AN APPLICATION OF GOAL PROGRAMMING TO ACADEMIC RESOURCE ALLOCATION PLANNING

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DEDICATION

This thesis is dedicated
to the memory of my late mother;
and to
my father
Both attached "Priority One" to
my life goals in their
Informal Life Goal Programming;

My wife
For her love, steadfastness and support;

And all my children
With a promise that I will attach
"Priority One" to their life goals
in my Formal Life Goal Programming.

ABSTRACT

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Adedoyin Soyibo

Since the last decade, universities in Nigeria have been experiencing a progressive decline in required inputs, like funds, materials and academic staff. In spite of this, there has been a continuing rise in the demand for their services, as shown by rising student enrolment figures (Nigeria, 1981). Confronted with such a problem, universities require more than ever before, formal decision models for planning the allocation of their scarce resources as efficiently as possible. This study applies goal programming for planning the academic resource allocation--a major input--of the University of Ibadan for 1982/83-1986/87. The goal programming model used modifies that of Schroeder (1974) by defining explicitly a student enrolment goal and introducing an academic staff level goal, which is designed to cater for academic staff advancement, at least according to the historical rate in each faculty. Furthermore, it redefines the academic rank distribution goal to incorporate the controversial 30%-40%-30% rank distribution ratios introduced

in 1981. The study seeks principally to determine the distribution of academic staff by rank, in each faculty/college, over a five-year period and recommend the planning implications of such a distribution. In addition, it attempts to find the effects of dropping the controversial rank distribution goal on the model solution.

The model was solved using the Revised Simplex Goal Programming Algorithm developed by Kang (1980) on an I.B.M. VM 370 computer in the University of Nebraska-Lincoln, U.S.A.

The analysis of the model solution:

- suggests that from a purely theoretical point of view, it is desirable to use a rank distribution goal for an optimization model of the type used in the study; otherwise, the model will select least cost allocation alternatives only and such a solution cannot be used effectively for planning. However, the distributional ratios to be used should not be rigid like the controversial ones of 1981, but should reflect the historical advancement rates in the respective faculties. The result of solving such a model should be used for indicative planning only;

- confirms the fear that the use of fixed rank distribution ratios might inhibit promotion rate;

- indicates that the Faculty of Agriculture and

Forestry appears to be operating very much below the minimum level of academic staff requirement to meet the student enrolment goal of that faculty as of now;

- suggests that by the beginning of 1986/87, the University of Ibadan will require a minimum of 1,133 academic staff of various ranks to meet its student enrolment goal. This is over 60% above the minimum requirement at the beginning of 1982/83;

- recommends that the University should pursue a vigorous Staff Development Programme in which the training of the best of its graduates--through a type of Junior Fellowship Programme--will be the core, as one approach of augmenting the supply of academic staff normally obtained through recruitment;

- corroborates the findings of Kang (1980) that CPU time of the Revised Simplex Goal Programming Algorithm, tends to increase with increasing negative deviational variables in the objective function.

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CERTIFICATION

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CHAPTER 1

INTRODUCTION

One of the major constraints to the effective implementation of Nigeria's National Development Plans since independence has consistently been identified as "lack of high-level manpower", or "lack of executive capacity"[1]. In a bid to tackle this problem, government increased, the number of universities from five to thirteen within the last decade. Consequent upon the increase in university places, there was an astronomical rise in student enrolment.[2] At first, because of the mirage of the oil boom, finance was not viewed as a problem. However, the glut in the world oil market in 1977/79 resulted in a financial crisis in the country and government was faced with the stark reality

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1. See various issues of Progress Report on Nigeria's Development Plans, e.g. National Development Plan, Progress Report 1964, p.60; Second National Development Plan 1970-74 First and Second Progress Reports p.104; Second Progress Report on the Third National Development Plan, p.122.

2. University enrolment increased from 31,511 in 1975-76 to 57,722 in 1979-80, The Third National Development Plan target enrolment of 53,000 was thus exceeded. See Fourth National Development Plan 1981-85, pp 255 and 270.

that finance could be a major constraint to any unplanned university expansion. Starting with the 1977/78 session, this crisis brought in its wake, a progressive decrease in educational inputs like funds, materials and academic staff required for the transformation of the astronomically rising student population into the required high-level manpower output. In spite of the decreasing resources, the Federal andssome state governments have opened new universities within the last three years. Furthermore, these governments have signified an intention of opening more universities in the eighties.

1.1 Need for the Study.

In the face of these declining resources and increasing demand for their services, universities require, more than ever, other means beyond management "judgement" or "experience" to aid them in the efficient allocation of their scarce resources. This study presents a formal decision model for the allocation of academic staff to the different faculties or colleges of a university subject to the budget and staff level goals of the university.

Nigerian universities which are major sources for supplying the much needed high-level manpower, are subject to this critical resource constraint. Thus, for the country to attain its manpower targets, careful academic manpower planning is desirable in each university. One way of doing this is to plan for meeting academic staff goals or targets subject to constraints like funds and demand for such staff, as will be done in this study. Also, universities are known for developing problem-solving models for other areas of the economy. For such problem-solving modelling to cover all areas of the economy, a look at university problems from within the university appears apposite.

1.2 Objectives of the Study

This study utilizes manpower flows (flow of academic staff within the academic hierarchy in specific academic units, i.e. faculty/college, and over the planning horizon) in a goal-oriented optimization model, using the University of Ibadan as a case study. The university has several goals or targets of its academic manpower strength and such a model can help determine the required number of academic staff for achieving, underachieving or overachieving these goals subject to budget constraints as well as demand and supply conditions.

Thus, the study is expected to determine, for example, the number of academic staff by rank in each faculty/college required to meet the staff goal levels of the unit subject to the budget and other constraints and to establish the required inputs (e.g. funds) necessary to meet the staff goal levels, as well as student enrolment goal levels set by the university. In this way, the model can show whether goals set are realistic or not by comparing the value of input resources determined from the model solution with actual resources made available to the university and by analyzing the value of the goal deviational variables determined by the model solution. These deviational variables can indicate whether goals are achieved, underachieved or overachieved.

Further, the study aims at establishing the number of academic staff that can be recruited in each faculty and year of the planning horizon subject to the financial as well as demand and supply constraints and determine whether this number can help meet the set targets or not. It also aims at performing extensive sensitivity analyses to evaluate various policy planning options available to the university. The sensitivity analyses will address issues like:

(i) the effects on the policy (decision) variables due to changes in goal levels;

(ii) the effects on the policy variables due to changes in budget levels;

(iii) the effects on the policy variables due to changes in priorities attached to the various goals;

(iv) the effects on the policy variables due to changes in academic rank distribution ratio; in particular, this study will evaluate the effects on academic rank distribution in various faculties of the University of Ibadan if the controversial 30%-40%-30% academic rank distribution ratio presented by the Vice-Chancellor's Press Release 46 of 1981 are implemented or withdrawn.

Specifically, the study aims at determining the following:

(a) X_{ijt}: the number of academic staff of rank i in faculty/college j at the beginning of period t.

(b) Y_{ijt}: the number of academic staff rank i, recruited at the beginning of period t in faculty/college j;

(c) deviational variables that show the extent of achievement of the various goals set by the different faculties over the planning horizon;

(d) the policy implications of changes in the budget levels, goal levels and goal priorities within faculties and in each year of the planning horizon;

(e) the amount of resource inputs like funds required to meet the various staff level goals and the extent to which the budget of the university is compatible with its staff level goals and student enrolment goals as well as the policy implication of such findings.

Finally, the study aims at making general policy recommendations for the efficient allocation of academic staff over a five-year planning horizon subject to budget, demand and supply constraints and academic staffing goals of the various faculties/college of this university.

1.3 A Review of Related Studies

Perhaps the genesis of the application of management science/operations research to education can be traced to Platt (1962). In this paper, Platt lamented the dearth of applications of O.R. to problem solving in education in spite of the fact that a sister discipline like economics had benefited immensely, as at that time, from "this relatively novel discipline." Further the degree of decision latitudes in education such as

(i) the portion of society's resources to be invested in education;

(ii) the way these resources should be allocated to achieve the objectives of the individual and the society; and

(iii) the types of technology to be used

call for a systems-approach, if suboptimization is to be avoided. Thus, management science/operations research, which is well-known for its systems orientation, is very well suited for problem solving in educational resource allocation.

Resource allocation models developed for university operations can be broadly classified into two:

(i) (Cost) Simulation Models

(ii) Analytic Models.

In Figure 1.1, this broad classification is broken down into smaller subclasses.

(Cost) Simulation Models like CAMPUS (Computerized Analytic Methods in Planning University Systems) and R.R.P.M. (Resource Requirement Prediction Model) simulate the resources required over the planning horizon for specified inputs like enrolment projections, student demand for courses, academic staff work-loads and cost factors like cost of courses including

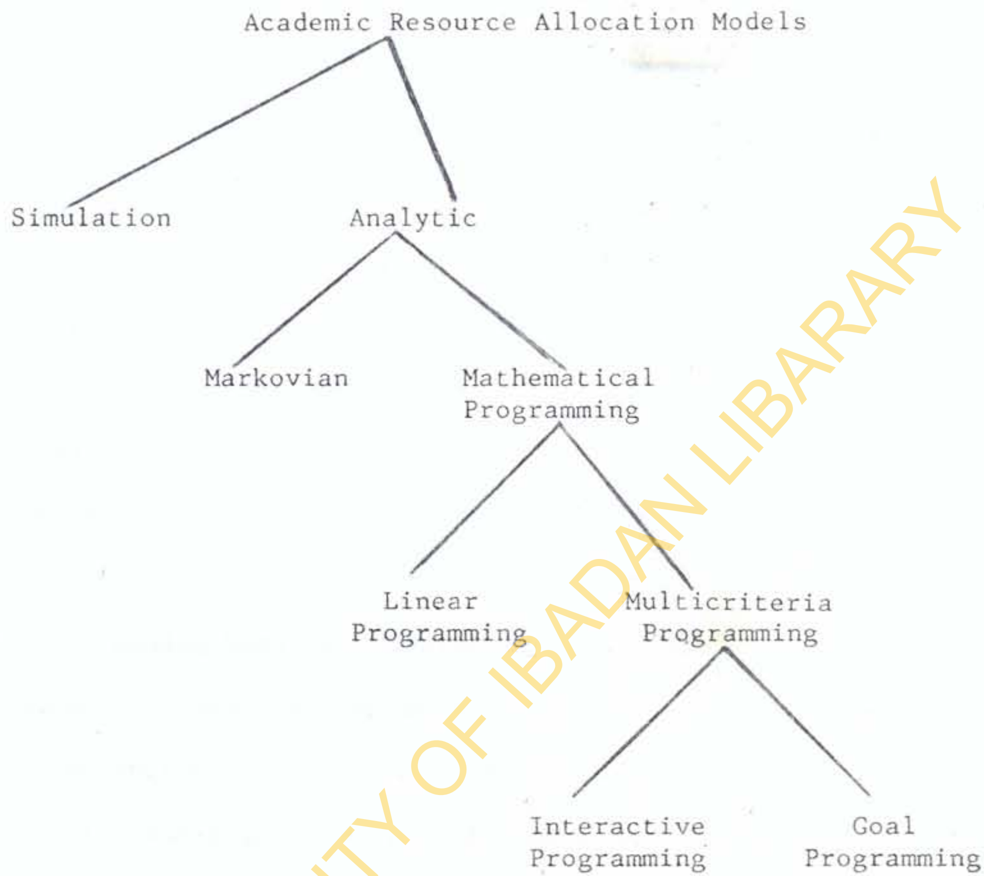

Fig. 1.1 A Classification of

Academic Resource Allocation Models

tuition payments, cost of staff and assistants, etc. Required resources are expressed in terms of the number of academic staff, facilities availabilities and costs (of these requirements).

The major difference between CAMPUS and R.R.P.M. is that R.R.P.M. is more aggregated and its data-inputting format is less flexible. It cannot, for example, simulate the level of individual courses. However, two major weaknesses of these simulation models are (i) total budget over the planning horizon are considered as model outputs rather than inputs, (ii) academic staff-to-enrolment ratio is considered fixed over the planning horizon. Schroeder (1973) presents a general survey of management science models used in university operations including resource allocation models.

Analytic University Resource Allocation models can be further subdivided into two classes: Markovian models and Mathematical Programming models. Markov chain modelling has been extensively applied to manpower planning in universities. For example, Branchflower (1970)--discussed in Grinold and Marshall (1977)--and Akinlade (1979) applied Markov chain to analyze the movement of academic staff within the academic hierarchies of the College of Engineering, University of California, Los Angeles and University of Ibadan respectively. While Branchflower's model used thirteen different ranks defined with respect to which step of the salary scale the incumbent was in as at the first of July of each year of the peiod 1960-1968, Akinlade's model was more aggregated and it distinguished only five different academic ranks of Assistant Lecturer, Lecturer, Senior Lecturer, Reader and Professor. The period covered by the study was 1969-79. Though in reality two grades of Lecturer, namely Lecturer I and Lecturer II, exist, the University of Ibadan records seem to bother little about distinguishing between the grades; hence the merging of the grades.

Both studies used the description of movement of academic staff to evaluate the effects of various hiring and promotion policies on rank structure. Furthermore, the Akinlade study made projections of the future pattern of academic staff mix and their associated costs under various assumptions. One major limitation of the two studies is that the stationarity of their transition probability matrices were assumed and never tested. If these matrices turned out not to be stationary, the various conclusions of the studies can be questionable. Zanakis and Maret (1980) indicate how the stationarity assumption of the transition probability matrix as well as the individual transition probabilities can be tested.

We can further divide Mathematical Programming academic resource allocation models into: linear programming models which use single objective function and multicriteria programming models which use multiple objective functions.

Bowles (1967), being reviewed for historical reasons--(i) it is perhaps one of the earliest answers to Platt's clarion call for the application of management science to education; (ii) it is perhaps the first management science application to the planning of education in any part of Nigeria--is not essentially a resource allocation application in the university system. It is a multiperiod linear programming model for planning the educational system of the former Northern Region of Nigeria over an eight-year period (1964-71). The model sought to maximize the economic benefits accruing to the society as a whole as a result of educating each category of labour. A proxy defined to measure this objective was defined by the study as (the sum of):

> the present value of estimated life-time earning streams of each category of labour minus the present value of life-time earning streams if the individuals in that category of labour had not received that level of education (opportunity cost) minus the present value of direct costs of education for each category of labour.[1]

The decision variables defined for the problem relate to the number of students in each of primary and secondary schools and universities, the number of teachers produced in the country,

1. Bolwes(1967), p.197, equations (3.1) and (3.2) expressed in words.

and the number of teachers to be recruited from abroad. The constraints of the problem dealt with the availabilities of the different levels of teachers from within and outside and country, finance and accomodation. Because of the dearth of historical data, a sample survey had to be conducted to get estimates of the various parameters used in the study. Another major limitation of this model is the fact that since we can have different proxies defined for the economic benefits of education, as many "optimal" solutions can be obtained for as many different proxies defined. Furthermore, it is known that the present value is very sensitive to the discount rates used. Thus, as many solutions can be obtained for as many discount rates used.

Koch (1973) is an adaptation of Bowles' model of resource allocation to Illinois State University in which the University is viewed as a "multiproduct firm" whose "products" are the graduates of its various programmes and whose objective should be the maximization of the difference between the value of its graduates which is attributable to higher education and the costs of educating them. In estimating the coefficients of the objective function, Koch made the following adjustments:

(i) an adjustment which reflects the fact that part of the income of graduates is not attributable to educational attainment but to differential student ability and motivation--

only 75% of the observed differential in earning was attributed to higher education;

(ii) an adjustment which recognizes the fact that not all university graduates remain in the labour force throughout their life time and, therefore, cannot earn income in certain years. Contemporary labour participation rates were used to approximate the sizes of those who remain in the labour force.

However, a major limitation of the application of this model to the Nigerian situation is the fact that government, being the largest employer of labour in this country, employs a great number of university graduates. Unfortunately, salaries paid to government employees are fixed periodically by law and do not seem to reflect the relative value attached by society to the graduates of the various disciplines. Thus the coefficients of each decision variable in the objective funcion might tend to be approximately the same, resulting in a trivial problem.

Two major classes of multicriteria programming models have been formulated to solve academic resource allocation problems. These are Interactive Programming and Goal Programming. In Interactive Programming, the decision maker (DM) interacts directly with the computer or with the analyst as an intermediary.

The DM's utility function is assumed unknown, but he can give information about its local properties and value tradeoffs between the various objectives as the solution interaction progresses. Geoffrion, et al (1972) developed an interactive programming multicriteria optimization model to solve an aggregated operating problem of the Graduate School of Management, UCLA, in which each academic staff member is viewed as engaging in three principal activities of formal teaching; departmental services (e.g. administration and curriculum development) and research; and student counselling. Allocation of academic staff effort among the three activities were done in Full Time Equivalent (F.T.E.) basis with one unit of F.T.E. defined as the amount of time and effort equivalent to teaching one "course section." The model maximized six departmental objectives. However, being a one-period, one-academic unit model, a problem of suboptimization might result from its solution because of the apparent neglect of interactions with other academic units within the university. Because model implementation requires a minimum level of familiarity with the computer on the part of the DM, this may inhibit its application in this country.

Goal Programming (GP) academic resource allocation models, apart from attempting to help the university in the attainment of its multiple and often conflicting goals (objectives) are capable of reflecting the judgements of the authorities about the priorities attached to the desired goals. In general in goal programming, goals are set a priori by the DM and all the model does is to minimize the deviations from these goals.

The GP model of Lee and Clayton (1972) is a one-year planning model for a relatively small college or faculty of a university. What the model seems to lose in terms of limited planning horizon, appears gained in terms of the level of detail of the decision variables. Four groups of decision variables are identifiable: numbers of instructors, and allied staff like graduate/teaching assistants; number of full-time academic staff; number of part-time academic staff; and number of support staff like secretaries. Three types of model solution were obtained. The first type determined the amount of resource inputs required to meet the different goals set for the college/ faculty. The other two types found the resulting values of the different decision variables subject to different priorities attached to different goals. In this way, the model serves both as a resource requirement and resource allocation model.

However, the lack of global feature (i.e. covering the whole university) in the model may result in some problem of sub-optimization. Also because the model is a one-period model, it cannot capture effectively the dynamic nature of the planning system as would do a multiperiod model, though the model can be run annually to determine the following period's solution.

Walters, et al (1976) developed a long range academic resource allocation planning model for a single academic unit (e.g. school or faculty) of a university using goal programming. The academic unit being modelled is assumed to have desired academic staffing goal levels within each area of specialization of teaching and research in the unit. These goals are measured in full time equivalents (F.T.E.). For example, a goal may be met by a professor devoting full time to an area or two professors devoting half-time to the same area. This appears to be attractive because the decision variables need not be restricted to take integer values only. Unfortunately, however, this model seems to be highly disaggregated for a strategic planning problem. Moreover, a lot of subjectivity is inputted in the estimation of parameters, e.g. the estimation of probabilities of promotion of staff requires the subjective inputs of several superior officers of each staff being considered rather than using Markovian estimates. Also because the model can only be used in one unit of the university, it might lead to some degree of suboptimization.

Schroeder (1974) is a multiperiod, multiacademic unit goal programming academic resource allocation model which appears to have rectified most of the limitations of the models discussed previously. It can be applied to the whole university at a time. It has little or no subjectively estimated model parameter and the information required for model application appears sufficiently aggregated for long range planning. In general, the model focused on the division of payroll budget between academic and related staff with a view to achieving or coming close, as much as is possible, to the prioritized goals of the various units that make up the university. A modified form of this model in which new goals are defined and some dropped will be used in this study. Further elaboration on the modified model will be presented in Chapter 3. However, one major limitation of this modified model is the implicit assumption of linear relationship between the decision variables and the model parameters. Furthermore, the decision variables are assumed to be continous. Ideally, integer goal programming should be have been used but this is very difficult to solve. Goal programming also assumes that goals can be easily quantified and ranked. There may be difficulties in doing this in practice.

Several methods have been proposed to solve goal programming problems. Ijiri (1965), proposed the generalized inverse technique. Lee (1972, 1976) modified the simplex procedure of linear programming to solve goal programming problems. In this algorithm the reduced cost row of the linear programming simplex tableau is replaced by a matrix of 'reduced costs' of the pre-emptive priority factors. There are as many rows of this matrix as there are priority factors. Lee's algorithm contains all the variables of the problem in addition to all priority factors. When it is realized that the deviational variables have coefficient of +1 and -1 in each row of the initial simplex tableau, it can be seen that a lot of computer space is wasted for storing only zeros, particularly when the problem is large scale.

Ignizio (1976) deletes the columns relating to the initial basic variables (i.e., the negative deviational variables) from the initial simplex tableau and adds one more priority at a time as optimal solutions to higher order priorities are found. However, the tableau is still relatively sparse for large-scale problems because each column of positive deviations in the initial form contains zeros only, except in one row.

