are used to describe the mechanism underlying the upward movement of water and/or the downward movement of gas into the perforations of a producing well.

Efforts have been directed at understanding the mechanism of water coning/cresting in vertical and horizontal wells. These efforts have led to the development of correlations for estimating critical oil rate to avoid water coning/cresting, the time to water breakthrough at production rates above the critical rates, and prediction of water cut behaviour after breakthrough.

The works on coning/cresting problems can be divided into two main groups: steady-state and transient solutions. The first group determines critical rate and the latter correlations for breakthrough time and post-breakthrough behaviour.

Muskat and Wyckoff[1] published the first paper on this subject. They determined critical coning rate analytically by solving Laplace equation for single-phase flow and for a partially penetrating well.

Wheatley[2] in his study of partially penetrating wells observed that Muskat and Wyckoff relation over predicted the critical oil production rate because neglected the presence of the cone when calculating the pressure distribution in the reservoir. He also observed that the value of the well radius does not significantly affect the critical production rate.

Schol[3] studied a fully penetrating well. He developed an empirical correlation for vertical wells based on results obtained from numerical simulator and laboratory experiments.

Guo and Lee[4] demonstrated that the existence of the unstable water cone depends on the vertical pressure gradient beneath the wellbore. An important result of their study is that the critical rate does not occur at zero wellbore penetration, but at a wellbore penetration about one-third of the total oil zone thickness for an isotropic reservoir. Sobocinski and Cornelius[5] developed a dimensionless plot which traces the rise of the cone from its build-up to breakthrough from experimental and simulation results. Critical rate and breakthrough time can be determined from the plot.

Bournazel and Jeanson[6] from this plot developed a simple and fast analytical expression to estimate the critical rate and breakthrough time.
Meyer and Garder[7] extended the study to simultaneous gas and water coning. They also gave an expression for the critical rate when a shale lens was present between the well and the fluids contact.

Chierici and Ciuci[8] and Chaney et al.[9] used a potentiometric model to predict the coning behaviour in vertical wells. Chierici and Ciuci's results are presented in dimensionless graphs that take into account the vertical and horizontal permeability. They determined from the diagrams the maximum oil production rate without water and /or gas coning and the optimum position of the perforated interval. Chaney et al. also developed a set of working curves for determining critical oil rate applying Muskat and Wyckoff theory to oil-water, gas-oil and gas-water systems.

Hoyland et al.[10] presented two methods for predicting critical oil rate for bottom water coning in anisotropic, homogeneous formations with the well completed from the top of the formation. They presented an analytical method that is based on the Muskat and Wyckoff theory. In a steady-state flow condition, the solution takes a simple form when it is combined with the method of images to give the boundary conditions such that the top and bottom of the oil column are 'no flow boundaries' and the sides of the reservoir are constant pressure boundaries. To predict the critical rate, the authors superimpose the same criteria as those of Muskat and Wyckoff on the single-phase solution and therefore, neglect the influence of the cone shape on the potential distribution.

Their second method was based on a large number of simulation runs with more than fifty critical rate values. The authors used a regression analysis routine to develop critical rate correlations.

Chaperon[11] studied the behaviour of cresting towards horizontal wells in an anisotropic formation. She assumed a constant interface elevation at a finite distance. Her approach is identical to that of Muskat and would give an over optimistic value of the critical rate due to the negligence of the floe restriction due to the immobile water in the crest.

Giger[12] presented an analytical two-dimensional method to determine the shape of the deformed oil-water contact and the value of the critical rate at three different mechanism; (i) lateral edge drive (ii) gas cap drive and (iii) bottom water drive. He assumed that fluid displacement is pistonlike and that capillary effects are negligible.

Joshi[13] used the derived expression for critical rate of horizontal wells using an effective wellbore radius concept, and concluded that for any situation, the critical rate for horizontal well is higher than that for vertical well. Efros[14] proposed a critical rate that is based on the assumption that the critical rate is nearly independent of the drainage radius. His correlation does not account for the effect of vertical permeability. Karcher[15] proposed a correlation that produces a critical oil flow rate value similar to that of Efros'. Also, he did not account for the vertical permeability.

Papatzacos et al[16] developed correlations for breakthrough time for horizontal wells for both single cone and two cone cases in an infinite acting reservoir. Their solution was obtained by two methods. In the first method, it was assumed that either the top gas or bottom water can be represented as a constant pressure boundary. Their second method considered gravity equilibrium in the cone instead of assuming constant pressure boundary.