Arthur (1977) and Arthur and Ravindran (1978) propose an algorithm that reduces the number of computation at each iteration by partitioning the goals according to priorities and using a variety of nested subproblems and variable elimination proceedures. It consists of three main procedures: partitioning, elimination and termination. It is more efficient than both Lee's and Ignizio's algorithms because it reduces the number of computations by modifying the matrix size, when the number of subproblems increases and by eliminating unnecessary non-basic variables. However, it does not provide the optimal simplex tableau which is required to carry out sensitivity analysis.

Rho (1976) formulated a decomposition algorithm for goal programming combining the techniques of Dantzig and Wolfe (1960) and Kurnai and Liptik (1965). This algorithm can be applied to resource allocation in decentralized organizations having multiple objectives.

Kang (1980) formulated the Revised simplex goal programming algorithm that combines the revised simplex method[1] in

1. The revised simplex method expresses the inverse of the current basis of the simplex tableau as a product of elementary matrices. Each of the elementary matrix is the identity matrix except one column. This non-unit column contains the coefficients of the pivot column at the current iteration. Only the non-unit columns of the elementary matrices are stored in the computer. In particular, only non-zero values of these columns are stored by indication of their column and row locations. This substantially improves time and storage costs because at each iteration only a single vector is stored and hence is very useful for large-scale problems.

product form (or Gauss-Jordan form), with Lee's modified simplex algorithm and Arthur's goal partitioning algorithm. This algorithm is more efficient than previous goal programming algorithms in terms of reduction in CPU time and storage particularly for large-scale problems. However, it was found that the CPU time of the algorithm tends to increase with increasing negative deviational variables in the objective function. The revised simplex goal programming algorithm will be used to solve the model formulated for this study.

In a recent paper, Lee and Gen (1982) propose a new algorithm based on the LU decomposition of the basis of the simplex tableau. In this algorithm, the basis is factored into a product of lower and upper triangular matrices L and U where L and U can respectively be decomposed into a product of elementary matrices which have 1's in the diagonals and only one non-zero column. The algorithm also uses sparsity techniques and may prove more efficient than the Kang algorithm for large scale problems because commercial linear programming codes use the LU factorization techniques and have proved to be more efficient than other codes based on other techniques.

1.4 Organization of the Thesis

The remaining chapters of this thesis will be organized as follows. Chapter 2 reviews the existing academic manpower planning system of the University of Ibadan while in Chapter 3, the theoretical framework and the empirical basis of the model used in the study are discussed. The model solution and its interpretation are reported in Chapters 5 and 6. In Chapter 7, the major findings, recommendations and conclusion of the study are recorded with suggestions for future research.

CHAPTER 2

A REVIEW OF THE EXISTING ACADEMIC PLANNING SYSTEM OF THE UNIVERSITY OF IBADAN

Planning in the University of Ibadan is done under the aegis of the Development and Planning Office headed by the Director of Planning. The duties of the Development and Planning Office include the coordination of the total university manpower planning system as well as planning for the physical development of the university. This study addresses an aspect of the university manpower planning that is concerned with the allocation of academic staff by rank to the various faculties and college of the University of Ibadan. Consequently, the discussion in this section will focus mainly on the existing academic resource allocation planning system in the University of Ibadan.

Each department makes requests for additional academic staff to the Planning Office annually. Each request is usually justified on the basis of expansion of existing programmes and/or addition of new programmes. In addition, the Director of Planning deposed that departments sometimes allude to what they construe as their 1976/77 established position in trying to justify requests for additional staff. 1976/77 represents, perhaps, the last of those heyday periods of adequate funding in Nigerian universities.

In evaluating requests for additional academic staff, the Development and Planning Office is guided by at least four main groups of factors:

(i) National University Commission (N.U.C.) guidelines,

(ii) National Development Planning goals of the University for the period under consideration,

(iii) performance of the units making the request, and

(iv) available funds.

The N.U.C. issues from time to time, guidelines representing government policy changes to universities. Such guidelines germane to this study include standard student/staff ratios used for computing the required staff strength using the headcount of students or the F.T.E. approach; and the maximum course units load per session that should be carried by an academic staff, if the university uses the contact hours load system in computing the required staff strength.

The standard student/staff ratios are discipline-dependent. For example, for science-based academic units, the N.U.C. guideline stipulates a standard student/staff ratio of 10:1

while for arts-based disciplines, a ratio of 15:1 is stipulated. In medicine, the ratio is 7:1, while in education, it is 25:1. For this purpose, the arts and social sciences are regarded as arts-based, while faculties of science, agriculture, technology and veterinary medicine are termed as science-based. However, in practice, the faculty of veterinary medicine operates as the College of Medicine. In the case where the university operates the course system and uses the contact hours system, the N.U.C. guideline stipulates that an academic staff member may carry a maximum of 400 credit or contact hours per session for science-based disciplines and 300 credit hours per session for arts-based disciplines.

The University of Ibadan operates course system in all disciplines except in the College of Medicine and in the Faculty of Veterinary Medicine. In practice, the Planning Office uses student enrolment headcount and the standard student/staff ratio to allocate academic staff in these two units. In the other six faculties, the office uses the F.T.E. approach and standard staff/student ratios. Sometimes also, the contact hours load approach is used for comparative purposes. Later in this chapter, we shall elaborate on how this is done.

The programmes of the University at a particular time are influenced to a great extent by the goals of the University for the existing National Development Plan period. For example, the philosophy guiding the preparation of the university's submission for the current National Development Plan (1980-85) is stated as "commitment to manpower at professional and academic levels, relevance to the needs of society and response to national and international obligations."[1]

In addition, during this plan period, this university seeks to:

"(a) consolidate existing undergraduate programmes,

(b) embark on new dimension of development of undergraduate programmes in the Faculties of Technology and Agriculture,

(c) gradually phase out subdegree programmes,

(d) emphasize postgraduate programmes and ultimately seek to achieve an undergraduate-postgraduate ratio of 3:1,

(e) commence professional degree programmes in Law, Business Management and Pharmacy, possibly within existing faculty structures, and where available resources permit such new growth.

(f) adopt a new college structure for the Faculty of Medicine,

(g) work towards eventual faculty status for Law, Pharmacy and Dentistry."[2]

Performance of each academic unit is measured using head-

1. Development and Planning Office File, University of Ibadan.

2. Ibid.

count student enrolment (where no course system is available) or using F.T.E. students. In the College of Medicine and Faculty of Veterinary Medicine where there is no course system, the desired number of academic staff is determined using the projected headcount student enrolment and the standard student/staff ratio. The projected student enrolment is determined using the long-term goals of the university during the plan period, as discussed earlier. If the existing staff cannot cope with the projected student enrolment and funds are available, then the Development and Planning Office will give its approval.

In contrast to headcount student enrolment, Full Time Equivalent (F.T.E.) students and contact hours are academic load measures which take cognizance of the fact that students can move within and between faculties during their courses of study in the University.

The F.T.E. approach looks at all the courses offered in each faculty/college of the university and computes for each course the product of the number of students registered and the credit units. For each academic unit, a summation of all such products for all courses is obtained. The resulting sum is divided by the average credit unit load that can be taken by a student per session to get the F.T.E. students. Under the semester system, it is assumed that, on the average, a student is expected to carry 28 units/session. Thus F.T.E. students can be computed as:

$$\text{F.T.E. Students} = \frac{\sum_{i=1}^{n_j} S_{ij}U_{ij}}{28} \quad \text{for all } j \quad (2.1)$$

where S_{ij} = number of students registered for course i in faculty j

U_{ij} = credit unit of course i in faculty j

n_j = total number of courses offered in faculty j.

Having obtained the F.T.E. students for a particular faculty/college, the required staff strength is obtained by dividing the value for F.T.E. students by the standard student/staff ratio for that faculty:

$$X_j = \frac{\text{F.T.E. students in faculty } j}{h_j} \quad (2.2)$$

where X_j is the number of academic staff required in faculty j and h_j is the standard student/staff ratio for faculty j.

For medium and long-term planning purposes, F.T.E. students in each faculty are projected bearing in mind the goals of the university during the given planning horizon as stated earlier.

The use of contact hours as load measure to evaluate performance in various faculties is similar in conception to F.T.E. students. In the University of Ibadan, a credit unit is equivalent to 15 hours of theoretical instruction and 45 hours of practical instruction. Using this simple rule, the total course units offered in a faculty can be converted to contact hours. On the assumption that an academic staff member cannot carry more than 300 contact hours in arts-based disciplines or 400 contact hours in science-based disciplines, the required number of academic staff can be determined using the equation:

$$X_j = \frac{15N_j + 45n_j}{C} \qquad (2.3)$$

where X_j = the number of academic staff required in faculty j

N_j = total number of credit units of theoretical instruction offered in faculty j

n_j = total number of credit units of practical instruction offered in faculty j

C = maximum contact hours that can be carried by each academic staff member. (C = 300 for arts-based disciplines and 400 for science-based disciplines).

Equation (2.3) shows a direct relation between X_j and both N_j and n_j. Thus, the greater the values of N_j and/or n_j the greater is X_j. Consequently, this method of evaluating performance and determining the required staff strength can encourage a proliferation of courses, irrespective of the number of students registering for such courses. For example, Adeyemi (1981) reported that in 1978, a study conducted by the Planning Office revealed that about 30% of all courses offered by the university enroled less than ten undergraduates. On disaggregation to departmental levels, it was found that in some departments, over 85% of courses offered enroled less than 10 students. This, perhaps, explains why the Planning Office uses the F.T.E. approach in preference to the contact hour approach to evaluate performance of departments.

Perhaps the most important of all the factors influencing the decision of the Development and Planning Office in granting requests for additional academic staff from the various faculties/ colleges is the availability of funds. For example, the crisis resulting from the oil glut of 1977/78 affected academic planning in the University of Ibadan drastically. Some of the measures adopted by the University include (Adeyemi, 1981):

(i) reduction of student enrolment;

(ii) suspension/compression of funds for staff development and general university research;

(iii) embargo on new staff positions except in proven cases of dire need;

(iv) freezing of all vacant positions and those that become vacant except in cases of extreme emergency.

Clearly, whatever the value of the required academic staff strength determined by the other three factors, the final decision of the Development and Planning Office will be heavily influenced by available funding, if past experience is anything to go by.

CHAPTER 3

THE THEORETICAL FRAMEWORK

The study uses a multiperiod goal programming model for academic resource allocation employing the University of Ibadan as a case-study for model application. First, a brief overview of goal programming is presented and later an attempt will be made to justify the choice of goal programming in preference to other models for carrying out the study.

3.1 A Short Description of Goal Programming

Goal Programming (GP) is one of the methods for solving problems with multiple objectives. Its origin can be traced to the early 60's, when Charnes and Cooper (1961) presented an algorithm for solving linear decision models having more than one objective function. The computation capabilities of GP have since been improved upon through the works of Ijiri (1965), Lee (1972), and Ignizio (1976). As of now, algorithms have been developed to handle not only nonlinear goal programming problems, but also integer and mixed integer goal programming problems (Ignizio, 1976). In GP, the DM sets goals and the model helps him to come as close as possible to these goals.

The linear goal programming model (which will be used in this study) can be formulated as:

Minimize $P(n + p)$ (3.1)

Subject to

$Ax + I(n-p) = g$ (3.2)

$Bx \leqq b$ (3.3)

$x \geqq o, \quad n \geqq o, \quad p \geqq o$ (3.4)

where

P = a k-row vector of goal priority weights;

p = a k-column vector of overachievement of goal levels;

n = a k-column vector of underachievement of goal levels;

g = a k-column vector of desired goal levels;

A = a k X n matrix of coefficients of goal constraints;

x = an n-column vector of decision variables;

I = a k x k identity matrix;

B = an m x n matrix of coefficients;

b = an m-column vector of resource levels.

The G.P. problem formulated in (3.1)-(3.4) has k goals and m non-goal constraints. (3.1) is the objective function and it minimizes a weighted combination of deviational variables. This equation can also be written as:

$$\text{Minimize} \quad \sum_{i=1}^{k} P_i \, (n_i + p_i) \qquad (3.5)$$

For optimality it is required that

$$n_i \cdot p_i = o \text{ for each } i=1,2,\ldots,k. \qquad (3.6)$$

Thus when $n_i > o$, $p_i = o$ for each i; and we have an underachievement of the i^{th} goal. Similarly, there is an overachievement of goal i when $p_i > 0$ and $n_i = 0$. Exact achievement implies that both n_i and p_i are zero for some i.

In general, P_i $(i = 1,2,\ldots,k)$ is taken as the ordinal ranking of priority attached to the i^{th} goal by the D.M. When this is the case, the problem is called a pre-emptive ordered G.P. The solution is obtained in sequence: goal(s) with priority 1 are achieved to the extent possible before goal(s) with priority 2 are considered; and goal(s) with priority 2 are satisfied to the extent possible before those with priority 3, etc. The pre-emptive ordered G.P. will be used in this study.

In practice, the resource constraints of equation (3.3) are converted to binding constraints by adding negative deviational variables and subtracting positive deviational variables as is done in (3.2). However, the type of priority attached depends on the type of problem one wants to solve. Ignizio (1976) suggests that if when a resource constraint is not satisfied, the solution becomes unimplementable, then priority 1 should be attached to the deviational variables in the objective function and the resulting goal is called an absolute objective or goal.

Lee (1972) suggests that the priorities to be attached to the deviational variables of (3.1) depend, in general, on three factors relating to:

(i) the identification of resource requirements to attain all the desired goals;

(ii) the degree of goal attainment with the given inputs;

(iii) the degree of goal attainment under various combinations of inputs and goal structure.

It can easily be seen that case (ii) of Lee's suggestion coincides with Ignizio's suggestion. The approach suggested by Lee (1972) will be adopted in this study.

3.2 Justifications for the Choice of GP

The university authorities, like all real-life decision-makers, have several conflicting and sometimes non-comensurable objectives. These objectives can often be expressed in terms of major goals and subgoals or multiple goals with different order of priorities. For example, the university may be required to produce a target number of graduates over a planning horizon using a specified (target) amount of resources like a given number of academic staff and a given amount of funds. In most cases, these targets will conflict. Those charged with decision-making in the university can re-order these goals on behalf of government and society. Goal programming is the only known method that can solve such problems of prioritized goals.

Secondly, goal programming is easier to use and relatively cheaper than other multicriteria programming models. Classical linear programming codes can be modified to solve the problems. Furthermore, large scale goal programming codes have been developed independently by Kang(1980) and Ignizio and Perlis (1979). Other multicriteria programming methods, like the methods of Zeleny (1974) and Evans and Steuer (1973), solve relatively small problems and do not seem to have been applied to many real-life problems. In fact, most of what is reported in the literature in terms of the computational experience of these methods are mere experiments for testing their computational properties (Cohon, 1978). Given the state-of-the-art of multicriteria problem-solving, therefore, goal programming is easily seen as a preferred choice for a large-scale problem of the type of this study.

Thirdly, in real-life, goal-setting is a common concept.

Thus goal programming is very close to real life. The problem of defining proxies to estimate the national social welfare function is absent in goal programming models. Finally, goal programming allows for the evaluations of marginal tradeoffs between possible courses of action and the opportunity costs of the various goals which are considered as constraints (Walters, et al 1976). This information is very useful to the DM. It is like the shadow price of classical linear programming, which determines the benefit derivable from an extra unit of a given resource.

3.3 Model Specification

This study will apply a modified version of Schroeder's (1974) model to academic resource allocation planning using the University of Ibadan as a case study. The model modifies Schroeder's in the following aspects:

(i) It attempts to utilize both academic staff flows and student flows over the planning horizon by defining explicitly a student enrolment goal constraint whereas Schroeder's uses only academic staff goals, though student enrolment is exogenously estimated to determine the desired academic staff strength.

(ii) The academic rank distribution goal is redefined to reflect the controversial proposal of 30%-40%-30% distribution between academic staff in the lecturer, senior lecturer and professorial grades as prescribed by the Vice Chancellor's Release 46 of 1981.

(iii) The model does not consider support staff like graduate assistants and secretaries because the cost of these categories of staff is often negligible, compared to that of academic staff. Besides, their supply is typically not a constraint involving staff development.

3.3.1 Model Formulation

A. Definition of Decision Variables

X_{ijt} = number of academic staff of rank i in faculty j at the beginning of period t.

Y_{ijt} = number of new academic staff of rank i recruited at the beginning of period t in faculty j.

where $1 \leq i \leq m$, $1 \leq j \leq n$, $1 \leq t \leq T$.

B. Definition of Constants (Parameters)

C_{ijt} = (average) salary per academic staff rank i, faculty j at period t.

B_t = total academic payroll budget available at the beginning of period t.

g_{jt} = academic staff goal level desired in faculty j at period t.

α_{ijt} = desired proportion of academic staff in rank i, faculty j at period t.

β_{ijt} = proportion of academic staff who stay from period t to t+1, rank i, faculty j.

γ_{ijt} = proportion of academic staff promoted from rank i-1 to rank i during period t in faculty j.

U_{jt} = upperbound on the number of academic staff that can be recruited in faculty j at period t.

S_{jt} = desired student enrolment in faculty j, period t.

h_{jt} = desired student/staff ratio in faculty j, period t.

C. Formulation of Non-goal Constraints

(i) Academic Staff Flow Constraint:

$$X_{ijt+1} = \beta_{ijt} X_{ijt} + Y_{ijt+1} + \gamma_{ijt} X_{i-1,jt} \qquad (3.7)$$

for all i, j, t.

Equation (3.7) states that the number of academic staff rank i, in faculty j in period t+1 is the sum of those who remain from the previous period ($\beta_{ijt} X_{ijt}$) plus those recruited at the beginning of period t+1 (Y_{ijt+1}) plus those promoted from rank i-1 to rank i ($\gamma_{ijt} X_{i-1,jt}$). We observe that the relation $Y_{ijt+1} \geqq 0$ implies that academic staff cannot be "laid off." Reduction in academic staff is achieved by normal attrition.

(ii) Maximum Hiring Constraint:

$$\sum_i Y_{ijt} \leqq U_{jt} \text{ for all } j,t. \qquad (3.8)$$

An upperbound is placed on the number of academic staff that can be recruited owing to such factors as supply and demand prospects and budget of the university.

(iii) Academic Payroll Budget Constraint:

$$\sum_i \sum_j C_{ijt} X_{ijt} \leq B_t \quad \text{for all } t \qquad (3.9)$$

The total amount available for academic staff salaries cannot exceed the budgeted salaries for each period. We observe that adding the cost of newly recruited staff will amount to double counting since we can always express X_{ijt} in terms of Y_{ijt} from equation (3.7).

D. Formulation of Goal Constraints

(i) Academic Staff Level Goal:

$$\sum_i X_{ijt} \geq g_{jt} \quad \text{for all } j,t. \qquad (3.10)$$

Defining n^a_{jt} as deviational variable correponding to the underachievement of g_{jt}; and p^a_{jt} as deviational variable corresponding to the overachievement of g_{jt}; then (3.10) can be re-written as

$$\sum_i X_{ijt} + n^a_{jt} - p^a_{jt} = g_{jt} \quad \text{for all } j,t. \qquad (3.11)$$

(3.10) and (3.11) require that the total academic staff strength in faculty j should be at least as much as the desired goal set in the faculty in period t.

(ii) Student enrolment goal:

$$\sum_i X_{ijt} \geq \frac{S_{jt}}{h_{jt}} \quad \text{for all } j,t. \qquad (3.12)$$

Adding deviational variables, (3.12) becomes

$$\sum_i X_{ijt} + n^b_{jt} - p^b_{jt} = \frac{S_{jt}}{h_{jt}} \quad \text{for all } j,t. \qquad (3.13)$$

where n^b_{jt} and p^b_{jt} are respectively negative and positive deviational variables corresponding to the student enrolment goal.

Equations (3.12) and (3.13) imply that the number of academic staff available in faculty j, period t, must be at least, that required for the desired student enrolment as specified by the staff/student ratio.

(iii) Academic Rank Distribution Goal:

$$\frac{\sum_{i\epsilon\rho} X_{ijt}}{\sum_i X_{ijt}} = \alpha_{ijt} \quad \text{for all } i\epsilon\rho\,,\ j,t \qquad (3.14)$$

where $\{\rho\}$ = { A,B,C } partitions the academic hierarchy into three mutually exclusive and collectively exhaustive sets defined by

(i) A = {i=1,2 } is the set of the lecturer grade consisting of assistant lecturers and lecturers II and I;

(ii) B = {i=3 } is the set of Senior Lecturer grade.

(iii) $C = \{ i=4,5 \}$ is the set of professorial grade consisting of Readers and Professors.

(3.14) is saying that the actual distribution of a particular grade of the hierarchy cannot exceed the desired proportion set for the faculty and period.

Adding deviational variables and linearizing, (3.14) becomes

$$\sum_{i \epsilon \rho} X_{ijt} - \alpha_{ijt} \sum_{i} X_{ijt} + n^{c}_{ijt} - p^{c}_{ijt} = 0 \qquad (3.15)$$

for all $i \epsilon \rho$, j, t.

where n^{c}_{ijt} and p^{c}_{ijt}, respectively, are the negative and positive deviational variables associated with the rank distribution goal.

E. Formulation of the Objective Function

Let P^{a}_{jt}, P^{b}_{jt} and P^{c}_{ijt} be the respective priority weights or factors attached by the DM to academic staff level goal, student enrolment goal and academic rank distribution goal. For model solution, as was pointed out in Section 3.1, the non-goal constraints are converted to equality constraints by adding deviational variables. Let n^{d}_{jt} and p^{d}_{jt}; and P^{d}_{jt} be the deviational variables as well as the priority factors attached to maximum hiring absolute objective (goal); and n^{e}_{t}, p^{e}_{t} as well as P^{e}_{t} be the deviational variables and priority factors attached to the Budget absolute objective. Then, the objective function which minimizes the weighted sum of the deviational variables can be written as:

Minimize

$$\sum_j \sum_t (P^a_{jt}(n^a_{jt} + p^a_{jt}) + P^b_{jt}(n^b_{jt} + p^b_{jt}) + P^d_{jt}(n^d_{jt} + p^d_{jt}))$$

$$+ \sum_t P^e_t (n^e_t + p^e_t) + \sum_i \sum_j \sum_t P^c_{ijt} (n^c_{ijt} + p^c_{ijt}) \tag{3.16}$$

In general, for model solution, not all the deviational variables will appear in the objective function. The appearance of any deviational variables in the objective function is dependent on the judgement of the DM. If, for example, he decides that underachieving a particular goal is desirable, then the negative deviational variables corresponding to this goal is dropped from the objective. Similarly, if overachievement is desirable to him, the positive deviational variables corresponding to the goal is dropped from the objective function. When exact achievement of a goal is desired, all the deviational variables corresponding to the goal are retained in the objective function. In Chapters 5 and 6, we shall discuss the details of how the objective functions used in this study were determined.

3.3.2 Model Modification

In its present form, the model can be very large when applied to planning a big university. To reduce the number of variables as well as the number of constraints, the academic staff flow equation (3.7) can be solved for X_{ijt}, giving

$$X_{ijt} = \frac{1}{\beta_{ijt}} (X_{ijt+1} - Y_{ijt+1} - \gamma_{ijt} X_{i-1,jt}) \qquad (3.17)$$

The expression is then substituted for X_{ijt} in (3.9), (3.11), (3.13) and (3.15) to get (3.18), (3.19), (3.20) and (3.21) respectively, thus:

$$\sum_i \sum_j \frac{c_{ijt}}{\beta_{ijt}} (-\gamma_{ijt} X_{i-1,jt} + X_{ijt+1} - Y_{ijt+1})$$

$$+ n^e_{jt} - p^e_{jt} = B_t \qquad (3.18)$$

for all t.

$$\sum_i \frac{1}{\beta_{ijt}} (- \gamma_{ijt} X_{i-1,jt} + X_{ijt+1} - Y_{ijt+1}) + n^a_{jt} -$$

$$p^a_{jt} = g_{jt} \qquad (3.19)$$

for all j,t.

$$\sum_{i} \frac{1}{\beta_{ijt}} (-\gamma_{ijt} X_{i-1,jt} + X_{ijt+1} - Y_{ijt})$$

$$+ n^{b}_{jt} - p^{b}_{jt} = \frac{S_{jt}}{h_{jt}} \text{ for all } j,t \qquad (3.20)$$

$$\sum_{i\epsilon\rho} \frac{1}{\beta_{ijt}} (- \gamma_{ijt} X_{i-1,jt} + X_{ijt+1} - Y_{ijt+1})$$

$$- \alpha_{ijt} \sum_{i} \frac{1}{\beta_{ijt}} (- \gamma_{ijt} X_{i-1,jt} + X_{ijt+1} - Y_{ijt+1})$$

$$+n^{c}_{ijt} - p^{c}_{ijt} = 0 \text{ for all } i\epsilon\rho, j, t. \qquad (3.21)$$

3.3.3 Model Summary

The model formulated in Section 3.3.1 and modified in section 3.3.2 will be summarized in this section in the form that is akin to the one that will be used in the study.

A. Objective Function

$$\text{Minimize} \sum_{j} \sum_{t} (P^{a}_{jt} (n^{a}_{jt} + p^{a}_{jt}) + P^{b}_{jt} (n^{b}_{jt} + p^{b}_{jt})$$

$$+ P^{d}_{jt} (n^{d}_{jt} + p^{d}_{jt})) + \sum_{t} P^{e}_{t} (n^{e}_{t} + p^{e}_{t})$$

$$+ \sum_{i} \sum_{j} \sum_{t} P^{c}_{ijt} (n^{c}_{ijt} + p^{c}_{ijt}) \qquad (3.22)$$

Subject to:

B. Goal Constraints

(i) Academic Staff Level Goal:

$$\sum_i \frac{1}{\beta_{ijt}} (-\gamma_{ijt} X_{i-1,jt} + X_{ijt+1} - Y_{ijt+1}) + n^a_{jt}$$

$$- p^a_{jt} = g_{jt} \qquad \text{for all } j,t \qquad (3.23)$$

(ii) Student Enrolment Goal:

$$\sum_i \frac{1}{\beta_{ijt}} (-\gamma_{ijt} X_{i-1,jt} + X_{ijt+1} - Y_{ijt+1})$$

$$+ n^b_{jt} - p^b_{jt} = \frac{S_{jt}}{h_{jt}} \qquad \text{for all } j,t \qquad (3.24)$$

(iii) Academic Rank Distribution Goal:

$$\sum_{i \epsilon \rho} \frac{1}{\beta_{ijt}} (-\gamma_{ijt} X_{i-1,jt} + X_{ijt+1} - Y_{ijt+1})$$

$$- \alpha_{ijt} \sum_i \frac{1}{\beta_{ijt}} (-\gamma_{ijt} X_{i-1,jt} + X_{ijt+1} - Y_{ijt+1})$$

$$+ n^c_{ijt} - p^c_{ijt} = 0 \quad \text{for all } i \epsilon \rho, j, t. \qquad (3.25)$$

C. Non-Goal Constraints

(i) Maximum Hiring Constraint:

$$\sum_i Y_{ijt} + n^d_{jt} - p^d_{jt} = U_{jt}$$

for all j,t. (3.26)

(ii) Academic Payroll Budget Constraint:

$$\sum_i \sum_j \frac{c_{ijt}}{\beta_{ijt}} \left(-\gamma_{ijt} X_{i-1,jt} + X_{ijt+1} - Y_{ijt+1}\right)$$

$$+ n_t^e - p_t^e = B_t \quad \text{for all } t. \qquad (3.27)$$

All decision and deviational variables are non-negative.

In its present form, the model can be related to the GP formulation of Section 3.1. The objective function (3.22) is similar to (3.1). Equations (3.23)-(3.25) correspond to (3.2). However, some of the entries of the vector g in equations (3.23)-(3.25) will be zeros. Finally, equations (3.27) and (3.28) of our formulation correspond to (3.3), though in the former, the constraints have been converted to goals by adding deviational variables.

3.3.4 Model Size Estimation

For policy recommendation purposes, two variants of the model will be considered. Variant I will be as formulated in Section 3.3.1, while Variant II will drop the controversial rank distribution goal. It is hoped that useful suggestions might emanate from considering two variants of the model. Table 3.1 estimates the sizes of the two variants of the model. Because of the mode of data available, five academic staff ranks

Table 3.1 Model size estimation

Model Structure Under Consideration	Type	Variant I Size	Variant II Size
Decision Variables	X_{ijt} (including t+1)	240 (5X8X6)	240
	Y_{ijt} "	240 "	240
		480	480
Deviational Variables	n^a_{jt} and p^a_{jt}	80 (8X5X2)	80
	n^b_{jt} and p^b_{jt}	80 "	80
	n^c_{ijt} and p^c_{ijt} (for $i \epsilon \rho$)	240 (3X8X5X2)	Not Applicable
	n^d_{jt} and p^d_{jt}	80 (8X5X2)	80
	n^e_t and p^e_t	10 (5X2)	10
		490	250
Constraints	Academic Staff Level Goal	40 (5X8)	40
	Student Enrolment Goal	40 "	40
	Rank Distribution Goal	120 (3X8X5)	Not Applicable
	Maximum Hiring Budget	40 (5X8)	40
		5	5
		245	125

will be considered as done in Akinlade (1979), namely, i=1 represents Assistant Lecturer grade, i=2 means Lecturer I and II grades combined, i=3 implies Senior Lecturer grade, while i=4 refers to the Reader grade and i=5 means the Professor grade. Since the University of Ibadan will be used as a case study, $1 \leq j \leq 8$, (i.e. there are eight faculties/colleges.) The following notation will be used to distinguish the faculties:

j = 1 represents College of Medicine

j = 2 means Faculty of Arts

j = 3 refers to Faculty of Science

j = 4 implies Faculty of Agriculture and Forestry

j = 5 means Faculty of Education

j = 6 represents Faculty of the Social Sciences

j = 7 refers to Faculty of Veterinary Medicine,

while j = 8 means Faculty of Technology.

A planning horizon of five years will be used in the study primarily because it coincides with the planning horizon used by the University Planning Office and partly because a period less than five years seems rather short for meaningful strategic planning.

Table 3.1 indicates that Variant I of the model has a maximum size of 245 constraints by 970 variables (including deviational variables) while that of Variant II is of the order of 125 constraints by 730 variables.

3.4 Data Types and Sources

3.4.1 Data Types

Most of the data needed for this study are defined in Section 3.3.1 and they can be split into two broad classes: financial or monetary data and non-financial data. Included in the class of financial data are (average) salary of academic staff of particular rank by faculty and each year of the planning horizon; and the total budget of the University for each year of the planning horizon. In the group of non-financial data are specified goals like academic staff level goal in each faculty for each year; standard or desired staff/student ratio for each faculty and year; student enrolment level for each faculty and year; and upperbound on the number of academic staff that can be recruited in each faculty and year of the planning horizon. Parameters like α_{ijt}, β_{ijt} and γ_{ijt} will be estimated from such data as historical size of acadmic staff by rank as well as movement between the various ranks of the academic hierarchy in each faculty for a ten-year period.

3.4.2. Data Sources

Financial data were obtained mainly from the files of the University Bursary, though annual publications like University of Ibadan Budget Estimates and University of Ibadan Audited Accounts were used as aids to make forecasts for each year of the planning horizon.

Data relating to historical size of the academic staff by rank and faculty were estimated from three sources:

(a) University of Ibadan Budget Estimates;

(b) files of the University Planning Office; and

(c) University of Ibadan Official Calendars.

The University Establishments Office supplied information relating to the movement of academic staff through the various ranks of the academic hierarchy, as well as the wastage rates of academic staff due to resignation, retirement, death, etc. Parameters like standard staff/student ratios and academic staff level goals by faculty and year as well as student enrolment goals by faculty and year, were obtained or estimated from data collected mainly from the files of the University Planning Office.

3.5 Limitations of the Model

This study has three main limitations:

(i) It is mainly concerned with resource allocation in higher education as an economic issue. However, education being a vehicle of social transformation, decisions affecting it, more often than not, have political undertones that have not been explicitly considered in the study. But the results of the study can provide an objective basis for making informed decisions by policy makers who, as the ultimate decision makers, can take into consideration political and social factors, if need be, to make the final decision.

(ii) The model is mainly deterministic. However, extensive sensitivity analysis can help take care of uncertainties in parameter estimation. Moreover, most of the parameters are estimated using the Markovian framework; thus giving some stochastic stance to the model.

(iii) A linear relationship between decision variables and parameters is assumed. Also decision variables are assumed to be continuous. Ideally, integer goal programmes should have been used, but the state-of-the-art of integer goal programming is still in its infancy. Even classical single criterion linear integer programming problems can be difficult to solve. However, the method adopted by the study seems justified by what obtains in the university in practice--the University Development and Planning Office uses the Full-Time-Equivalent (F.T.E.) approach for allocating academic resources and this assumes that decision variables are continuous.

CHAPTER 4

EMPIRICAL ESTIMATION OF PARAMETERS

The methods of parameter estimation adopted in this study can be broadly classified into two:

(a) Markovian parameter estimation, and

(b) simple statistical estimations.

However, for certain parameters, a combination of both methods was employed while in some cases, some parameters were standard values fixed by policy decisions of the N.U.C. or the university.

In the first group were parameters like:

β_{ijt}: the proportion of academic staff of rank i who stay from period t to period t+1 in faculty j;

γ_{ijt}: proportion of academic staff promoted from rank i-1 to rank i during period t in faculty j.

The second group parameters include:

S_{jt}: desired student enrolment (headcount or F.T.E.) in faculty j, period t.

U_{jt}: upperbound on the number of academic staff that can be recruited in faculty j, period t.

The parameters estimated using a combination of both Markovian and simple statistical estimation procedures include:

B_t: total academic payroll budget at the beginning of period t.

g_{jt}: academic staff level goal desired in faculty j at period t.

C_{ijt}: average salary per academic staff type by faculty and year.

Parameters like h_{jt}: the desired student/staff ratio in faculty j, period t and α_{ijt}: desired proportion of academic staff of rank i, faculty j at period t, are standard values fixed by policy decisions of the N.U.C. or the University.

4.1 Markovian Parameter Estimation

A system can be modelled using a first-order Markov chain if it satisfies the following properties[1]

(i) The set of possible outcomes is finite.

(ii) The probability of the next outcome depends only on the outcome immediately before.

(iii) The probabilities are constant over time.

A manpower planning system often satisfies fully the first and third conditions; however, the second condition is usually only approximated because the probability of promotion from one grade to the other in the system depends also on other factors like differential individual ability and educational background, etc. This approximation notwithstanding, Markov chain modelling has been used successfully for manpower planning

1. Shamblin and Stevens (1974), p.53.

in organization, e.g. Zanakis and Maret (1980)[1].

For estimation of some parameters in this study, the academic hierarchy was divided into five with each grade representing a state of the Markov chain. The grades of the hierarchy have been elaborated upon in the last chapter. An additional state, representing wastage was also defined. This state is an absorbing state because once entered, transition from it, is not possible. The wastage state defines the proportion of people leaving the manpower system as a result of resignation, death, dismissal or retirement.

To estimate the transition probability matrix (T.P.M.) of each faculty/college, historical data of movements between the various grades of the academic hierarchy were collected for the ten year period 1970/71-1979/80 from the files of the Development and Planning Office as well as from those of the Establishment Office, the University of Ibadan Annual Budget Estimates and the University Calendars. The raw data showing the transition between the various ranks of the academic hierarchy and the wastage state are contained in Appendices 1A-1H.

In estimating the T.P.M. for each faculty, the following notation is introduced:

m = total number of absorbing and nonabsorbing states;

a = total number of nonabsorbing states;

1. See also Akinlade (1979), Grinold and Marshall (1977); Roland and Sovereign (1969), Hopes (1973), Nelson and Young (1973), and Merch (1970).

T = total number of time periods for which historical data were collected;

$N_{ij}(t)$ = number of persons moving from state i to state j in period t.

Then

$$N_i(t) = \sum_{j=1}^{m} N_{ij}(t) \qquad (4.1)$$

gives the total number of people available in state i at the beginning of period t.

$$\hat{p}_{ij}(t) = \frac{N_{ij}(t)}{N_i(t)} \qquad (4.2)$$

is the proportion of people that moved from state i to state j during period t.

$$\hat{p}_{ij} = \frac{\sum_{t=1}^{T} N_{ij}(t)}{\sum_{t=1}^{T} \sum_{j=1}^{m} N_{ij}(t)} \qquad (4.3)$$

gives theestimate of the transition probability from state i to state j. This is assumed constant overtime but the validity of this assumption will be tested at a given significance level.

Appendices 1A-1H give the values of $N_{ij}(t)$ for various faculties. The last rows of the appendices give sums of $N_i(t)$ for the whole period for each faculty. The numerator in equation (4.3) is given by the appropriate entries of Appendix 2 while the row total of the tables in this appendix give the values of the denominator in the equation. Therefore, dividing each entry of this appendix by appropriate row totals will give estimates of the transitional probabilities. The estimated transition probability matrix for each faculty/college is shown in Table 4.1.

One important assumption of Markov chain modelling that needs testing for the purpose of this study is the stationarity assumption of the T.P.M. and each transition probability. The stationary assumption hypothesizes that the T.P.M. and each individual transitional probability is constant over time and hence is time-independent. The χ^2 test can be used to test the stationarity assumption (Zanakis and Maret, 1980) as follows. At α significance level, the $(i,j)^{th}$ transition probability is constant over time if

$$\frac{\sum_{t=1}^{T} N_i(t)\,[\hat{p}_{ij}(t) - \hat{p}_{ij}]^2}{\hat{p}_{ij}} < \chi^2_{\alpha[T-1]} \qquad (4.4)$$

The entire T.P.M. is constant over time if

$$\frac{\sum_{i=1}^{a} \sum_{j=1}^{m} \sum_{t=1}^{T} N_i(t)\,[\hat{p}_{ij}(t) - \hat{p}_{ij}]^2}{\hat{p}_{ij}} < \chi^2_{\alpha[a(m-1)(T-1)]} \qquad (4.5)$$

TABLE 4.1

Transition probability matrices
By faculty/college

	A.L.	LECT.	S.L.	READ.	PROF.	WASTAGE
			(a) Medicine			
A.L.	0.7619	0.2143	0	0	0	0.0238
LECT.	0	0.8267	0.1487	0	0	0.0246
S.L.	0	0	0.9119	0.0313	0.0478	0.0090
READ.	0	0	0	0.8667	0.1000	0.0333
PROF.	0	0	0	0	0.9858	0.0142
WASTAGE	0	0	0	0	0	1.0000
			(b) Arts			
A.L.	0.7857	0.1905	0	0	0	0.0238
LECT.	0	0.9164	0.0538	0	0	0.0298
S.L.	0	0	0.9203	0.0319	0.0239	0.0239
READ.	0	0	0	0.7667	0.0333	0.2000
PROF.	0	0	0	0	0.9580	0.0420
WASTAGE	0	0	0	0	0	1.0000
			(c) Science			
A.L.	0.7800	0.1200	0	0	0	0.1000
LECT.	0	0.8716	0.0750	0	0	0.0534
S.L.	0	0	0.9084	0.0393	0.0288	0.0236
READ.	0	0	0	0.9032	0.0430	0.0538
PROF.	0	0	0	0	0.9490	0.0510
WASTAGE	0	0	0	0	0	1.0000
			(d) Agric. & Forestry			
A.L.	0.6818	0.1364	0	0	0	0.1818
LECT.	0	0.8481	0.1224	0	0	0.0295
S.L.	0	0	0.8466	0.0797	0.0413	0.0324
READ.	0	0	0	0.8736	0.0690	0.0574
PROF.	0	0	0	0	0.9575	0.0425
WASTAGE	0	0	0	0	0	1.0000

TABLE 4.1 (continued)

	A.L.	LECT.	S.L.	READ.	PROF.	WASTAGE
			(e) Education			
A.L.	0.7566	0.1778	0	0	0	0.0666
LECT.	0	0.8621	0.1035	0	0	0.0344
S.L.	0	0	0.9059	0.0412	0.0235	0.0294
READ.	0	0	0	0.7368	0.1579	0.1053
PROF.	0	0	0	0	0.9348	0.0652
WASTAGE	0	0	0	0	0	1.0000
			(f) Social Sciences			
A.L.	0.7000	0	0	0	0	0.3000
LECT.	0	0.8623	0.1199	0	0	0.0178
S.L.	0	0	0.9020	0.0412	0.0516	0.0052
READ.	0	0	0	0.8235	0.1177	0.0588
PROF.	0	0	0	0	0.9886	0.0114
WASTAGE	0	0	0	0	0	1.0000
			(g) Vet. Medicine			
A.L.	0	0	0	0	0	0
LECT.	0	0.8732	0.0976	0	0	0.0292
S.L.	0	0	0.8429	0.0429	0.1142	0
READ.	0	0	0	0.7000	0.2000	0.1000
PROF.	0	0	0	0	0.9063	0.0937
WASTAGE	0	0	0	0	0	1.0000
			(h) Technology			
A.L.	0.8333	0.1667	0	0	0	0
LECT.	0	0.8846	0.0962	0	0	0.0192
S.L.	0	0	0.9394	0.0303	0.0303	0
READ.	0	0	0	1.0000	0	0
PROF.	0	0	0	0	1.0000	0
WASTAGE	0	0	0	0	0	1.0000

The X^2 tests indicate that the entire T.P.M. is stationary for all faculties at 0.01 level of significance. Table 4.2 gives the computed X^2 values for each T.P.M. by faculty/college. The critical X^2 value at 0.01 and 245 degrees of freedom is 282.50.