Ozkan and Raghavan[17] developed a similar approach to that used by Papatzacos et al. They investigated the time dependent performance of horizontal wells subject to bottom water drive. They assumed that the reservoir boundary at the top of the formation and the boundaries at the lateral extent of the formation to be impermeable, and that an active aquifer at the bottom of the reservoir would yield an effect identical to that of a constant pressure boundary located at the original oil-water contact. Furthermore, they assumed that the density difference between the oil and water is negligible. They graphically correlated the sweep efficiency with the dimensionless well length and dimensionless vertical distance.

Kuo and Desbrisay[18] applied the material balance equation to predict the rise in the oil-water contact in a homogeneous reservoir and correlated their numerical result in terms of dimensionless water cut, dimensionless breakthrough time and the dimensionless limiting water cut. Addington[19] also developed correlations using a radial grid for gas coning in the Prudhoe Bay field. He developed correlations to calculate the average oil column height above the perforations at breakthrough time and for post-breakthrough behaviour.

Yang and Wattenbarger[20] derived numerical correlations for both vertical and horizontal wells. They used the same definition as Addington and found correlations for breakthrough time, WOR, and critical rate for a particular time.

Mechanism of water coning

As oil is produced from the well, pressure gradient tend to elevate the water-oil contact in the immediate vicinity of the well. Counterbalancing the gradient is the tendency of the water to remain below the oil zone because of its higher density. This counterbalancing force tends to deform the water-oil contact into a cone shape. There are essentially three forces that affect the flow of fluids around the wellbore. They are capillary, gravity, and viscous forces. Capillary forces usually have negligible effect on water coning and are therefore neglected. Gravity forces are directed in the vertical direction and arise from fluid density differences. They are dominant before production. When production commences, viscous forces which results from pressure gradients associated with fluid flow increases. This viscous force continues to increase until it achieves a balance with the gravitational force at points on and away from the completion interval. When the viscous forces exceed the gravitational forces, the cone is dragged up until it breaks into the well. The shape and nature of the cone depends on several factors such as production rate, mobility ratio, horizontal and vertical permeability, well penetration and viscous forces.

This present study develops simple correlations to evaluate the critical production rate and the breakthrough time.

Steady-State flow to Horizontal Wells

The following assumptions apply: homogenous oil reservoir, horizontal and vertical directions are the principal axes of permeability, wellbore is long such that two dimensional flow dominates in the reservoir, steady state flow, capillary and relative permeability effects are neglected, formation underlain by water, sharp interface exists between oil and water.

The flow pattern to a horizontal well differs markedly from that for a completely penetrating vertical well. For a horizontal well, the flow is much more complex because it is constrained by the horizontal reservoir boundaries. The desired solution is obtained by conformally mapping the wells as shown (in figure 2) in the appendix using the transformation:

$$w = u + iv = e^{2\pi z/h} \quad \text{.............................1}$$

where u and v are the coordinates of the map and

$$z = x + iy \quad \text{.......................................2}$$

where x and y are the coordinates of the reservoir.

The mapping results in points along the y axis falling upon a circle of unit radius and the production well falling at the point $w = 1$. Each of the image wells falls coincidentally upon the production well.

The flow equation to a horizontal well in terms of the upward dynamic force caused by wellbore drawdown is:

$$P_x - P_w = \frac{Q_o \mu_o B_o}{2kLh}\left(y_e + \frac{h}{\pi}\ln\frac{h}{2\pi r_w}\right) \text{......3}$$

Determination of Critical Rate

The upward dynamic force must be equal or less than the pressure drop due to gravity to have a stable cone, thus avoiding water breakthrough. Pressure drop due to gravity occurs as a result of density contrast between oil and water expressed as:

$$\Delta P_g = \Delta \rho g h_{wc} \text{ ....4}$$

Combining the upward dynamic force and pressure drop due to gravity yields the critical rate above which water will break into the well. In field units, this critical rate is expressed as:

$$Q_{oc} = \frac{1.566*10^{-5} kLh(\rho_w - \rho_o)D_b}{\mu_o B_o\left(y_e + \frac{h}{\pi}\ln\frac{h}{2\pi r_w}\right)} \text{ ....5}$$

where

$$k = k_{eff} = \sqrt{k_h k_v} \text{ and } D_b = h_{wc}$$

Determination of Breakthrough Time

At water breakthrough, the reservoir is in the depletion stage at which a closed outer boundary will exist. At this stage, the energy from the expansion of oil, water and rock due to reservoir voidage through oil production and pressure drop due to gravity controls the movement of oil and water in the reservoir.