TABLE 4.2

Stationary test; values of computed chi-square for each T.P.M. by faculty/college

Faculty/College	Computed Chi-square
Medicine	215.56
Arts	181.75
Science	112.46
Education	100.97
Agric. & Forestry	88.22
Social Sciences	86.39
Vet. Medicine	64.59
Technology	44.17

All the 120 individual transitional probabilities are stationary at 0.01 level except five: two in the faculty of Arts, two in the College of Medicine and one in the faculty of Technology. These probabilities are made up as follows:

(a) Faculty of Arts--probability of transition from Lecturer grade to Wastage with a computed chi-square of 67.79 and probability of transition from Senior Lecturer grade to Wastage with a computed chi-square of 25.74.

(b) College of Medicine--Probability of transition from Lecturer to Senior Lecturer with a computed chi-square of 74.95 and probability of transition from Senior Lecturer to Reader with a computed chi-square of 28.68.

(c) Faculty of Technology--probability of transition from Lecturer grade to Wastage with a computed chi-square of 24.02.

However, only two of these five non-stationary probabilities are relevant model parameters or useful in estimating other model parameters. Since all the T.P.M.'s are stationary, it is assumed that the results will not be affected much by the non-stationarity of just two transition probabilities. Appendix 3 shows the computed chi-square for all the transition probabilties.

From the T.P.M. for each faculty, we can now obtain parameters like β_{ijt} and γ_{ijt}. The stationarity test performed in the foregoing, establishes that both are time-invariant. β_{ijt} is found in the diagonal of the T.P.M. corresponding to faculty j while γ_{ijt} is given in the appropriate upper triangular portion of the T.P.M. of faculty j (see Table 4.1).

4.2 Simple Statistical Estimation of Parameters

Two groups of parameters were estimated using simple statistical techniques of taking averages and percentages of certain quantities. These are desired student enrolment by faculty and year, and the upperbound on the number of academic staff that can be recruited in a given faculty and year of the planning horizon.

4.2.1 Projected Student Enrolment by Faculty and Year (S_{jt})

The University of Ibadan Development and Planning Office have in their files projected student enrolment by headcount where no course system exists and by F.T.E. where there is course system for only two years of the planning horizon of this study: 1984/85-1985/86. In addition, they also have aggregated figures for the 1990's. However, actual enrolments for 1979/80 are also available. On the assumption of a constant annual percentage increase between 1979/80 and 1984/85, the projected student enrolment for each year was determined using the equation

$$S_{jt} = S_{jo} (1 + t r_j) \qquad (4.6)$$

where

S_{jt} = projected student enrolment in faculty j during year t

S_{jo} = actual student enrolment in faculty j in the base year, i.e. 1979/80

t = number of years with t=0 referring to 1979/80, t=1 is 1980/81, etc.

r_j = constant annual rate of increase of student enrolment in faculty j.

Table 4.3 shows the projected student enrolment obtained using equation (4.6). The staff strength determined from the projected

TABLE 4.3

Projected student enrolment by college/faculty and year*

College/Faculty	Base Year Enrolment (1979/80)	Annual Growth Rate	1981/82	1982/83	1983/84	1984/85	1985/86
Medicine	1493	0.05	1642	1717	1792	1846	1870
Arts	1061	0.10	1273	1379	1485	1575	1602
Science	1180	0.18	1604	1817	2010	2221	2258
Agric. & Forestry	805	0.26	1224	1433	1642	1838	1874
Education	975	0.06	1034	1092	1151	1262	1277
Social Sciences	767	0.41	1396	1710	2025	2344	2360
Vet. Medicine	279	0.07	318	338	357	381	399
Technology	294	0.35	450	603	706	815	831

*Figures for the College of Medicine and the Faculty of Vet. Medicine were based on headcount while others were determined using F.T.E.

student enrolment of Table 4.3 is shown in Table 4.4. For certain faculties, like Arts, Education and Veterinary Medicine, the standard student/staff ratios were more or less long-term goals because if they were used, some academic staff would have to be retrenched. In the Faculty of Arts, the actual ratio used was 11:1 while in Education, it was 16:1, and in Veterinary Medicine a ratio of 7:1 was used. This conforms with the practice of the Development and Planning Office. However, for other faculties, the standard ratio laid down by the N.U.C. guidelines was used.

TABLE 4.4

Staff strength determined from
the projected student enrolment

College/Faculty	1981/82	1982/83	1983/84	1984/85	1985/86
Medicine	235	245	256	264	267
Arts	115	125	135	143	150
Science	160	181	201	222	226
Agric. & Forestry	122	143	162	184	187
Education	64	68	72	78	78
Social Sciences	93	114	135	156	157
Vet. Medicine	45	48	51	76	76
Technology	45	60	71	82	83

4.2.2 Upper-bound on the Number of Academic Staff That Can be Recruited in a given Year and Faculty (U_{jt}).

From the period 1969/70 to 1978/79, 15% of the academic staff (i.e., 796 members of the academic staff out of a total of 5126) were recruited (Akinlade, 1979). (See Table 4.5.) Thus on the average, it can be said that, university-wide, 15% of the staff are recruited annually. Table 4.6 shows the estimated upper bound on the number of academic staff that can be recruited using this criterion on the projected staff strength of Table 4.4.

TABLE 4.5

Distribution and recruitment of academic staff by rank during 1969/70-1978/79

Academic Staff Rank	Total Number of Staff	Total Number Recruited	Percentage
Assistant Lecturer	246	103	42
Lecturer	2661	630	24
Senior Lecturer	1333	43	3
Reader	180	2	1
Professor	706	13	2
Total All Grades	5126	791	15

Source: Computed from Tables I & II of Akinlade (1979).

TABLE 4.6

Upper bounds on the number of academic staff that can be recruited by faculty and year

College/Faculty	1981/82	1982/83	1983/84	1984/85	1985/86
Medicine	35	37	38	40	40
Arts	17	19	20	22	23
Science	24	27	30	33	34
Agric. & Forestry	18	22	24	28	28
Education	10	10	11	12	12
Social Sciences	14	17	20	23	24
Vet. Medicine	7	7	8	11	11
Technology	7	9	11	12	13

4.3 Combination of Markovian and Simple Statistical Estimation

Parameters estimated by a combination of Markovian and simple statistical estimation procedures and (i) average salary of each academic staff by rank in each faculty and year of the planning horizon; (ii) total payroll budget at the beginning of each year, and (iii) academic staff level goal desired in each faculty and year.

4.3.1 Academic Staff Level Goal Desired in Each Faculty and Year (g_{jt})

The academic staff level goal was estimated using the staff-flow equation (3.7). We rewrite this equation here for convenience:

$$X_{ijt+1} = \beta_{ijt}X_{ijt} + Y_{ijt+1} + \gamma_{ijt}X_{i-1,jt} \quad \text{for all } i,j,t.$$

This equation states that the number of academic staff of a given rank in year t+1 is made up of those remaining on that grade plus those recruited and those promoted from a lower rank. Two of the terms, namely $\beta_{ijt}X_{ijt}$ and $\gamma_{ijt}X_{i-1,jt}$ are Markovian and each is estimated using the appropriate transition probability in the corresponding T.P.M. Since we have established that the T.P.M.'s and the transition probabilities are stationary, β_{ijt} and γ_{ijt} are, therefore, time-invariant. To estimate Y_{ijt+1}, we use the secondary data of Table 4.5. For example, for the Assistant Lecturer grade, on the average, 42% of staff on this grade are recruited annually while only 2% of Professors are recruited each year. However, there is the possibility of the forecast estimates of the number of staff recruited in the Assistant Lecturer grade being bloated. The forecast estimates of the staff level goals using the staff flow equation are shown in Table 4.7. These estimates can be interpreted as the desired academic staff level goals assuming that the current rates of advancement and recruitment of staff are maintained. However, it does not take into consideration whether enough students will be available for such staff to teach. Neither does it take

TABLE 4.7

Forecast estimates of academic staff
level goals by faculty and year

College/Faculty	1981/82	1982/83	1983/84	1984/85	1985/86
Medicine	238	256	275	296	321
Arts	149	167	189	214	246
Science	100	112	124	138	153
Agric. & Forestry	76	83	89	95	101
Education	68	79	92	107	123
Social Sciences	77	84	91	99	107
Vet. Medicine	82	91	101	111	122
Technology	31	37	44	50	59

cognizance of the availabilities of necessary infra-structures and other facilities required for use by such students. From the point of view of healthy labour relations, however, it appears desirable to have such a goal because it incorporates the goals and desires of employees into the planning process. It is from this viewpoint that this goal does not seem to be superfluous.

A comparison of tables 4.4 and 4.7 will show some differences in the forecast estimates. While in some faculties, forecast estimates of the staff level goals are much greater than the estimated staff strength based on student enrolment, the reverse is the case in certain faculties. For example, in the Faculties of Arts, Education, Veterinary Medicine and the

College of Medicine, the estimated staff level goals are larger (sometimes substantially) than the estimated staff strength based on projected student enrolment. In the Faculties of Science, Agriculture and Forestry, Social Sciences, and Technology, the staff level goals are much smaller. One advantage of the staff flow equation is that it is much easier to estimate total payroll budget estimates more objectively using the academic staff flow equation because it disaggregates the hierarchy into ranks. The estimated payroll budget using the staff equation will be about the same if it were possible to use the student enrolment because where one method under-estimates, the other method overestimates and vice versa.

4.3.2 Average Salary of Academic Staff by Rank, Faculty and Year (C_{ijt})

Initially, it was planned that this parameter will be estimated by extrapolating a simple linear trend using ten-year data for each rank and faculty. However, within the last decade, universities in this country have had two salary reviews: the Udoji Salaries Review Commission and the Cookey Commission with attendant jumps in average salaries. Since linear trend is a simple regression technique and since regression is always towards the mean, the jumps resulting from the reviews, might just be treated as outliers if a linear trend is used. Thus the resulting forecast estimate will probably underestimate the average cost.

On the assumption that the University System Scale (U.S.S.),which is now in use, will not change during the planning horizon, data on the individual staff were collected by research assistants from the bursary. Such data indicated the step of the scale each academic staff member was in the 1981/82 session. The total salary collected by each rank in each faculty for the session was computed and the average salary was found.

To make forecasts of the total salary for each rank and faculty for other years, the academic staff flow equation (3.7) was used. For each rank, the academic staff that remained on the same grade were assumed to have advanced to the step nearest the average salary of the previous year. Those promoted from the next lower rank and those recruited were assumed to start on Step 1 of the scale corresponding to the rank. Appendix 4 shows the University System Scale while the forecast average salary by rank, faculty and year is shown in Table 4.8.

TABLE 4.8

Forecast average cost by rank and faculty
for each year of the planning horizon
(Naira)

	Med.	Arts	Sci.	Agric. & Fores.	Educ.	Soc. Sci.	Vet. Med.	Tech.
				1981/82				
Asst. Lect.	6792	6723	6368	6336	6624	6537	--	6896
Lecturer	8656	8896	9125	9113	8871	8788	8802	8946
Snr. Lect.	11796	11652	11920	11678	11748	11729	11779	11868
Reader	13722	13722	13722	13832	13392	13557	13612	14052
Professor	15625	15524	15595	15678	15360	15609	15480	15720
				1982/83				
Asst. Lect.	6720	6706	6456	6336	6720	6566	--	6912
Lecturer	8632	8276	8927	8938	8929	8433	8975	8884
Snr. Lect.	11868	11868	12297	11840	12213	12367	12295	12286
Reader	13887	13832	14548	13788	13612	13722	13612	14712
Professor	15528	15655	15547	15560	15720	15450	15336	15720
				1983/84				
Asst. Lect.	6703	6709	6592	6432	6703	6566	--	6797
Lecturer	8643	8948	8942	8964	8908	8951	8977	8832
Snr. Lect.	11866	12355	12732	12286	12255	12876	12269	12307
Reader	13887	13832	14548	13812	13356	13722	13612	14712
Professor	15567	15655	15560	15568	15360	15493	15400	15720

TABLE 4.8 (continued)

	Med.	Arts	Sci.	Agric. & Fores.	Educ.	Soc. Sci.	Vet. Med.	Tech.
				1984/85				
Asst. Lect.	6562	6710	6580	6528	6691	6566	--	6848
Lecturer	8638	8953	8931	8946	8899	8987	8954	8908
Snr. Lect.	12364	12867	12772	12295	12250	12838	12756	12324
Reader	13887	13832	14548	14382	13356	13722	13612	14712
Professor	15562	15655	15571	15576	15432	15524	15432	15720
				1985/86				
Asst. Lect.	6477	6720	6583	6528	6696	6566	--	6816
Lecturer	8638	8942	8949	8979	8922	8969	8970	8921
Snr. Lect.	12362	12816	12804	12772	12702	12852	12714	12269
Reader	13887	13675	14548	14407	13356	13722	13612	14712
Professor	15560	15655	15581	15583	15458	15547	15458	15720

4.3.3 Total Academic Payroll Budget

To forecast the total payroll budget, the total cost estimated in the last section was cumulated over ranks and for each faculty and each year of the planning horizon. This gives the estimated actual emoluments for each faculty and year of the planning horizon. Each estimate was then multiplied by the average ratio of budget/actual expenditure determined from historical data for the periods 1970/71-1979/80. (See Appendix 5.) The estimates for all faculties were then added to give the payroll budget for the whole university for the particular year under consideration. The payroll budget so forecast by faculty and year, as well as the total forecast payroll budget for the whole university in each year of the planning horizon, are shown in Table 4.9.

4.4 Parameters Fixed by Policy Decisions

Two parameters belong to this class: the standard student/staff ratio fixed by the policy decision of the N.U.C. and the academic rank distribution proportions, α_{ijt} which are fixed by the policy decision of the University Council and made public by the Release 46 of 1981. Table 4.10 shows the standard student/staff ratio in each faculty. The proportion of academic staff of particular rank as fixed by the University Council is faculty-and time-invariant. Using the notation introduced in Chapter 3, the proportions are given by:

α_{Ajt} = 30% for all j and t

α_{Bjt} = 40% for all j and t

α_{Cjt} = 30% for all j and t.

TABLE 4.9

Forecast payroll budget by faculty and year
(Naira)

	1981/82	1982/83	1983/84	1984/85	1985/86
Medicine	3,434,681	3,699,832	3,968,138	4,323,628	4,669,761
Arts	1,887,472	2,028,792	2,359,481	2,659,541	2,948,566
Science	1,386,982	1,544,504	1,700,664	1,889,170	2,074,691
Agric. & Forestry	1,025,996	1,106,641	1,201,806	1,287,675	1,383,060
Education	850,420	1,009,237	1,174,127	1,369,613	1,589,441
Social Sciences	1,100,577	1,213,460	1,362,435	1,489,188	1,615,838
Vet. Medicine	1,087,252	1,235,677	1,377,303	1,527,237	1,673,228
Technology	570,542	681,395	729,581	894,852	1,042,288
Total	11,343,922	12,519,538	13,873,535	15,440,904	16,996,873

TABLE 4.10

Standard student/staff ratios

College/Faculty	Student/Staff Ratios	Criterion for Applying Ratios
Medicine	7:1	Headcount
Arts	15:1	F.T.E.
Science	10:1	F.T.E.
Agric. & Forestry	10:1	F.T.E.
Education	25:1	F.T.E.
Social Sciences	15:1	F.T.E.
Vet. Medicine	10:1	Headcount
Technology	10:1	F.T.E.

4.5 Substitution of Estimated Parameters into the Model

The model summarized in Section 3.3.3 was expanded by substituting for values of i, j and t such that $1 \leq i \leq 5$, $1 \leq j \leq 8$, and $1 \leq t \leq 5$. The coefficients of like terms of the model decision variables were collected in such a way that the decision variables were arranged sequentially in increasing order of their subscripts. It was in this form that the model parameters estimated in the foregoing sections of this chapter were substituted. The detailed model obtained after this substitution is shown in Appendix 6.

CHAPTER 5

BASIC MODEL SOLUTION AND DISCUSSION OF RESULTS

The model formulated for this study was solved using the revised simplex goal programming algorithm developed by Kang (1980) under the supervision of Professor Sang M. Lee on an IBM VM 370 computer in the University of Nebraska-Lincoln, U.S.A. The original programme was coded to handle 350 variables (including deviational variables), and 150 constraints and 15 priorities. It consists of a main programme and eight subroutines which carry out varying functions ranging from selecting the pivot column to printing out the output. The code was redimensioned and modified to handle models as big as 250 rows by 1,000 variables (including deviational variables) by Seung Ho Lee. In this form, it requires a memory size of the order of 600K.

As mentioned in Section 3.1, the goal programming model provides three types of solutions (Lee, 1972):

(i) the identification of input (resource) requirements to attain all desired goals;

(ii) the degree of goal attainment with the given inputs; and

(iii) the degree of goal attainment under various combinations of inputs and goal structures.

For this study, two variants of the model formulated in Chapter 3 will be solved. Each will be solved to obtain the types of solutions specified above, as much as possible.

5.1 Variant I, Run I.

In Variant I Run I, the aim is to identify the input requirements to attain all the desired goals. The priority structure of this run is as follows:

P_1 : Ensure that the university has adequate academic staff to meet the student enrolment goal in each college/faculty at the beginning of each year of the planning horizon.

P_2 : Attain as much as possible the academic rank distribution goal.

P_3 : The maximum hiring constraint should not be exceeded and the academic staff level goal should also not be exceeded. However, the goal of not exceeding the maximum hiring constraint is twice as important as that of not exceeding the academic staff level goal.

P_4 : All the foregoing goals should be achieved as much as possible with minimum budget.

The objective function associated with this priority structure can be stated as

$$\text{Minimize } P_1 \sum_{j=1}^{8} \sum_{t=1}^{5} n_{jt}^{b} + P_2 \sum_{i \varepsilon \rho} \sum_{j=1}^{8} \sum_{t=1}^{5} p_{ijt}^{c}$$

$$+ 2 P_3 \sum_{j=1}^{8} \sum_{t=1}^{5} p_{jt}^{d} + P_3 \sum_{j=1}^{8} \sum_{t=1}^{5} p_{jt}^{a}$$

$$+ P_4 \sum_{t=1}^{5} p_{t}^{e} \qquad (5.4)$$

The first group of terms in the objective function indicates that over-achievement of goals with priority one is desirable to the DM. This is reasonable since we wish to have as many teachers as possible to teach students. However, this can lead to excess number of academic staff. Ideally, we should aim at achieving the goal exactly, in which case both positive and negative deviational variables will appear in the objective function. Initially, the idea was to achieve both P_1 and P_2 exactly, in which case for the two priority levels both deviational variables appeared in the objective function. However, when the model was run, no solution was obtained after over 35 minutes of CPU time. When positive deviations were dropped from P_1 (40 of them) and negative deviations (120 of them) were dropped from P_2, a solution was obtained at about 20 minutes of CPU time. This corroborates the findings of Kang (1980) that the CPU time of the revised simplex goal programming algorithm tends to increase with the number of negative deviational variables in the objective function. The memory core utilized by the model in this form was 584K.

At priority 2 of the objective function (5.4), only the positive deviational variables will be minimized and so, negative deviational variables can appear in the solution. This is also true of all the lower order priorities, 3 and 4.

5.1.1 Analysis of Goal Attainment

Table 5.1 shows the result of analysis of the objective function of the goals stated earlier.

TABLE 5.1

Analysis of the objective function of Variant I, Run I

Goals	Degree of Attainment
Student Enrolment	Achieved
Academic Rank Distritution	Achieved
Maximum Hiring and Academic Staff Level	Unattained (898)
Minimize Budget	Achieved

From Table 5.1, we can see that the objective of minimizing the negative deviational variables at priority 1 is achieved. In fact, an examination of the deviational variables relating to these goals, indicates that the positive deviational variables are also minimized. The interpretation of this is that the number of academic staff determined by this model and under its various assumptions, will just be enough to achieve the student enrolment goal at the beginning of each year of the planning horizon.

Table 5.1 also indicates that the academic rank distribution goal is achieved. What this means is that in each year of the planning horizon, the distribution of academic staff is such that the percentage distribution prescribed by the press release No. 46 of 1981 is not exceeded. This goal can be under-achieved and, as will be seen later in the chapter, there are examples where zero percentage allocation can be made to some ranks.

The priority 3 goals are not achieved, i.e. the goals of not exceeding maximum hiring constraint and that of minimizing the overachievement of the academic staff level goal. However, an examination of the deviational variables appearing in the solution shows that all the positive deviational variables associated with the maximum hiring goal are zero. Thus, it is only the staff level goal that is unattained. This means that there are certain faculties, where the total number of academic staff allocated exceeded what it should be if the current rate of promotion and recruitment is maintained. The figure in Table 5.1 (i.e. 898) indicates that over the planning horizon, a total of 898 academic staff members are allocated over and above what they should be in certain faculties if the current rate of recruitment and promotion are to be maintained. It would not have been possible to see this result had we dropped the academic staff level goal and this type of result can be a potent negotiating weapon in favour of the University authority in any industrial negotiation between it and the Academic Staff Union of Universities (ASUU). Of course, a closer examination of the deviational variables of certain faculties will reveal that there is underachievement of the staff level goal in these faculties, i.e. the allocation of academic staff is below the current rate of promotion and recruitment in these faculties. This can be expected because the allocation of academic staff depends not only on the historical rate of recruitment and promotion, but also on other factors like shift in emphasis of government in funding of certain programmes and the demand for particular courses, etc.

Table 5.1 also indicates that the budget goal is completely achieved: academic staff are allocated between the various faculties and in each year of the planning horizon in such a way that the budget allocated to staff salaries in each year is not exceeded. This also means that the budget goal can be underachieved. A detailed discussion on this will be seen towards the end of the next subsection.

5.1.2 Analysis of Deviations from Stated Goals

Table 5.2 contains the values of the deviational variables from the academic staff level goal. As will be seen, for each faculty and year, there can only be one non-zero value of the positive deviation variable,p, and negative deviational variable,

TABLE 5.2

Values of deviational variables corresponding to the staff level goal by type, year and faculty/college

Faculty/ College	1981/82 p	n	1982/83 p	n	1983/84 p	n	1984/85 p	n	1985/86 p	n	Total p	n
Medicine	-	3	-	11	-	19	-	32	-	54	-	119
Arts	-	34	-	42	-	54	-	120	-	96	-	346
Science	0	0	69	-	77	-	84	-	73	-	303	-
Agric. & Forestry	0	0	60	-	73	-	89	-	86	-	308	-
Education	-	4	-	11	-	20	-	29	-	45	-	109
Social Sciences	-	12	30	-	44	-	57	-	50	-	181	12
Vet. Med.	-	37	-	43	-	50	-	35	-	46	-	211
Technology	0	0	23	-	27	-	32	-	24	-	106	-
Total	-	90	182	107	221	143	262	216	233	241	898	797

n. This conforms with equation (3.6). However, university-wide, both can be non-zero (see column totals of Table 5.2).

Only one faculty (the Faculty of the Social Sciences) has both non-zero values of positive and negative deviation over the five-year plan period. For the 1981/82 session, the underachievement of the academic staff level goal for this faculty is 12. Starting from 1982/83, it is expected that the historical rate of recruitment, promotion, and allocation of academic staff to the faculty will be exceeded by 30, reaching 50 by 1985/86. This appears plausible and seems to be in agreement with the long-term goals of the University for the current 1980/85 National Development Plan Period (see Chapter 2) given the fact that new programmes like Law, Banking and Finance, MILR and the MBA, which have just taken off in this faculty, will be expected to be "taking shape" during the planning horizon of this study.

Other faculties in which a shift of emphasis in programmes appears to have influenced the allocation of academic staff by the model are Science (with a total of 303 over the historical rate during the planning horizon); Agriculture and Forestry (308 over and above the historical rate during the planning horizon); and Technology (106 over and above the current rate for the period 81/82-85/86).

For the College of Medicine, Faculties of Arts, Education, and Veterinary Medicine, Table 5.2 tells a different story. Starting with the 1981/82 session, the slow-down in the allocation of academic staff in the College of Medicine below the historical rate in that college, takes a value of three and increases slowly at first to 19 by 1983/84 and almost doubles to 32 in 1984/85, reaching a value of 54 at the end of the planning horizon. The worst hit is the Faculty of Arts, which has a total of 346 in allocation of staff below its historical rate over the planning horizon. Thus, it can be seen that an analysis of deviational variables can reveal certain structural changes resulting from resource allocation as a result of shift in emphasis and development of new programmes. The faculties that have over-achievement of the academic staff level goal are those that are now operating new programmes or are expected to start new ones during the planning horizon of this study. The column totals reveal the relationship between over- and under-achievement of this goal by year university-wide. On the whole, throughout the planning horizon, 898 academic staff members are allocated over and above the historical rate while there will be a slow-down in allocation below the current rate by a value of 797 over the planning horizon resulting in a ratio of p:n of 1.13. This can be interpreted by saying that the percentage of allocation over the historical rate during the planning horizon will be nearly 13%.

The model achieved the academic rank distribution goal for the lecturer grade exactly in all faculties except the

Faculty of Veterinary Medicine. Thus, in all faculties except this one, both the positive and negative deviational variables are zero. Table 5.3 gives the values of the under-achievement of this goal for the Faculty of Veterinary Medicine.

TABLE 5.3

Values of the negative deviational variables for lecturer grade of the academic rank distribution goal in the Faculty of Veterinary Medicine by year

Year	n
1981/82	14
1982/83	14
1983/84	15
1984/85	23
1985/86	23

Given the budget, the structure of movement between the academic hierarchy as given by the TPM's of Table 4.1 and our goal structure, there is no way by which the model could allocate enough senior lecturers in such a way as to achieve the rank distribution structure of 40% during the planning horizon. Table 5.4 shows substantial underachievement of this goal in all faculties and in each year of the planning

TABLE 5.4

Values of the negative deviational variables for the senior lecturer grade of the academic rank distribution goal by faculty and year

Faculty/ College	1981/82	1982/83	1983/84	1984/85	1985/86
Medicine	94	97	102	106	107
Arts	46	50	54	38	60
Science	40	72	80	89	90
Agric. & Forestry	49	57	65	74	75
Education	26	27	29	31	31
Soc. Sci.	37	46	54	62	39
Vet. Med.	18	19	20	30	30
Technology	18	24	28	33	33

horizon. This table seems to justify the fear of members of the academic staff that the introduction of rigid percentage allocation of staff by rank irrespective of how productive an academic is, may not be in the best interest of academics. Furthermore, it goes on to suggest that making the senior lecturer grade a career grade in the academic hierarchy by allotting a higher percentage to it than the other two grades appears rather long-term and may only be achieved perhaps in a life-time. Of course, then a lot of frustration must have been caused academics due to prolonged underachievement of the goal.

The rank distribution goal for this professorial grade was achieved in nearly all faculties except the Faculty of Technology for all years of the planning horizon, the Faculty of Agriculture and Forestry from 1982/83 until the end of the planning horizon and the Faculty of Arts, for only 1984/85 (Table 5.5). The case of the Faculty of Technology can easily be

TABLE 5.5

Values of the negative deviational variables for the professorial grade of the academic rank distribution goal by faculty and year

Faculty/ College	1981/82	1982/83	1983/84	1984/85	1985/86
Medicine	0	0	0	0	0
Arts	0	0	0	33	0
Science	0	0	0	0	0
Agric. & Forestry	0	49	61	70	72
Education	0	0	0	0	0
Soc. Sci.	0	0	0	0	0
Vet. Med.	0	0	0	0	0
Technology	14	20	24	28	29

explained given the age of the faculty and the special structure of its TPM which has two apparent absorbing states (Reader and Professor) in addition to regular absorbing state of wastage (Table 4.1(h)). However, it is not as easy to explain the circimstances surrounding the vast underachievement of this goal in the Faculty of Agriculture and Forestry for almost every period of the planning horizon. Later on in this chapter, efforts will be made to demonstrate that the results do satisfy the constraints of the model and, as well, give a plausible explanation.

Table 5.6 indicates that there was only a case of exact achievement of the maximum hiring constraints (Education in 1985/86). In fact, the model suggested hiring new academic staff members only in two faculties, viz., Agriculture and Forestry in 1982/83 and Education in 1985/86 (see next section).

Table 5.7 gives the value of the unspent portion of the budget with the allocation made by the model as well as the respective relative value in relation to the budget of the University for each year of the planning horizon. The table suggests that the forecast budget for each year of the planning horizon is adequate.

TABLE 5.6

Values of the negative deviational variables
for the maximum hiring constraint
by faculty and year

Faculty/ College	1981/82	1982/83	1983/84	1984/85	1985/86
Medicine	35	37	38	40	40
Arts	17	19	20	22	23
Science	24	27	30	33	34
Agric. & Forestry	18	14	24	28	28
Education	10	10	11	12	0
Soc. Sci.	14	17	20	23	24
Vet. Med.	7	7	8	11	11
Technology	7	9	11	12	13

TABLE 5.7

Values of the negative deviational variables for the payroll budget goal by year

Year	n	% Unspent Budget
1981/82	₦ 2,368,616	21
1982/83	1,844,493	15
1983/84	2,025,170	15
1984/85	2,741,836	18
1985/86	3,419,795	20

5.1.3 Analysis of Decision Variables

In order to give the right interpretations to the values of the decision variables, reference will have to be made to their definitions and the forms of the formulation of the constraints of the model in Section 3.3.1.

In subsection A of this section, it will be seen that the decision variables are defined as the number of academic staff by rank at the beginning of particular years. Furthermore, in subsection D, the definition of the student enrolment goal--which has priority 1 in the model solution and hence the most important--states that the total number of academic staff obtained from the model solution must be the minimum number that is required for the projected student enrolment. Therefore, it must be borne in mind that the values determined by this model represent the minimum number of academic staff by rank to satisfy, in particular, the student enrolment goal, the budget constraint, the academic rank distribution goal and the maximum hiring constraint. The staff level goal is not completely satisfied by the solution. This result reveals the existence of conflicting goals in the University, as is expected of all real life organizations. The best that can be done is to achieve the goals as much as possible subject to the resources available. As advocated by Simon (1979), organizations should seek to satifice rather than optimize because global optimization is rather difficult to achieve due to the existence of conflicts of objectives in a world of limited resources and unlimited wants. Thus, the results of the model solution represent minimum, satisficing values only, for each rank, faculty and year of the planning horizon.

Tables 5.8–5.12 give the values of the decision variables distributed by rank and faculty for the years 1982/83–1986/87. Two of the academic ranks: lecturer grade and professorial grade are made up of two ranks as defined by this

TABLE 5.8

Distribution of minimum academic staff requirement by rank and faculty at the beginning of 1982/83

Faculty/ College	Lecturer Grade	Sen. Lect. Grade	Profess. Grade	New Recruitment	Total*
Medicine	54	86	61	-	201
Arts	27	42	27	-	96
Science	23	36	27	-	86
Agric. & Forestry	25	41	44	8 (Prof.)	110
Education	15	23	14	-	52
Soc. Sci.	20	34	23	-	77
Vet. Med.	-	27	10	-	37
Technology	11	32	-	-	43
Total	175	321	206	8	702

* Excludes new recruitment

TABLE 5.9

Distribution of minimum academic staff requirement by rank and faculty at the beginning of 1983/84

Faculty/ College	Lecturer Grade	Sen. Lect. Grade	Profess. Grade	New Recruitment	Total
Medicine	67	89	76	–	232
Arts	34	46	32	–	112
Science	45	66	52	–	163
Agric. & Forestry	32	89	–	–	121
Education	18	25	18	–	61
Soc. Sci.	25	41	33	–	99
Vet. Med.	–	28	15	–	43
Technology	17	44	–	–	61
Total	238	428	226	–	892

TABLE 5.10

Distribution of minimum academic staff requirement by rank and faculty at the beginning of 1984/85

Faculty/ College	Lecturer Grade	Sen. Lect. Grade	Profess. Grade	New Recruitment	Total
Medicine	72	93	80	-	245
Arts	37	50	34	-	121
Science	52	74	60	-	186
Agric. & Forestry	37	106	-	-	143
Education	19	26	20	-	65
Soc. Sci.	28	49	40	-	117
Vet. Med.	-	30	17	-	47
Technology	20	52	-	-	72
Total	265	480	251	-	996

TABLE 5.11

Distribution of minimum academic staff requirement by rank and faculty at the beginning of 1985/86

Faculty/ College	Lecturer Grade	Sen. Lect. Grade	Profess. Grade	New Recruitment	Total*
Medicine	75	96	83	-	254
Arts	28	58	-	-	86
Science	58	82	67	-	207
Agric. & Forestry	42	121	-	-	163
Education	21	60	6	12 (S.L.)	87
Soc. Sci.	33	56	47	-	136
Vet. Med.	-	45	23	-	68
Technology	24	61	-	-	85
Total	281	579	226	12	1086

* Excludes new recruitment

TABLE 5.12

Distribution of minimum academic staff requirement by rank and faculty at the beginning of 1986/87

Faculty/ College	Lecturer Grade	Sen. Lect. Grade	Profess. Grade	New Recruitment	Total
Medicine	76	97	84	–	257
Arts	40	55	38	–	133
Science	59	83	68	–	210
Agric. & Forestry	43	125	–	–	168
Education	21	28	21	–	70
Soc. Sci.	33	57	48	–	138
Vet. Med.	–	45	25	–	70
Technology	25	62	–	–	87
Total	297	552	284	–	1133

TABLE 5.13

Distribution of existing academic staff
by rank and faculty as at the end of 1981/82

Faculty/ College	Lecturer Grade	Sen. Lect. Grade	Profess. Grade	Total
Medicine	69	92	77	233
Arts	83	38	28	149
Science	46	31	23	100
Agric. & Forestry	28	22	26	76
Education	45	15	8	68
Soc. Sci.	30	30	17	77
Vet. Med.	42	25	15	82
Technology	19	8	4	31
Total	362	261	193	816

model. The lecturer grade is made up of assistant lecturers and lecturers--the latter having been combined from Lecturer I and II (see Chapter 3). The professorial grade is made up of the readers and professors. This type of presentation of result agrees with the practice of the Development and Planning Office. Besides, because of the goal of minimization of budget, the model allocated nearly all academic staff in these grades, in most cases, to the least cost choices, i.e. assistant lecturers in the lecturer grade and readers in the professorial grade. The truth is that these two grades in real life are the least populated in the University. This is due in part to the way the academic rank distribution is defined. For that goal, the academic hierarchy has to be broken into three mutually exclusive sets of lecturer grade, senior lecturer grade and professorial grades in order to be able to use the proportions specified by the University Council. Ideally, to get optimal distributions into the various cadres, proportions may have to be specified for each rank. However, Variant I seeks to investigate what the distribution would be like, if the rank distribution used is as specified by the University Council. In interpreting the result, therefore, the values for assistant lecturer and lecturer ranks were combined where both occur in the solution or for the assistant lecturer rank is taken as representing the lecturer grade where it is the only value occurring in the solution. A similar interpretation was given to the professorial grade. Because of this interpretation, the underachievement of the budget in each year is expected to be a little less than the model has shown, in real-life application. However, the budgeted values, it is clear, will be sufficient for the distribution, if they can be made available.

The model recommended that there should be recruitment only in two years of the planning horizon and in only two faculties and ranks. In Table 5.8, the model recommended that eight professors be recruited in the Faculty of Agriculture and Forestry at the beginning of 1982/83 session to make up for the fall in rank structure and meeting the student enrolment goal. In Table 5.11, the model recommended that 12 senior lecturers be recruited in the Faculty of Education at the beginning of the 1985/86 session. From the definition of the decision variables in Section 3.3.1 and the formulations of the academic staff flow equation (3.7), it will be seen that the row totals of all academic staff in a given faculty or throughout the University in Tables 5.8 and 5.11 should not include the newly recruited staff because it will amount to double counting. Thus, of the 44 professors recommended by the model for 1982/83 in the Faculty of Agriculture, eight must be newly recruited. Similarly, of the 60 senior lecturers recommended by the model for the Faculty of Education in 1985/86, twelve must be newly recruited.

On the whole, the model recommended that a minimum number of 201 academic staff will be required in the College of Medicine at the beginning of 1982/83 to meet the desired academic staff level goal of the college. The interpretation of the values for other faculties is similar.

Comparing the distribution of Table 5.8 with that of the existing academic staff determined from the Bursary records as at the end of 1981/82 (Table 5.13), it will be seen that the model results suggest that in terms of total number of academic staff available in each faculty, the existing number of academic staff exceeds the minimum number required to meet the forecast desired staff level goal for 1982/83 in nearly all the faculties. However, there is a wide variation in the amount by which the minimum required total number of academic staff is exceeded. This conclusion implicitly assumes that there is substitutability between skills among and between academic staff ranks. This being not necessarily so, it is possible that in a faculty where the existing number of academic staff exceeds the minimum number as determined by this model that requests for academic staff having skills not already available can be made and granted.

However, in the Faculty of Agriculture and Forestry, the existing staff is very much below the minimum amount required for meeting the student enrolment goal of that faculty by about 31%. This suggests that it is possible that under the existing arrangement, the academic staff in that faculty might be overstretched. In the Faculty of Arts, the number of existing staff exceeds the minimum number allocated by the model by nearly 55% while in the Faculty of Education, the minimum number is exceeded by about 31%.

Tables 5.8 and 5.13 also seem to justify the fear of academic staff about the use of the academic rank distribution proposals. From these tables, it will be seen that if this proposal is followed, there may be no promotion to certain ranks in some faculties for many years. For example, in the Faculty of Medicine, as at the end of 1981/82, there are 77 academic staff members in the professorial grade. However, this model determines that if the academic rank distribution proposal is adopted, given the present rate of promotion, by 1983/84, the number of academic staff in the professorial rank will be 76. It is only after that year that promotion can be made to the rank of professor, i.e. only at the beginning of 1984/85 can promotion be made to the rank of professor in the college if the proposal is to be in force. Therefore, irrespective of how productive an academic is, he may have to be in the same rank for nearly three years more if the rank distribution goal is used.

In the Faculty of Arts, the model solution suggests that there is a preponderance of academic staff in the lecturer grade as compared with those in the other grades as at the end of 1981/82. In spite of this, if the academic rank distribution proposal is used, staff on the lecturer grade will apparently be the losers. This is because there are 38 senior lecturers now and there are 83 staff in the lecturer grade. However, using the rank distribution proposal, there should be 42 senior lecturers at the beginning of 1982/83, i.e. only four lecturers should be promoted irrespective of productivity. Given that the model suggests that the existing number of academic staff in the Faculty of Arts is more than enough for its enrolment goal over the planning horizon (Tables 5.12 and 5.13), and that the University does not retrench academic staff, the 83 members in the lecturer grade will be moving up slowly at an average of 4-5 per year over the planning horizon. Thus by the end of the planning horizon, at most 20 of them would have become senior lecturers irrespective of their academic achievements and productivity.

The proposal appears to be beneficial to only very few faculties according to the model solution, for example, the Faculty of Agriculture and Forestry from 1982/83 and the Faculty of Science from 1983/84 if the solution is taken in its numerical face-value only. This is because in the case of the Faculty of Agriculture and Forestry, the higher number academic staff of the rank of senior lecturer and professorial grade allocated by the model for 1982/83 may be due in part to the fact that the model has identified that it seems that there is under-allocation of required staff to achieve the student enrolment goal under the present dispensation. Thus, the result of the model solution seems to confirm very clearly the fear of the academic staff members that the use of the academic rank distribution may not likely be in their best interests.

A look at the allocation made to the Faculty of Agriculture from 1983/84 will reveal that no allocation is made again to the professorial grade. This looks rather unreal as all the 44 members in the rank the previous year could not have been fired or resigned or died. Further, one might be tempted that the allocation given to the senior lecturer rank for the year exceeds the mandatory 40% as the ratio of 89 to the row total of 121 is about 74%. Later in this chapter, it will be demonstrated that the solution has not violated any constraint. This type of allocation is repeated for the Faculty for the remaining part of the planning horizon. However, the total number of required staff was not affected. This result and two others: allocation to professorial rank in the Faculties of Arts and Education show some of the major limitations of the model, i.e. the fact that it implicitly assumes substituability of skills and experience between the various ranks and hence that academic staff can be allocated only on the basis of the goals, budget and current rate of advancement in the hierarchy. Of course, this is not so in real life. Certain essential functions, like administrative functions, require a minimum level of experience and an appropriate rank and certain teaching functions, for example, supervision of graduate students, can only be carried out by staff at a particular rank. These are not explicitly taken into consideration by the model and they might account, to a great extent, for the type of results just described. It is also possible that the huge allocation of nearly 70% to the professorial grade in the Faculty of Agriculture over and above what exists now in the Faculty (see Tables 5.8 and 5.13) by the model at the beginning of the 1982/83 session might account for some of these "strange" allocations in later years of the planning horizon. We now demonstrate that these results are perfectly compatible with the model constraints.