Taking a material balance by equating the reservoir voidage due to oil production with the expansion of the remaining oil, water and rock:

$$N_p B_o = Ah\phi c_t (P_i - P)/5.615 \text{ ....6}$$

For a well producing at a constant rate of Q_o STB/Day for a time period of t_{bt} before water breakthrough, then the cumulative oil produced is:

$$N_p = Q_o t_{bt} \text{ ....7}$$

$$Q_o B_o t_{bt} = Ah\phi c_t (P_i - P)/5.615 \text{ ....8}$$

The above equation in terms upward dynamic pressure is:

$$(P_i - P) = \frac{5.615 Q_o B_o t_{bt}}{Ah\phi c_t} \text{ ....9}$$

The upward dynamic pressure must overcome pressure drop due to gravity (equation 4) to have water break into the well.

Combining equation 4 and equation 9 yields an expression for breakthrough time at supercritical rates of production as:

$$t_{bt} = \frac{\Delta \rho g D_b Ah\phi c_t}{5.615 Q_o B_o} \text{ ....10}$$

where

$$D_b = h_{wc}$$

Applications and Discussions

To validate the correlations developed, the sample data in the appendix is used. Using equation 5 and the sample data, the critical rate estimate is 296 STB/day and it shows a favourable comparison with Joshi's correlation. Also, equation 10 was used to estimate the water breakthrough time for a well producing 5000 STB/day, the breakthrough time estimate is 4.8 years which also compares well with Yang and Wattengargers' correlation. However, it is markedly different from Ozkan and Raghavan, and Papatzacos' correlation because these neglected the pore volume of the reservoir in their correlation. The cumulative production with the estimated breakthrough time is $8.75*10^6$ STB which is economically attractive.

Parameter Sensitivity Analysis

Critical Rate: the sensitivity of various reservoir and fluid properties on the critical rate of horizontal wells was investigated. Figure 3 shows that well length has a direct influence on critical rate and consequently on the overall productivity of the well. Figure 4 shows that oil viscosity has a inverse influence on critical rate. Critical rate decreases with increasing viscosity. However, it tends to a constant at high values of viscosity. This implies that water cresting is a severe problem in heavy oil reservoirs.

As evident in figures 3 and 4, oil column thickness has a direct influence on critical rate. Thus, the thicker the oil column, the higher the critical rate.

Anisotropy

For a reservoir with different horizontal and vertical permeability, we can write the diffusivity equation as:

$$k_h \frac{\partial^2 P}{\partial x^2} + k_v \frac{\partial^2 P}{\partial y^2} = 0 \qquad \text{.........................11}$$

It can be rewritten as:

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y'^2} = 0 \qquad \text{..............................12}$$

Where:

$$y' = y\sqrt{\frac{k_h}{k_v}} \qquad \text{.....................................13}$$

And effective reservoir permeability is defined as:

$$k_{eff} = \sqrt{k_v k_h} \qquad \text{.....................................14}$$

Thus, the influence of reservoir anisotropy can be accounted for by modifying the oil column thickness as:

$$h' = h\sqrt{\frac{k_h}{k_v}} \qquad \text{.....................................15}$$

From figure 5, the critical rate decreases as the ratio k_h/k_v decreases, that is as it moves from anisotropy towards isotropy for. This implies that anisotropy favours high production, especially when the horizontal permeability k_h is far greater than the vertical permeability k_v.

Breakthrough Time: sensitivity analysis was also done on various reservoir and fluid properties affecting the breakthrough time.
Production rate has an inverse effect on breakthrough time. Figure 6 shows that with increasing production rate, breakthrough time decreases. Since cumulative production at breakthrough $N_p = Q_o t_{bt}$, it is evident that cumulative production is the same at low and high production rates at their respective breakthrough time. Therefore, it is advantageous to produce at high rates so as to quicken recovery and ultimately reduce cost.

Oil column thickness has a direct influence on breakthrough time. Figure 6 shows that breakthrough time increases with increasing oil column thickness.

Conclusion

1. A fast and simple correlation has been developed to estimate the critical oil production rate for a horizontal well in an oil reservoir underlain by water.
2. A simple correlation was also developed to estimate the water breakthrough time in an oil reservoir underlain by water for well producing at supercritical rates.
3. The breakthrough time correlation is applicable to both horizontal and vertical wells.
4. The applicability, simplicity and accuracy of the correlations has been demonstrated using a sample data.

Nomenclature

Q_o = oil rate (STB/day)
Q_{oc} = critical oil rate (STB/day)
k = permeability (md)
k_h = horizontal permeability (md)
k_v = vertical permeability (md)
k_{eff} = effective permeability (md)
h = oil column height (ft)
h_{wc} = height of water crest (ft)
x = main horizontal direction
y = main vertical direction
z = complex plane
$\Delta\rho$ = density contrast (lb/ft^3)
ρ_o = oil density (lb/ft^3)
ρ_w = water density (lb/ft^3)
μ_o = oil viscosity (cp)
B_o = oil formation volume factor (rb/STB)

r_w = wellbore radius (ft)
L = horizontal well length (ft)
y_e = half distance between two lines of horizontal wells (ft)
t_{bt} = water breakthrough time (days)
ϕ = formation porosity (fraction)
P = pressure (psia)
$\overline{P}$ = average pressure (psia)
P_w = wellbore pressure (psia)
ΔP_g = pressure drop due to gravity (psia)
A = reservoir area (ft^2)
N_p = cumulative oil production (STB)
c_t = total compressibility
g = acceleration due to gravity (32ft/s^2)
F = complex potential at w
Φ = potential at w
Ψ = stream function at w

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Appendix

Figure 1 shows a vertical section cut at right angles to a long horizontal well located centrally within the reservoir. The horizontal reservoir boundaries are shown as solid lines. The effect of these boundaries can be shown by imagining the horizontal well to be one of a series of such wells stacked vertically above each other with spacing h. The lines marked reservoir boundary are planes of symmetry and there will be no flow across them.

Let $F = \Phi + i\Psi$ A1

$$F = \frac{Q_o\mu_o B_o}{2\pi kL}\ln(w-1) - \frac{1}{2}\frac{Q_o\mu_o B_o}{2\pi kL}\ln(w)$$

$$F = \frac{Q_o\mu_o B_o}{2\pi kL}\ln\left(\frac{e^{2\pi z/h}-1}{e^{\pi z/h}}\right)$$

$$F = \frac{Q_o\mu_o B_o}{2\pi kL}\ln(2\sinh(\pi z/h))\text{.............A2}$$

The factor 2 within the logarithm of equation A2 is dropped since we are only interested in differences in potential. Thus:

$$F = \frac{Q_o\mu_o B_o}{2\pi kL}\ln(\sinh(\pi z/h))\text{...............A3}$$

$$F = \frac{Q_o\mu_o B_o}{2\pi kL}\ln\begin{pmatrix}\sinh\dfrac{\pi x}{h}\cos\dfrac{\pi y}{h}\\ +i\cosh\dfrac{\pi x}{h}\sin\dfrac{\pi y}{h}\end{pmatrix}\text{.....A4}$$

Φ from equation A1 is the real part of the complex potential F in equation A4 and is given by:

$$\Phi = \frac{Q_o\mu_o B_o}{4\pi kL}\ln\left(\cosh\frac{2\pi x}{h} - \cos\frac{2\pi y}{h}\right)\text{..A5}$$

Taking the difference in potential between a point in the x-axis with coordinates $(x,0)$ and a point on the circumference on the wellbore with coordinates $(r_w,0)$, we have:

$$\Phi_x - \Phi_w = \frac{Q_o\mu_o B_o}{4\pi kL}\begin{pmatrix}\ln\left(\dfrac{\cosh 2\pi x}{h}-1\right)\\ -\ln\left(\dfrac{\cosh 2\pi r_w}{h}-1\right)\end{pmatrix}$$

....A6

As x/h increases, the first logarithmic term in equation A6 becomes linear. Also, since the argument is small in the second logarithmic term, $\cosh\theta - 1$ can be approximated by $\theta^2/2$. Substituting into equation A6:

$$\Phi_x - \Phi_w = \frac{Q_o\mu_o B_o}{2kLh}\left(|x| + \frac{h}{\pi}\ln\frac{h}{2\pi r_w}\right)\text{.A7}$$

Where

$|x| = y_e$.