Consider equation (57) of Appendix 6 which is the student enrolment goal for the Faculty of Agriculture (j = 4) for 1982/83 (t = 2). We shall use only the basic variables since the non-basics are zero. The basic variables in this equation can be found from Tables 5.8 and 5.9. They are X_{142}, X_{143}, X_{342}, X_{343}, since we have just seen that allocations were made by the model only to the lecturer and senior lecturer grades for these years. We wish to demonstrate that

$$- 0.1608X_{142} + 1.4667X_{143} - 0.3144X_{342} + 1.1812X_{343}$$
$$= 143.$$

Using the values from the printouts rounded to 2 decimal places, we have

$$\text{L.H.S.} = - 0.1608\ (24.95) + 1.4667\ (31.99)$$
$$- 0.1344\ (41.31) + 1.1812\ (89.44) = 143.00 = \text{R.H.S.}$$

To verify whether the academic rank distribution goal is violated for the senior lecturer grade in the Faculty of Agriculture, we use equation (137) of Appendix 6. This is the senior lecturer rank distribution goal (i = B = {3}), for the Faculty of Agriculture (j = 4) for the year 1982/83 (t = 2). Using the non-zero variables, the equation that we wish to verify is

$0.0643X_{142} - 0.5867X_{143} + 0.0538X_{342} - 0.4725X_{342} + n^{c}_{B42} = 0.$

We note that since this goal is underachieved (Table 5.4), only the value of the negative deviational variable is non-zero. Substituting for the values, we have:

L.H.S.

$0.0643\ (24.95) - 0.5867\ (31.99) + 0.0538\ (41.31)$

$- 0.4725\ (89.44) + 57.20 = - 0.002$

L.H.S. = zero to 2 decimal places = R.H.S.

One may ask, "if 89 allocated to the sneior lecturer grade in 1983/84 is not to be compared with its row sum in table 5.9, with what then must it be compared to verify that the amount allocated to this grade satisfies the rank distribution apart from the substitution done above?"

The value will have to be compared to the following sum: allocation to lecturer grade + allocation to senior lecturer grade + underachievement of the goal for the senior lecturer grade + underachievement of the goal for the professorial grade (for 1982/83). Using Tables 5.9, 5.4, and 5.5, this value can be determined as:

32 + 89 + 57 + 49 = 227

When compared with this number, the allocation made to the senior lecturer grade will be found to be 0.39 which is still within the bounds set by the rank distribution goal. The interpretation that can be given to this value is that 227 is the long term goal allocation of total academic staff to the Faculty of Agriculture based on the current promotion rates, budget, etc., and the rank distribution but this goal cannot be achieved at the expense of the student enrolment goal which has priority 1.

Similar tests as shown in the preceding paragraphs were performed to show that the allocations made to Education and Arts in 1985/86 do not violate any of the constraints and that mathematically, the solutions are in order. However, it was felt that a change in the objective function of the problem might help to eliminate the discrepancies discussed in the foregoing sections. Therefore, the goal at P_2 was changed to achieving <u>exactly</u> the academic rank distribution goal. This means that 120 negative deviational variables will be added to the objective function. On running the model in this form for over 50 minutes (150% above the time we obtained previous solutions), no solution was obtained and was, therefore, discontinued. Therefore, the solution reported in this section can be regarded as the best we can get in present circumstances, given the state-of-the-art of large scale goal programming problem solving.

CHAPTER 6

MODIFICATIONS OF THE BASIC MODEL: SOLUTION AND INTERPRETATION OF RESULTS.

In this chapter we discuss and interprete the solution of runs II and III of variant I; and the different runs of variant II of the model.

6.1. Variant I Run II

Run II of Variant I solves what the Director of Planning of the University, as the major decision maker, thinks is the desirable priority structure for the University over the planning horizon. Accordingly, the aim of the run is to determine how far the university can achieve its various goals over the planning horizon, on the assumption that the forecast budget of this model will be made available. The new priority structure is:

P_1 : Ensure that the university has enough academic staff to meet the desired student enrolment goal.

P_2 : Minimize budget.

P_3 : Ensure that the academic rank distribution goal is not exceeded.

P_4 : Maximum hiring constraint and academic staff level goal should not be exceeded. However, twice weight is attached to the achievement of the maximum hiring constraint.

The objective function associated with this priority structure can be written as:

$$\text{Minimize } P_1 \sum_{j=1}^{8} \sum_{5=1}^{5} n_{jt}^{b} + P_2 \sum_{t=1}^{5} p_{t}^{e}$$

$$+ P_3 \sum_{i\varepsilon\rho} \sum_{j=1}^{8} \sum_{t=1}^{5} p_{ijt}^{c} + 2P_4 \sum_{j=1}^{8} \sum_{t=1}^{5} p_{jt}^{d}$$

$$+ P_4 \sum_{j=1}^{8} \sum_{t=1}^{5} p_{jt}^{a} \quad (5.5)$$

The CPU time for running this version of variant I model is about 20 minutes and its solution was exactly the same as that of variant I, Run I.

6.2 Variant I Run III

Run III of Variant I can be regarded as testing whether the solution will change at all with change in priority level of some of the goals, i.e. a kind of sensitivity analysis. For the priority structure of Run III, the first two priorities of Run II are retained while P_3 and P_4 are interchanged. This means that academic staff level goal and maximum hiring are now at P_3 while academic rank distribution goes to P_4. The resultant objective function can be written as:

$$\text{Minimize } P_1 \sum_{j=1}^{8} \sum_{t=1}^{5} n_{jt}^{b} + P_2 \sum_{t=1}^{5} p_{t}^{e}$$

$$+ 2P_3 \sum_{j=1}^{8} \sum_{t=1}^{5} p_{jt}^{d} + P_3 \sum_{j=1}^{8} \sum_{t=1}^{5} p_{jt}^{a}$$

$$+ P_4 \sum_{i\varepsilon\rho} \sum_{j=1}^{8} \sum_{t=1}^{5} P_{ijt}^{c} \qquad (5.6)$$

Running this variant with this objective function took exactly the same time as in Run II and the answers are also the same as in Runs I and II. The conclusion that can be drawn is that the solution will be invariant to changes in the priority structure as long as the student enrolment goal is at priority 1.

6.3 Variant II Run I

Variant II of the model does not include the academic rank distribution goals. The aim of solving it is to see the effect on academic rank structure if the controversial rank distribution proposal was dropped totally. The priority structure for the first run is as follows:

P_1 : as in the first run of Variant I, priority 1 was attached to the achievement of the student enrolment goal;

P_2 : is now attached to the academic staff level goal and the maximum hiring goal with maximum hiring constraint having twice weight;

P_3 : minimization of budget.

Unfortunately, the model in this form performed poorly, both as a resource allocation model and in determining the minimum number of academic staff required to achieve the student enrolment goal. Because of the cost-minimization objective, the model without the rank distribution chose the easiest way out--the least cost combination. In this way, the values given for most faculties relating to the structure of academic ranks include at most two of the ranks only. In some cases, only one rank structure is chosen. In this form, the model solution is an impractical one. The answers given are merely academic. This goes to suggest that rank distribution has to be used to be able to get a realistic proposal from an optimization model of this form. The question then is "what form and pattern of the rank distribution ratio will be acceptable to the generality of the academic staff?" Variant I of the model has indicated that a rigid ratio of distribution between ranks will likely hurt most academic staff members in terms of moving up the academic hierarchy.

A plausible alternative to this in order to be able to use this model effectively for planning in the Variant I form will be to base the rank distribution ratio on the historical rate of advancement through the hierarchy and in each faculty. This can be determined by at least two methods:

(i) determining the average of;ratio of each rank over a given period (e.g. ten years) for each faculty. This value can be assumed constant and hence will be time-invariant over the planning horizon. This approach will take into cognizance the differing rate of advancement in each faculty and will not necessarily penalize old faculties in favour of new ones and vice-versa.

(ii) The average determined by (i) may not be assumed constant for all faculties. For young faculties, it may be necessary to adjust this average ratios between the various ranks over the years of the planning horizon to take into consideration the fact that in spite of the age of such faculties, they may need to have a change in their rank structure, particularly at the professorial levels in a given planning horizon.

The rank structure obtained from such a solution should be taken as the results of an <u>indicative</u> planning process: the result of the model solution is only a means to an end, that is providing the decision maker with relevant and objective facts to make an <u>informed decision</u>. No attempt should be made to rigidly implement such a rank structure. Academics must be allowed to advance according to their productivity. Otherwise, the type of opposition that greeted the Press Release 46 of 1981 will recur.

From Table 6.1, it will be seen that even in terms of minimum number of academic staff to meet the goals of the

University in each faculty, Variant II also performed more poorly than Variant I. It underestimated the minimum number of academic staff requirement of each faculty over the planning horizon.

When compared with Table 5.13, except for the Faculty of Science, it gave the impression that all faculties are seriously overstaffed. It is not until 1983/84, when it requires the doubling of the staff in the Faculty of Agriculture that it gives the impression that the present staff of the Faculty of Agriculture appears overstretched. This impression was given by Variant I in 1982/83 (Table 5.8).

Because of the fact that the least cost alternative is chosen, a substantial part of the budget is left unspent. Furthermore, an examination of the deviational variables indicates that achievement of the student enrolment goal degraded the historical advancement of academic staff as indicated by the ratio of total positive deviation to the total negative deviation from the achievement of this goal (Table 6.2). Under Variant I, total sum of the overachievement variables for this goal, exceeds the underachievement, whereas in the case of Variant II, the reverse is the case. The ratio of the values is 0.91 for Variant II whereas it is 1.13 for Variant I. This can be interpreted by saying using Variant II of the model, throughout the University, there will be a decline in the rate of allocation of total staff over the historical advancement rate by 10%.

Schroeder (1974), in a short illustrative example on the possible application of the original model modified for this study, also dropped the rank distribution goal but all new academic staff were hired at the least cost level, i.e. assistant professors, in his case. He asserted that "in this case, it was not necessary to specify the desired faculty distribution goals, since the distribution was fixed by hiring assumption"[1] His suggestion was also adopted in solving a version of Variant II of the model. All recruitment variables (Y_{ijt}) were dropped for i = 3, 4, 5, i.e. recruitment was assumed to be done only at the assistant lecturer and lecturer grades: the least cost alternatives. This assumption is in line with what obtains in practice in the University. Akinlade (1979) reported that for the period covered by her study, about 93% of recruitments were at the lecturer grade level (i.e. assistant lecturers and lecturers). It was disappointing to note, however, that the model in this form still gave the same solution as the original Variant II.

1. Shroeder (1974), p.706.

TABLE 6.1

Distribution of the minimum academic staff requirement by faculty and year as determined by Variant II of the model

Faculty/ College	1982/83	1983/84	1984/85	1985/86	1986/87
Medicine	179	222	238	248	253
Arts	88	98	109	111	119
Science	135	150	173	192	197
Agric. & Forestry	52	103	121	139	143
Education	47	56	60	65	66
Soc. Sci.	65	80	95	109	110
Vet. Med.	32	39	42	60	62
Technology	12	54	68	79	82
Total	610	667	906	1003	1032

TABLE 6.2

Values of the deviational variables corresponding to the academic staff level goal as determined by Variant II

Faculty/ College	1981/82 p	n	1982/83 p	n	1983/84 p	n	1984/85 p	n	1985/86 p	n	Total p	n
Medicine	-	3	-	11	-	19	-	32	-	54	-	119
Arts	-	34	-	42	-	54	-	223	-	96	-	449
Science	-	-	69	-	77	-	84	-	73	-	303	-
Agric. & Forestry	-	-	60	-	73	-	89	-	86	-	308	-
Education	-	4	-	11	-	20	-	29	-	45	-	109
Soc. Sci.	-	77	30	-	44	-	57	-	50	-	181	77
Vet. Med.	-	37	-	43	-	50	-	35	-	46	-	211
Technology	-	-	-	-	27	-	32	-	24	-	83	-
Total	-	155	159	107	221	143	262	319	233	241	875	965

Thus, it can be concluded that recruitment at least cost level does not necessarily fix academic rank distribution and that it has to be explicitly specified in the model.

From a theoretical point of view, therefore, it is desirable to have a distribution ratio so as to have a realistic allocation between academic ranks for use in an _indicative manner_ in the planning process. The ratio must be such that it recognizes the differing advancement rates across and within faculties and must not be rigid.

6.4 Variant II, Other Runs

Three other runs of Variant II apart from the two reported above were made to see if the model solution is sensitive to changes in the priority levels of the various goals. In the first of these runs, the priority levels of the budget goal and the staff level goal and maximum hiring were interchanged. In the second of the runs, the Staff Level and Maximum Hiring Goal were given first priority. Enrolment was at P_2 while Budget was at P_3. In these two forms, the decision variables and their values remained the same, with the budget and priority 1 goals always achieved.

However, when Staff Level Goal was attached priority 1 by itself alone and enrolment was made to be at priority 2 with maximum hiring constraint at level 3 and budget is the least important (i.e. P_4), the values of some decision variables in particular faculties that have high student enrolment goals, e.g. Agriculture and Technology, have lower allocation under this version because their staff level goals are small. However, this result is only academic since it is absurd to allocate teachers to teach non-existing students and at the same time to deny faculties with students their required allocation.

Table 6.3 summarises the various models and results.

TABLE 6.3

Summary of various models and results

Model Variant	Major Characteristics	Major Observations of results		
		C.P.U. time	Type of Solution	Major Findings and Policy recommendations
1. Variant I, Run I (Basic Model)	(i) Contains all goals and constraints, viz. Academic Staff level goal, student enrolment goal, rank distribution goal, maximum hiring and budget constraints. (ii) Aims at identifying input requirements to achieve the goals, hence budget goal has the least priority.	19 minutes, 50 seconds.	Implementable solution.	(i) All goals achieved except academic staff level goal. (ii) Minimum academic staff requirements for the whole University by the end of the planning horizon will be about 60% over and above the requirements at the beginning of the plan period. (iii) The University will have to pursue a vigorous staff development programme to be able to meet this requirement during the plan period. (iv) Use of the rank distribution ratios introduced in 1981 will inhibit promotion

TABLE 6.3 Contd.

Model Variant	Major Characteristics	Major observations of results		Major Findings and Policy recommendations
		C.P.U. time	Type of Solution	
				(v) The staff of the Faculty of agriculture appear overstretched because they are operating about 30% below the minimum requirements specified by the solution. (vi) In Faculties where there will be increasing student enrolment, a total of 898 academic staff will be allocated over and above the historical rate during the plan period. (vii) The annual forecast budget estimates used for solving the model are adequate.
2. Variant I, Run II	(i) Contains all goals and constraints (ii) Attempts to determine the degree of goal attainment with given inputs. Budget goal is now at priority 2.	19 mins. 51 sec.	Implementable	Same as in Variant I, Run I.

TABLE 6.3 Contd.

Model Variant	Major Characteristic	Major observations of results		
		C.P.U. time	Type of Solution	Major Findings and Policy Recommendations
3. Variant I, Run III	(i) Contains all goals and constraints (ii) Aims at determining whether the degree of goal attainment and the solution will change by altering the priority levels of some of the goals. (Rank distribution goal is at priority 4 while maximum hiring and staff level goal are at priority 3. Others remain as in run II above.)	19 mins.51 sec.	Implementable	(i) Degree of goal attainments and values of the decision variables are exactly the same as in the first two runs. (ii) Policy recommendations same as in the first two runs (iii) The solution of variant I is invariant with changes in priority levels as long as student enrolment goal is at priority I.

TABLE 6.3 Contd.

Model Variant	Major Characteristics	Major Observations of results		
		C.P.U. time	Type of solution	Major Findings and Policy Recommendations
4. Variant II, Run I	(i) Rank distribution goal dropped. (ii) Aims at finding the effect dropping the controversial rank distribution ratios, on rank structure and identifying the input requirements of the resultant distribution of staff. (iii) Budget goal is at least priority	3 min. 45 sec.	Unimplementable	(i) All goals achieved except staff level goal (ii) Model chose the least cost allocation alternatives i.e. Assistant Lecturers and Readers. (iii) There will be a decline in allocation of staff below the historical rate by 10%. (iv) Use of rank distribution goal in a model of this type is desirable. (v) The type of rank distribution ratios to be used must reflect the differential advancement rates in the various faculties and the rank structure obtained from the solution of the model should be taken as the results of an indicative planning process.

TABLE 6.3 Contd.

Model Variant	Major Characteristics	Major Observations of results		
		C.P.U. time	Type of Solution	Major Finding and Policy Recommendations
5. Variant II, Other Runs.	(i) Rank distribution goal dropped (ii) Attempt to determine the degree of goal attainment with given inputs and whether this will change when the priority levels of the goals are altered.	Average of 3 mins. 8 sec.	Unimplementable	(i) Same as Variant II, Run I, (ii) Solution is invariant of the model with changes in priority level as long as student enrolment goal is at the highest priority level.

6.5 Postoptimal Sensitivity Analysis

Postoptimal sensitivity analysis addresses the effect of changes in the values of the parameters of a given model on the decision variables after an optimal solution has been found. Goal programming is a deterministic model, assuming that the values of parameters are known for certain. In real life, this is not often so. It is possible that there may be changes in the priorities the DM attaches to the objective function after the model has been solved. It may also be possible to discover some errors in the parameter estimation after having obtained an "optimal" solution. The coefficients of the decision variables may change or it may be discovered that an important decision variable was omitted or that a new system/structural (or non-goal) constraint has to be added to the model. The effects of these changes on the values of the decision variables are what postoptimal sensitivity analysis seeks to analyze. A short review of the theory of postoptimal sensitivity analysis is given in Appendix 7.

As can be seen from Appendix 7, postoptimal sensitivity analysis involving changes in the r.h.s. value or in the coefficient of the goals require matrix multiplication with the transformation matrix, $\bar{T}$, which by definition is obtained from the final simplex tableau. Since the matrix of this model is extremely big (245X970), and will be difficult to manipulate, the empirical examination of these types of sensitivity analysis was not done. Ideally, the computer should do it. However, given the state-of-the-art of goal programming problem-solving, this cannot be done.

This leaves us with changes in the weights of the priority levels. For this model only at one priority level do we have differing weight and so, there was no empirical alteration in weights. Rather, what was done was to alter the level of priorities attached to the goals to see how sensitive the solution will be to the change. As was reported in Sections 6.1 and 6.2 the optimal solution mix and its value remain the same as long as the student enrolment goal is at priority level 1.

6.6 Planning Implication of the Empirical Findings

Table 6.4 summarizes the minimum academic staff requirement of the University of Ibadan over the planning horizon of this study. By the end of the plan-period, to meet its student enrolment goal, the University is expected to have a minimum number of academic staff of 1,133 in total. A close examination of Table 6.4 will reveal that there is a critical demand for academic staff in certain faculties over the plan period. For example, by the beginning of 1986/87, the minimum requirement in the Faculty of Science is nearly 2.5 times its minimum requirement at the beginning of 1982/83 (Table 6.5). When compared with Table 5.13, Table 6.4 indicates that other faculties where the demand for academic staff will be critical over the planning horizon are Agriculture, Social Sciences, and Technology.

A relevant question to ask is "How and where will the University get the qualified people to satisfy its minimum requirement of academic staff in 1986/87 by 61% over and above its minimum requirement in 1982/83?" Two major alternatives readily come to mind:

(i) recruitment, and

(ii) staff development.

TABLE 6.4

Minimum academic staff requirement of the University of Ibadan by faculty as determined by the model solution for the five-year period 1982/83-1986/87

Faculty/ College	1982/83	1983/84	1984/85	1985/86	1986/87
Medicine	201	232	245	254	257
Arts	96	112	121	86	133
Science	86	163	186	207	210
Agric. & Forestry	110	121	143	163	168
Education	52	61	65	87	70
Soc. Sci.	77	99	117	136	138
Vet. Med.	37	43	47	68	70
Technology	43	61	72	85	87
Total	702	892	996	1086	1133

TABLE 6.5

Ratio of minimum academic staff requirement
at the end of the plan period to that
at the beginning by faculty

Faculty/College	Ratio
Medicine	1.28
Arts	1.39
Science	2.44
Agriculture & Forestry	1.53
Education	1.35
Social Sciences	1.79
Veterinary Medicine	1.89
Technology	2.02
University-wide	1.61

Given the fact that during the plan period covered by this study as many as twenty or more universities may be in operation in Nigeria, and these institutions may likely recruit many of their foundation staff from the University of Ibadan because of its age, the first alternative may not yield the desired result. Therefore, a major planning implication of the solution of the model is that the University of Ibadan should consider very seriously the staff development option. Three possible options can also be considered under staff development:

(i) Employ new graduates of the University and train them in the University;

(ii) Employ new graduates of the University and train them abroad in special areas;

(iii) Upgraduate other academic staff who have no doctorates through training at home and abroad.

It seems as if the first two options need to be vigorously pursued by the University using a type of Junior Fellowship programme in order that it may be attractive enough to hold back brilliant products of the University. This means that the budget of the University in relation to staff development may need substantial increase over the plan period. The third option will aid the rate of promotion within the academic hierarchy more than causing a change in the minimum level of academic staff required.

The analysis of goal attainments in section 5.1.1 indicates that the goal of minimization of academic staff payroll budget is achieved. The planning implication of this solution is that the forecast payroll budget used in the study is adequate for each year of the planning horizon. The annual payroll budget for academic staff recommended by the study is as follows:

1981/82	₦11,343,922
1982/83	₦12,519,538
1983/84	₦13,873,535
1984/85	₦15,440,094
1985/86	₦16,996,873.

CHAPTER 7

SUMMARY OF MAJOR FINDINGS AND IMPLICATIONS OF STUDY

7.1 Existing Academic Planning System vs. System Suggested by Study

As will be seen from Chapter 2, all the approaches used for planning in the University at present, namely the F.T.E. and headcount in conjunction with standard student/staff ratios determined by N.U.C. guidelines provide some of the basic inputs into the model used for the study. The present system first determines what the required academic staff strength should be based on projected student enrolment, then based on the available budget, approval for recruiting such size of staff may be given or not.

Goal Programming does more than that. Projected budget and the required staff strength goal are inputs into the model. Further, other goals can be incorporated into the model reflecting the rate of historical movement within the academic hierarchy in each faculty. It looks at the University problem from a holistic perspective, thus emphasizing the interdependencies and inter-relationships between the various units making up the University. This type of approach is absent in the present approach which appears to treat each faculty as an entity.

Furthermore, an analysis of the deviational variables of GP can reveal structural changes in the goals of the various faculties as shown in the analysis of Chapter 5. This capability is not available in the present system.

7.2 Summary of Major Findings

1. From a purely theoretical point of view, in order to obtain a satisfactory solution to the GP model, it is desirable to have a rank distribution goal; otherwise, the model will choose the least cost alternatives and the result of the solution will be impracticable to implement. Given that fixed distribution ratio introduced by the Press Release No. 46 of 1981 resulted in a labour crisis in the University, the form of rank distribution suggested by this study is one that will not be rigid and will reflect the differing historical advancement rate in each faculty. Such a ratio will take care of the needs of the old faculties as well as young faculties in terms of the advancement of academic staff.

2. The analysis of the deviational variables reveal that in faculties where there will be increasing student enrolment

over the planning horizon mainly because of development of new programmes and partly because of the expansion of existing ones, a total of 898 academic staff will be allocated over and above the historical rate during the plan-period. The faculties are:

Science	303
Agriculture & Forestry	308
Social Sciences	181
Technology	106

However, in Medicine, Arts, Education, and Veterinary Medicine, the allocation will fall below the historical rate up to the tune of 797 during the period 81/82-85/86. University-wide, this means that the percentage allocation over the historical rate will increase by about 13%.

3. The fear of the academic staff about the controversial rank distribution ratio of 30%-40%-30% between the lecturer grade, senior lecturer grade and professorial grade seems justified by the model solution. Firstly, the senior lecturer grade that is envisaged by the proposal to be the career grade for academics, was substantially underachieved in all faculties throughout the planning horizon. This suggests that in most faculties, complete achievement of the rank distribution goal for the senior lecturer cadre is not possible during the plan-period. Therefore, a substantial number of lecturers will

find it difficult to get promoted if the proposal were to take effect.

Secondly, the analysis of the decision variables also reveal that in some faculties and ranks, there may not be promotion in certain years if the proposal were to be in force. For example, in the College of Medicine, given the existing staff at the professorial rank, there would be no promotion into this rank until the beginning of the 1984/85 session, irrespective of the productivity of academic staff. Similarly, in the Faculty of Arts, out of 83 lecturers presently in position in the lecturer grade, a maximum of 20 may get promoted to the senior lecturer grade by 1986/87, if the rank ratio proposal were to be in force.

4. A comparison of the existing number of academic staff in the Faculty of Agriculture and Forestry and the minimum requirement determined by the model suggests that the academic staff in that faculty appear to be overstretched because they are operating at nearly one third below their minimum required allocation to meet the student enrolment goal. It is suggested that the University authorities conduct a special study to confirm or refute this finding. In contrast, the model solution suggests that the Faculty of Arts is operating substantially above the minimum requirement. However, this is not totally

favourable to the academics there because it is the major reason why many of their lecturers would be unable to move up were the academic rank ratio proposal to be operative.

5. The model solution also reveals that to meet its student enrolment goal over the next five years, the University of Ibadan will require a minimum number of 1133 academic staff by 1986/87. This is over 60% above the minimum requirement for the 1982/83 session. Broken down by faculty, the minimum requirement of each faculty by the beginning of 1986/87 is as follows:

Medicine	257
Arts	133
Science	210
Agriculture and Forestry	168
Education	87
Social Sciences	138
Veterinary Medicine	70
Technology	87

The study, therefore, recommends that the University should consider, as a matter of urgency, the implementation of a virile Staff Development Programme in which the training of new graduates of the University under a Junior Fellowship Programme will be the focus. Under such a programme, graduates of the University with a minimum of an Upper Second Honours Degree will be awarded a Junior Fellowship (distinct from Graduate Assistantship) and will be trained by the University in the University or elsewhere. Otherwise, it will likely be very difficult for the University to obtain the required minimum number of academic staff during the plan period, given the high rate of demand for University teachers now in the country as a result of the opening of new Universities, most of which look up to the University of Ibadan to obtain their foundation staff. The faculties of Science, Agriculture and Forestry, the Social Sciences, and Technology require careful monitoring because their requirements over the planning horizon are substantially higher than in other faculties.

6. To meet the cost of the academic resource allocation recommended by the model solution, the model also suggests the following academic payroll budget for the whole University:

1981/82	₦ 11,343,922
1982/83	₦ 12,519,538
1983/84	₦ 13,873,535
1984/85	₦ 15,440,094
1985/86	₦ 16,996,873

7. The model solution runs corroborated the findings of Kang (1980) that the CPU time of the Revised Simplex Goal

Programming Algorithm tends to increase with increasing number of deviational variables in the objective function.

7.3 Suggestions for Future Studies

In this study, the budget has been aggregated for the whole university. In this way, it is possible that the allocation of academic staff to a given faculty by the model exceeds that required by its Budget. In practice, academic resources are allocated subject to the budget of the faculty and no **transfers** of budgets across faculties are allowed. But in its present form, the model allocati on can result in transfer of funds across faculty. Therefore, a much closer representation of what obtains in practice is to decompose the budget according to the faculties. However, this will result in 40 constraints instead of five with 80 deviational variables. The size of the model will be bigger. It is hoped that with further research on the development of new algorithms like the LU factorization technique, size will not be a constraint to solving a goal programming model.

In this study, the student enrolment goal did not distinguish between undergraduate student enrolment and graduate student enrolment. Given the fact that the demands for academic staff of the two groups of students vary, this type of aggregation can conceal some interesting results. Therefore, an area for further study is to consider disaggregating the student enrolment goal in such a way that it reflects both undergraduate and graduate enrolment.

At the tactical and operational levels, a related area for future investigation is a model for scheduling academic staff between departments and courses in a given faculty. Such a model may incorporate constraints relating to allocation of office to staff and classroom to courses. However, it will result in a mixed-integer goal programming problem and may be difficult to solve. It is possible that because it is limited to just a faculty, the size may not be too large.

The development of more efficient large scale goal programming algorithms that will enhance storage and CPU time is a related area requiring future investigation, if real life application in GP is to assume the dimension it has taken in linear programming. Given that most commercial linear programming codes use the LU factorization technique, it is possible that such a development will occur in goal programming via the LU factorization and sparsity technique of Lee and Gen (1982).

The model used in this study did not take cognizance of the semi-autonomous status of the College of Medicine. A model recognizing this would incorporate the concepts of decentralized organization by treating the college as a semi-autonomous unit within the University with its own departments. The resulting model will be very large and can be solved using goal programming decomposition algorithm (Rho, 1978).

The model used in this study focused only, on academic staff. An area for future investigation can consider, in addition the allocation of other support staff like secretaries, graduate assistants and other administrators. The resulting model will be quite large and may have to be solved using more efficient algorithms.

Future studies can also consider the macro form of this model at the level of the National University Commission. Such a study will allocate staff to the different academic units in each of the Federal Universities subject to the differential advancement rates in these units and each university as well as the academic payroll budget allocated to the different universities. The resulting model is obviously going to be very large. The older Universities which can generate enough data can be used in a pilot study at first.

Goal Programming has been criticized for its use of prioritized goals; e.g. Morse (1976, 1978). The criticisms revolve round the contention that decision makers may find it difficult to quantify their goals and that if it is even possible to quantify them, ranking of goals to reflect preferences may be difficult. Therefore, one area of future research will involve methods for easy quantification and ranking of goals by decision makers and analysts.

This study did not take into account the wastage rate of students in estimating the enrolment goal. An area of future study can take this into account. Of course the resulting model will likely be more complex and large.

Finally, efforts on the part of the University of Ibadan authorities should be directed to operationalizing this study. The Development and Planning Office will have a major role to play in this regard.

Deans of the various faculties and the heads of various departments can be contacted by the office about the results of the study and their comments sought. In this way, their confidence and cooperation can be secured about the possibility of implementing it. Where there are doubts, other studies might have to be done to clear them.

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APPENDICES

APPENDIX 1A

Transition between the different states by rank and year for the College of Medicine

	IN POSITION					PROMOTED					WASTAGE				
	Prof.	Read.	Sn. Lect.	Lect.	Asst. Lect.	Prof. from Read.	Prof. from S.L.	Read.	S.L.	Lect.	Prof.	Read.	S.L.	Lect.	A.L.
70/71	24	5	21	41	2	2	4	5	7	0	0	0	1	3	0
71/72	30	7	27	54	2	2	0	1	12	1	0	0	1	4	0
72/73	32	8	38	54	4	0	0	1	18	0	0	0	1	1	0
73/74	31	9	51	73	5	1	5	0	7	0	1	0	0	3	0
74/75	37	8	50	55	3	3	5	2	7	2	0	2	1	0	0
75/76	44	10	71	98	6	2	5	4	19	1	1	0	0	3	0
76/77	49	15	96	88	5	2	6	0	15	0	2	1	0	1	1
77/78	53	14	86	70	2	0	4	5	15	4	1	1	1	3	0
78/79	56	14	80	58	0	0	3	5	10	0	1	0	1	2	0
79/80	59	14	91	82	3	0	0	0	11	1	0	0	0	0	0
TOTAL	415	104	611	673	32	12	32	21	121	9	6	4	6	20	1

APPENDIX 1B

Transition between the various states by rank and year for the Faculty of Arts

	IN POSITION					PROMOTED					WASTAGE				
	Prof.	Read.	Sn. Lect.	Lect.	Asst. Lect.	Prof. from Read.	Prof. from S.L.	Read.	S.L.	Lect.	Prof.	Read.	S.L.	Lect.	A.L.
70/71	2	1	14	47	1	0	1	2	2	0	0	0	0	0	0
71/72	13	3	11	49	2	0	0	0	0	1	0	0	3	0	0
72/73	12	3	11	45	2	0	0	0	6	0	1	0	0	5	0
73/74	12	3	17	50	1	0	0	0	4	1	0	0	0	0	0
74/75	15	1	19	47	1	0	3	1	6	0	0	2	1	0	0
75/76	15	1	26	67	2	1	1	3	6	0	1	0	0	1	[illegible]
76/77	15	3	31	87	4	0	1	1	1	1	2	1	0	1	1
77/78	15	3	32	88	5	0	0	0	5	1	1	1	1	3	0
78/79	14	3	36	88	5	0	0	1	2	2	1	0	1	2	0
79/80	14	2	34	79	10	0	0	0	6	2	0	2	2	9	0
TOTAL	137	23	231	647	33	1	6	8	38	8	6	6	6	21	1

APPENDIX 1C

Transition between the various states by rank and year for the Faculty of Science

	IN POSITION					PROMOTED					WASTAGE				
	Prof.	Read.	Sn. Lect.	Lect.	Asst. Lect.	Prof. from Read.	Prof. from S.L.	Read.	S.L.	Lect.	Prof.	Read.	S.L.	Lect.	A.L.
70/71	10	3	14	35	0	0	0	1	4	1	1	1	2	1	0
71/72	10	4	17	49	0	1	0	0	4	0	4	0	1	1	0
72/73	11	4	21	53	0	0	0	3	6	0	0	0	0	0	0
73/74	11	7	27	68	2	0	3	1	10	0	1	0	0	7	1
74/75	14	7	36	64	1	1	0	3	5	1	0	1	1	4	1
75/76	15	9	38	70	5	1	1	3	7	2	0	1	3	5	2
76/77	15	11	45	72	5	1	4	3	4	0	2	1	0	8	1
77/78	20	13	49	69	8	0	0	0	4	2	0	1	0	3	0
78/79	20	13	52	63	9	0	0	0	4	0	0	0	1	6	0
79/80	23	13	48	61	9	0	3	1	4	0	0	0	1	2	0
TOTAL	149	84	347	604	39	4	11	15	52	6	8	5	9	37	5

APPENDIX 1D

Transition between the various states by rank and year for the Faculty of Agriculture and Forestry

	IN POSITION					PROMOTED					WASTAGE				
	Prof.	Read.	Sn. Lect.	Lect.	Asst. Lect.	Prof. from Read.	Prof. from S.L.	Read.	S.L.	Lect.	Prof.	Read.	S.L.	Lect.	A.L.
70/71	7	0	9	38	1	0	0	2	6	1	1	0	0	0	0
71/72	7	2	13	44	0	0	1	1	4	0	0	0	0	0	1
72/73	8	3	16	38	0	0	0	0	10	0	0	0	1	0	0
73/74	8	3	23	47	1	0	1	2	6	0	0	0	0	4	1
74/75	9	4	29	43	4	3	2	3	8	1	1	1	0	0	1
75/76	13	7	37	42	2	2	2	4	1	0	1	1	1	2	0
76/77	16	10	37	40	2	1	4	3	4	0	1	1	1	3	0
77/78	21	12	39	40	2	0	3	4	6	0	1	1	2	0	0
78/79	23	16	42	35	1	0	1	4	3	0	1	0	3	5	1
79/80	23	19	42	35	2	0	0	4	10	1	0	1	3	0	0
TOTAL	135	76	282	402	15	6	14	27	58	3	6	5	11	14	4

APPENDIX 1E

Transition between the various states by rank and year for the Faculty of Education

	IN POSITION					PROMOTED					WASTAGE				
	Prof.	Read.	Sn. Lect.	Lect.	Asst. Lect.	Prof. from Read.	Prof. from S.L.	Read.	S.L.	Lect.	Prof.	Read.	S.L.	Lect.	A.L.
70/71	1	2	6	16	0	0	0	1	0	2	1	0	1	0	0
71/72	3	2	5	15	2	0	2	0	2	0	0	0	1	1	0
72/73	3	0	7	15	2	2	0	0	4	0	0	0	0	1	0
73/74	5	0	10	20	3	0	0	1	5	0	0	0	0	0	0
74/75	5	1	15	20	2	0	0	2	1	1	0	0	1	2	1
75/76	4	2	17	21	5	1	1	2	4	0	1	1	1	1	0
76/77	5	2	21	25	6	0	0	0	3	0	1	2	1	1	0
77/78	5	2	23	25	6	0	1	0	1	1	0	0	0	3	1
78/79	6	2	24	35	5	0	0	0	3	2	0	0	0	0	1
79/80	6	1	26	33	3	0	0	1	4	2	0	1	0	0	0
TOTAL	43	14	154	225	34	3	4	7	27	8	3	2	5	9	2

APPENDIX 1F

Transition between the various states by rank and year for the Faculty of the Social Sciences

	IN POSITION					PROMOTED					WASTAGE				
	Prof.	Read.	Sn. Lect.	Lect.	Asst. Lect.	Prof. from Read.	Prof. from S.L.	Read.	S.L.	Lect.	Prof.	Read.	S.L.	Lect.	A.L.
70/71	4	1	5	27	0	0	1	2	0	0	0	0	0	0	0
71/72	5	2	4	28	0	1	1	0	5	0	0	0	0	0	0
72/73	7	1	9	30	0	0	0	0	2	0	0	1	0	0	0
73/74	7	0	10	30	0	0	0	1	8	0	0	0	0	1	0
74/75	7	1	17	33	2	0	1	0	3	0	0	0	0	0	0
75/76	7	1	15	40	3	0	4	1	10	0	1	0	0	3	0
76/77	11	2	24	44	1	0	0	0	6	0	0	0	0	1	2
77/78	11	2	27	37	1	1	1	1	6	0	0	0	1	1	0
78/79	13	2	31	34	0	0	2	0	2	0	0	0	0	1	1
79/80	15	2	33	35	0	0	0	3	5	0	0	0	0	0	0
TOTAL	87	14	175	338	7	2	10	8	47	0	1	1	1	7	3

APPENDIX 1G

Transition between the various states by rank and year for the Faculty of Veterinary Medicine

	IN POSITION					PROMOTED					WASTAGE				
	Prof.	Read.	Sn. Lect.	Lect.	Asst. Lect.	Prof. from Read.	Prof. from S.L.	Read.	S.L.	Lect.	Prof.	Read.	S.L.	Lect.	A.L.
70/71	2	0	0	8	0	0	1	0	0	0	0	0	0	0	0
71/72	1	0	1	8	0	0	0	0	1	0	1	0	0	0	0
72/73	2	0	2	13	0	0	0	0	1	0	0	0	0	0	0
73/74	2	0	4	8	0	0	0	0	3	0	0	0	0	2	0
74/75	1	0	6	10	0	0	1	2	2	0	1	0	0	0	0
75/76	2	2	7	18	0	0	1	1	0	0	0	0	0	0	0
76/77	3	3	7	18	0	0	1	0	1	0	0	0	0	3	0
77/78	3	2	8	25	0	2	0	0	4	0	1	1	0	1	0
78/79	5	0	12	32	0	0	1	0	3	0	0	0	0	0	0
79/80	8	0	12	39	0	0	3	0	5	0	0	0	0	0	0
TOTAL	29	7	59	179	0	2	8	3	20	0	3	1	0	6	0

APPENDIX 1H

Transition between the various states by rank and year for the Faculty of Technology

	IN POSITION					PROMOTED					WASTAGE				
	Prof.	Read.	Sn. Lect.	Lect.	Asst. Lect.	Prof. from Read.	Prof. from S.L.	Read.	S.L.	Lect.	Prof.	Read.	S.L.	Lect.	A.L.
70/71															
71/72															
72/73	0	0	0	4	0	0	0	0	0	0	0	0	0	0	0
73/74	0	0	1	8	0	0	0	0	0	0	0	0	0	0	0
74/75	0	0	1	6	0	0	0	0	0	0	0	0	0	2	0
75/76	0	0	3	15	0	0	0	0	2	0	0	0	0	0	0
76/77	1	0	4	15	0	0	1	0	1	0	0	0	0	0	0
77/78	1	0	4	12	2	0	0	1	4	1	0	0	0	0	0
78/79	1	1	9	16	4	0	0	0	2	0	0	0	0	0	0
79/80	1	1	9	16	4	0	0	0	1	1	0	0	0	0	0
TOTAL	4	2	31	92	10	0	1	1	10	2	0	0	0	2	0

APPENDIX 2

Transition between states 1970/71-79/80
by faculty/college

	Asst.Lect.	Lect.	Sn.Lect.	Read.	Prof.	Wastage	Total
1. College of Medicine							
Asst.Lect.	32	9	0	0	0	1	42
Lect.	0	673	121	0	0	21	814
Sn.Lect.	0	0	611	21	32	6	670
Read.	0	0	0	104	12	4	120
Prof.	0	0	0	0	415	6	421
2. Faculty of Arts							
Asst.Lect.	34	8	0	0	0	1	42
Lect.	0	647	38	0	0	21	706
Sn.Lect.	0	0	231	8	6	6	257
Read.	0	0	0	23	1	6	30
Prof.	0	0	0	0	137	6	143
3. Faculty of Science							
Asst.Lect.	39	6	0	0	0	5	50
Lect.	0	604	52	0	0	37	693
Sn.Lect.	0	0	347	15	11	9	382
Read.	0	0	0	84	4	5	93
Prof.	0	0	0	0	149	8	157
4. Faculty of Agriculture and Forestry							
Asst.Lect.	15	3	0	0	0	4	22
Lect.	0	402	58	0	0	14	474
Sn.Lect.	0	0	287	27	14	11	339
Read.	0	0	0	76	6	5	87
Prof.	0	0	0	0	135	6	141
5. Faculty of Education							
Asst.Lect.	34	8	0	0	0	3	45
Lect.	0	225	27	0	0	9	261
Sn.Lect.	0	0	154	7	4	5	170
Read.	0	0	0	14	3	2	19
Prof.	0	0	0	0	43	3	46
6. Faculty of the Social Sciences							
Asst.Lect.	7	0	0	0	0	3	10
Lect.	0	338	47	0	0	7	392
Sn.Lect.	0	0	175	8	10	1	194
Read.	0	0	0	14	2	1	17
Prof.	0	0	0	0	87	1	88

APPENDIX 2 (continued)

	Asst.Lect.	Lect.	Sn.Lect.	Read.	Prof.	Wastage	Total
7. Faculty of Veterinary Medicine							
Asst.Lect.	0	0	0	0	0	0	0
Lect.	0	179	20	0	0	6	205
Sn.Lect.	0	0	59	3	8	0	70
Read.	0	0	0	7	2	1	10
Prof.	0	0	0	0	29	3	32
8. Faculty of Technology							
Asst.Lect.	10	2	0	0	0	0	12
Lect.	0	92	10	0	0	2	104
Sn.Lect.	0	0	31	1	1	0	33
Read.	0	0	0	2	0	0	2
Prof.	0	0	0	0	4	0	4

APPENDIX 3

Computed chi-square for transition probabilities*

College/ Faculty	From Asst. Lect. to: A.L.	Lect.	Wast.	From Lect. to: Lect.	S.L.	Wast.	From Snr. Lect. to: S.L.	Read.	Prof.	Wast.	From Reader to: Read.	Prof.	Wast.	From Prof. to: Prof.	Wast.
Medicine	2.60	10.78	5.06	8.90	74.95	11.55	3.07	28.68	16.80	8.57	2.08	11.64	8.91	0.08	22.07
Arts	0.82	3.88	1.00	2.43	15.06	67.79	1.32	11.94	14.86	25.74	2.22	19.52	8.92	0.26	6.00
Science	2.60	5.81	7.78	2.56	4.57	10.72	1.31	12.57	16.43	18.11	0.85	7.14	6.16	1.21	14.63
Agric. & Forestry	1.87	4.33	7.91	1.46	12.70	22.66	0.34	6.49	6.56	6.14	2.26	7.31	2.58	0.20	5.42
Education	3.62	12.67	4.50	1.06	8.55	10.08	1.76	9.16	21.07	8.56	3.08	3.57	10.83	0.72	10.29
Social Sciences	2.05	0	4.78	2.21	10.81	7.86	1.70	14.42	15.13	5.41	0.77	3.66	7.50	0.12	9.97
Vet. Medicine	0	0	0	1.75	6.77	19.69	1.80	9.95	9.72	0	1.29	2.00	1.00	1.07	9.56
Technology	0.24	1.20	0	0.79	6.43	24.02	0.30	5.60	5.60	0	0	0	0	0	0

*Critical value of chi-square, 9 degrees of freedom at .01 level is 23.59.