In a horizontal plane, we may replace Φ with P, hence:

$$\Phi_x - \Phi_w = P_x - P_w \text{..........................A8}$$

Therefore, equation A7can be written as:

$$P_x - P_w = \frac{Q_o \mu_o B_o}{2kLh}\left(y_e + \frac{h}{\pi}\ln\frac{h}{2\pi r_w}\right) \text{...A9}$$

Sample data (Ref. 22):

A = 160 acre

$L = 1640\text{ft}$

$k_h = k_v = 70md$

$y_e = 1320 ft$

$\mu_o = 0.42cp$

$\phi = 0.15cp$

$c_t = 15 * 10^{-6}$

$\Delta\rho = 18.7lb / ft^3$

$h = 80 ft$

$B_o = 1.1rb / STB$

$r_w = 0.3 ft$

$D_b = 72 ft$

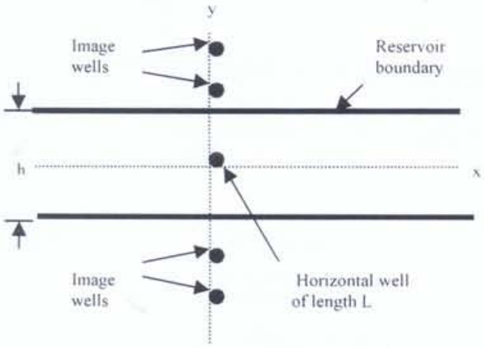

Figure 1: Vertical section through reservoir.

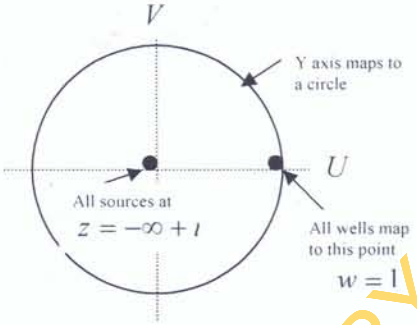

Figure 2: Conformal map of reservoir.

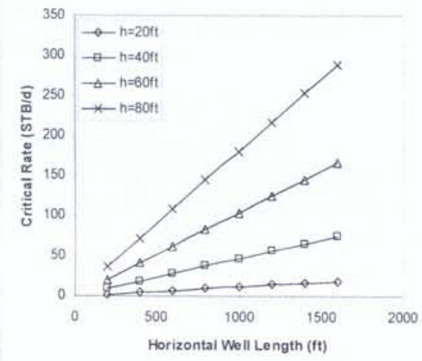

Figure 3: Effect of horizontal well length on critical rate at varying reservoir thicknesses.

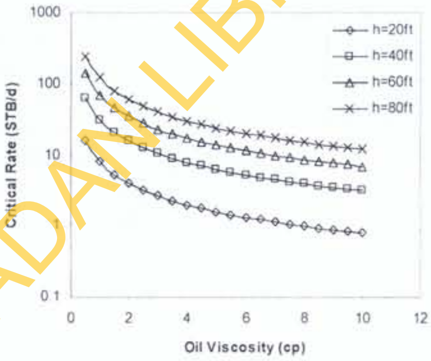

Figure 4: Effect of oil viscosity on critical rate at varying reservoir thicknesses.

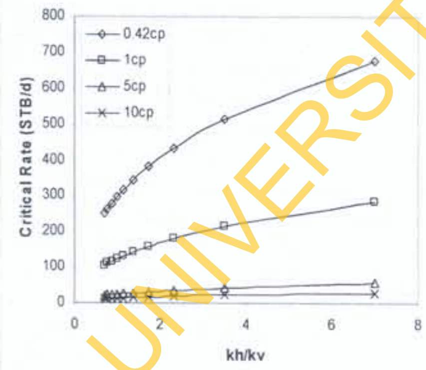

Figure 5: Effect of reservoir anisotropy on critical rate at oil viscosities.

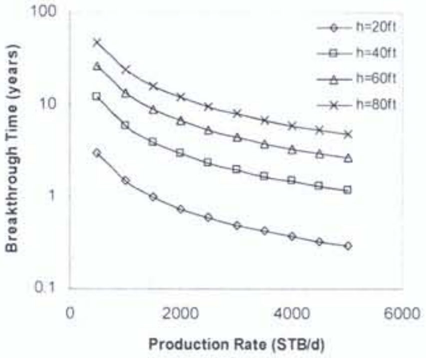

Figure 6: Effect of production rate on breakthrough time at varying reservoir thicknesses.