APPENDIX 4

University system scale (U.S.S.) for academic staff
(Naira)

	1	2	3	4	5	6	7
U.S.S. 8 Assistant Lecturer	6336	6528	6720	6912	7104	7296	7488
U.S.S. 9 Lecturer II	7550	7632	7836	8040			
U.S.S. 11 Lecturer I	9000	9360	9720	10080			
U.S.S. 13A Senior Lecturer	11364	11940	12516	13092	13668	14244	14820
U.S.S. 14 Reader	12732	13392	14052	14712	15372		
U.S.S. 15 Professor	14280	15000	15720				

APPENDIX 5

Ratio of Budget for Emoluments/Actual Expenditure by faculty (1970/71-79/80)

	Med.	Arts	Science	Agric.	Educ.	Soc. Sci.	Vet.	Tech.
70/71	1.1054	1.0151	1.0583	1.0699	1.1921	1.1220	-	-
71/72	1.2224	1.0851	1.2027	1.0975	1.2588	1.1921	-	-
72/73	1.1217	1.0666	1.0538	1.0200	1.0347	1.0830	-	-
73/74	1.2534	1.2478	1.2782	1.1296	1.3330	1.2619	-	2.1690
74/75	1.3890	1.2606	1.2808	1.0816	1.2906	1.3698	-	2.2340
75/76	1.3924	1.3142	1.2966	1.2213	1.4362	1.4911	1.1334	1.5838
76/77	1.3050	1.1715	1.2810	1.2129	1.5341	1.5335	1.3952	2.2826
77/78	1.1439	1.0999	1.0570	1.0919	1.1320	1.3197	1.2885	1.3759
78/79	1.0085	1.4830	1.3194	1.3315	1.1035	1.0689	1.0653	1.1090
Total	10.9417	10.7438	10.8277	10.2562	11.3150	11.4420	4.8824	10.7543
Average	1.2158	1.1938	1.2031	1.1396	1.2573	1.2714	1.2206	1.7924

Source: Computed from Annual Audited Accounts of the University of Ibadan, various issues.

APPENDIX 6

THE MODEL

APPENDIX 6

The Model

(i) Academic Staff Level Goals (40)

(1) $j = 1, \quad t = 1$

$$- 0.2592X_{111} + 1.3125X_{112} - 1.3125Y_{112} - 0.1302X_{211} + 1.2096X_{212} - 1.2096Y_{212} - 0.0846X_{311} + 1.0966X_{312} - 1.0966Y_{312} - 0.1014X_{411} + 1.1538X_{412} - 1.1538Y_{412} + 1.0144X_{512} - 1.0144Y_{512} + n^{a}_{11} - p^{a}_{11} = 238$$

(2) $j = 1, \quad t = 2$

$$- 0.2592X_{112} + 1.3125X_{113} - 1.3125Y_{113} - 0.1302X_{212} + 1.2096X_{213} - 1.2096Y_{213} - 0.0846X_{312} + 1.0966X_{313} - 1.0966Y_{313} - 0.1014X_{412} + 1.1538X_{413} \quad 1.538Y_{413} + 1.0144X_{513} - 1.0144Y_{513} + n^{a}_{12} - p^{a}_{12} = 256$$

(3) $j = 1, \quad t = 3$

$$-0.2592X_{113} + 1.3125X_{114} - 1.3125Y_{114} - 0.1302X_{213} + 1.2096X_{214} - 1.2096Y_{214} - 0.0846X_{313} + 1.0966X_{314} - 1.0966Y_{314} - 0.1014X_{413} + 1.1538X_{414} - 1.1538Y_{414} + 1.0144X_{514} - 1.0144Y_{514} + n^{a}_{13} - p^{a}_{13} = 275$$

(4) $j = 1, \quad t = 4$

$$- 0.2592X_{114} + 1.3125X_{115} - 1.3125Y_{115} - 0.1302X_{214}$$
$$+ 1.2096X_{215} - 1.2096Y_{215} - 0.0846X_{314} + 1.0966X_{315}$$
$$- 1.0966Y_{315} - 0.1014X_{414} + 1.1538X_{415} - 1.1538Y_{415}$$
$$+ 1.0144X_{515} - 1.0144Y_{515} + n^a_{14} - p^a_{14} = 296$$

(5) $j = 1, \quad t = 5$

$$- 0.2592X_{115} + 1.3125X_{116} - 1.3125Y_{116} - 0.1302X_{215}$$
$$+ 1.2096X_{216} - 1.2096Y_{216} - 0.0846X_{315} + 1.0966X_{316}$$
$$- 1.0966Y_{316} - 0.1014X_{415} + 1.1538X_{416} - 1.1538Y_{416}$$
$$+ 1.0144X_{516} - 1.0144Y_{516} + n^a_{15} - p^a_{15} = 321$$

(6) $j = 2, \quad t = 1$

$$-0.2079X_{121} + 1.2728X_{122} - 1.2728Y_{122} - 0.0585X_{221}$$
$$+ 1.0912X_{222} - 1.0912Y_{222} - 0.0666X_{321} + 1.0867X_{322}$$
$$- 1.0867Y_{322} - 0.0348X_{421} + 1.3043X_{422} - 1.3043Y_{422}$$
$$+ 1.0438X_{522} - 1.0438Y_{522} + n^a_{21} - p^a_{21} = 149$$

(7) $j = 2, \quad t = 2$

$$- 0.2079X_{122} + 1.2728X_{123} - 1.2728Y_{123} - 0.0585X_{222}$$
$$+ 1.0912X_{223} - 1.0912Y_{223} - 0.0666X_{322} + 1.0867X_{323}$$
$$-1.0867Y_{323} - 0.0348X_{422} + 1.3043X_{423} - 1.3043Y_{423}$$
$$+ 1.0438X_{523} - 1.0438Y_{523} + n^a_{22} - p^a_{22} = 167$$

(8) $j = 2, \quad t = 3$

$$- 0.2079X_{123} + 1.2728X_{124} - 1.2728Y_{124} - 0.0585X_{223} + 1.0912X_{224} - 1.0912Y_{224} - 0.0666X_{323} + 1.0867X_{324} - 1.0867Y_{324} - 0.0348X_{423} + 1.3043X_{424} - 1.3043Y_{424} + 1.0438X_{524} - 1.0438Y_{524} + n^a_{23} - p^a_{23} = 189$$

(9) $j = 2, \quad t = 4$

$$- 0.2079X_{124} + 1.2728X_{125} - 1.2728Y_{125} - 0.0585X_{224} + 1.0912X_{225} - 1.0912Y_{225} - 0.0666X_{324} + 1.0867X_{325} - 1.0867Y_{325} - 0.0348X_{424} + 1.3043X_{425} - 1.3043Y_{425} + 1.0438X_{525} - 1.0438Y_{525} + n^a_{24} - p^a_{24} = 214$$

(10) $j = 2, \quad t = 5$

$$- 0.2079X_{125} + 1.2728X_{126} - 1.2728Y_{126} - 0.0585X_{225} + 1.0912X_{226} - 1.0912Y_{226} - 0.0666X_{325} + 1.0867X_{326} - 1.0867Y_{326} - 0.0348X_{425} + 1.3043X_{426} - 1.3043Y_{426} + 1.0438X_{526} - 1.0438Y_{526} + n^a_{25} - p^a_{25} = 246$$

(11) $j = 3, \quad t = 1$

$$- 0.1377X_{131} + 1.2821X_{132} - 1.2821Y_{132} - 0.0826X_{231} + 1.1474X_{232} - 1.1474Y_{232} - 0.0739X_{331} + 1.1008X_{332} - 1.1008Y_{332} - 0.0453X_{431} + 1.1072X_{432} - 1.1072Y_{432} + 1.0537X_{532} - 1.0537Y_{532} + n^a_{31} - p^a_{31} = 100$$

(12) $j = 3, \quad t = 2$

$$- 0.1377X_{132} + 1.2821X_{133} - 1.2821Y_{133} - 0.0826X_{232} + 1.1474X_{233} - 1.1474Y_{233} - 0.0739X_{332} + 1.1008X_{333} - 1.1008Y_{333} - 0.0453X_{432} + 1.1072X_{433} - 1.1072Y_{433} + 1.0537X_{532} - 1.0537Y_{533} + n^a_{32} - p^a_{32} = 112$$

(13) $j = 3, \quad t = 3$

$$- 0.1377X_{133} + 1.2821X_{134} - 1.2821Y_{134} - 0.0826X_{233} + 1.1474X_{234} - 1.1474Y_{234} - 0.0739X_{333} + 1.1008X_{334} - 1.1008Y_{334} - 0.0453X_{433} + 1.1072X_{434} - 1.1072Y_{434} + 1.0537X_{534} - 1.0537Y_{534} + n^a_{33} - p^a_{33} = 124$$

(14) $j = 3, \quad t = 4$

$$- 0.1377X_{134} + 1.2821X_{135} - 1.2821Y_{135} - 0.0826X_{234} + 1.1474X_{235} - 1.1474Y_{235} - 0.0739X_{334} + 1.1008X_{335} - 1.1008Y_{335} - 0.0453X_{434} + 1.1072X_{435} - 1.1072Y_{435} + 1.0537X_{535} - 1.0537Y_{535} + n^a_{34} - p^a_{34} = 138$$

(15) $j = 3, \quad t = 5$

$$- 0.1377X_{135} + 1.2821X_{136} - 1.2821Y_{136} - 0.0826X_{235} + 1.1474X_{236} - 1.1474Y_{236} - 0.0739X_{335} + 1.1008X_{336} - 1.1008Y_{336} - 0.0453X_{435} + 1.1072X_{436} - 1.1072Y_{436} + 1.0537X_{536} - 1.0537Y_{536} + n^a_{35} - p^a_{35} = 153$$

(16) $j = 4, \quad t = 1$

$$- 0.1608X_{141} + 1.4667X_{142} - 1.4667Y_{142} - 0.1446X_{241} + 1.1791X_{242} - 1.1791Y_{242} - 0.1344X_{341} + 1.1812X_{342} - 1.1812Y_{342} - 0.0721X_{441} + 1.1447X_{442} - 1.1447Y_{442} + 1.0444X_{542} - 1.0444Y_{542} + n^a_{41} - p^a_{41} = 76$$

(17) $j = 4, \quad t = 2$

$$- 0.1608X_{142} + 1.4667X_{143} - 1.4667Y_{143} - 0.1446X_{242} + 1.1791X_{243} - 1.1791Y_{243} - 0.1344X_{342} + 1.1812X_{343} - 1.1812Y_{343} - 0.0721X_{442} + 1.1447X_{443} - 1.1447Y_{443} + 1.0444X_{543} - 1.0444Y_{543} + n^a_{42} - p^a_{42} = 83$$

(18) $j = 4, \quad t = 3$

$$- 0.1608X_{143} + 1.4667X_{144} - 1.4667Y_{144} - 0.1446X_{243} + 1.1791X_{244} - 1.1791Y_{244} - 0.1344X_{343} + 1.1812X_{344} - 1.1812Y_{344} - 0.0721X_{443} + 1.1447X_{444} - 1.1447Y_{444} + 1.0444X_{544} - 1.0444Y_{544} + n^a_{43} - p^a_{43} = 89$$

(19) $j = 4, \quad t = 4$

$$- 0.1608X_{144} + 1.4667X_{145} - 1.4667Y_{145} - 0.1446X_{244} + 1.1791X_{245} - 1.1791Y_{245} - 0.1344X_{344} + 1.1812X_{345} - 1.1812Y_{345} - 0.0721X_{444} + 1.1447X_{445} - 1.1447Y_{445} + 1.0444X_{545} - 1.0444Y_{545} + n^a_{44} - p^a_{44} = 95$$

(20) j = 4, t = 5

$$- 0.1608X_{145} + 1.4667X_{146} - 1.4667Y_{146} - 0.1446X_{245} + 1.1791X_{246} - 1.1791Y_{246} - 0.1344X_{345} + 1.1812X_{346} - 1.1812Y_{346} - 0.0721X_{445} + 1.1447X_{446} - 1.1447Y_{446} + 1.0444X_{546} - 1.0444Y_{546} + n^{a}_{45} - p^{a}_{45} = 101$$

(21) j = 5, t = 1

$$- 0.2062X_{151} + 1.3235X_{152} - 1.3235Y_{152} - 0.1143X_{251} + 1.1600X_{252} - 1.1600Y_{252} - 0.0811X_{351} + 1.1039X_{352} - 1.1039Y_{352} - 0.1689X_{451} + 1.3572X_{452} - 1.3572Y_{452} + 1.0697X_{552} - 1.0697Y_{552} + n^{a}_{51} - p^{a}_{51} = 68$$

(22) j = 5, t = 2

$$- 0.2062X_{152} + 1.3235X_{153} - 1.3235Y_{153} - 0.1143X_{252} + 1.1600X_{253} - 1.1600Y_{253} - 0.0811X_{352} + 1.1039X_{353} - 1.1039Y_{353} - 0.1689X_{452} + 1.3572X_{453} - 1.3572Y_{453} + 1.0697X_{553} - 1.0697Y_{553} + n^{a}_{52} - p^{a}_{52} = 79$$

(23) j = 5, t = 3

$$- 0.2062X_{153} + 1.3235X_{154} - 1.3235Y_{154} - 0.1143X_{253} + 1.1600X_{254} - 1.1600Y_{254} - 0.0811X_{353} + 1.1039X_{354} - 1.1039Y_{354} - 0.1689X_{453} + 1.3572X_{454} - 1.3572Y_{454} + 1.0697X_{554} - 1.0697Y_{554} + n^{a}_{53} - p^{a}_{53} = 92$$

(24) j = 5, t = 4

$$- 0.2062X_{154} + 1.3235X_{155} - 1.3235Y_{155} - 0.1143X_{254} + 1.1600X_{255} - 1.1600Y_{255} - 0.0811X_{354} + 1.1039X_{355} - 1.1039Y_{355} - 0.1689X_{454} + 1.3572X_{455} - 1.3572Y_{455} + 1.0697X_{555} - 1.0697Y_{555} + n^a_{54} - p^a_{54} = 107$$

(25) j = 5, t = 5

$$- 0.2062X_{155} + 1.3235X_{156} - 1.3235Y_{156} - 0.1143X_{255} + 1.1600X_{256} - 1.1600Y_{256} - 0.0811X_{355} + 1.1039X_{356} - 1.1039Y_{356} - 0.1689X_{455} + 1.3572X_{456} - 1.3592Y_{456} + 1.0697X_{556} - 1.0697Y_{556} + n^a_{55} - p^a_{55} = 123$$

(26) j = 6, t = 1

$$0.0X_{161} + 1.4286X_{162} - 1.4286Y_{162} - 0.1329X_{261} + 1.1597X_{262} - 1.1597Y_{262} - 0.1022X_{361} + 1.1087X_{362} - 1.1087Y_{362} - 0.1191X_{461} + 1.2143X_{462} - 1.2143Y_{462} + 1.0115X_{562} - 1.0115Y_{562} + n^a_{61} - p^a_{61} = 77$$

(27) j = 6, t = 2

$$0.0X_{162} + 1.4286X_{163} - 1.4286Y_{163} - 0.1329X_{262} + 1.1597X_{263} - 1.1597Y_{263} - 0.1022X_{362} + 1.1087X_{363} - 1.1087Y_{363} - 0.1191X_{462} + 1.2143X_{463} - 1.2143Y_{463} + 1.0115X_{563} - 1.0115Y_{563} + n^a_{62} - p^a_{62} = 84$$

(28) $j = 6, \quad t = 3$

$$0.0X_{163} + 1.4286X_{164} - 1.4286Y_{164} - 0.1329X_{263} + 1.1597X_{264} - 1.1597Y_{264} - 0.1022X_{363} + 1.1087X_{364} - 1.1087Y_{364} - 0.1191X_{463} + 1.2143X_{464} - 1.2143Y_{464} + 1.0115X_{564} - 1.0115Y_{564} + n^a_{63} - p^a_{63} = 91$$

(29) $j = 6, \quad t = 4$

$$0.0X_{164} + 1.4286X_{165} - 1.4286Y_{165} - 0.1329X_{264} + 1.1597X_{265} - 1.1597Y_{265} - 0.1022X_{364} + 1.1087X_{365} - 1.1087Y_{365} - 0.1191X_{464} + 1.2143X_{465} - 1.2143Y_{465} + 1.0115X_{565} - 1.0115Y_{565} + n^a_{64} - p^a_{64} = 99$$

(30) $j = 6, \quad t = 5$

$$0.0X_{165} + 1.4286X_{166} - 1.4286Y_{166} - 0.1329X_{265} + 1.1597X_{266} - 1.1597Y_{266} - 0.1022X_{365} + 1.1087X_{366} - 1.1087Y_{366} - 0.1191X_{465} + 1.2143X_{466} - 1.2143Y_{466} + 1.0115X_{566} - 1.0115Y_{566} + n^a_{65} - p^a_{65} = 107$$

(31) $j = 7, \quad t = 1$

$$0.0X_{171} + 0.0X_{172} - 0.0Y_{172} - 0.1158X_{271} + 1.1452X_{272} - 1.1452Y_{272} - 0.1873X_{371} + 1.1864X_{372} - 1.1864Y_{372} - 0.2207X_{471} + 1.4286X_{472} - 1.4286Y_{472} + 1.1034X_{572} - 1.1034Y_{572} + n^a_{71} - p^a_{71} = 82$$

(32) $j = 7, \quad t = 2$

$$0.0X_{172} + 0.0X_{173} - 0.0Y_{173} - 0.1158X_{272} + 1.1452X_{273} - 1.1452Y_{273} - 0.1873X_{372} + 1.1864X_{373} - 1.1864Y_{373} - 0.2207X_{472} + 1.4286X_{473} - 1.4286Y_{473} + 1.1034X_{573} - 1.1034Y_{573} + n^a_{72} - p^a_{72} = 91$$

(33) $j = 7, \quad t = 3$

$$0.0X_{173} + 0.0X_{174} - 0.0Y_{174} - 0.1158X_{273} + 1.1452X_{274} - 1.1452Y_{274} - 0.1873X_{373} + 1.1864X_{374} - 1.1864Y_{374} - 0.2207X_{473} + 1.4286X_{474} - 1.4286Y_{474} + 1.1034X_{574} - 1.1034Y_{574} + n^a_{73} - p^a_{73} = 101$$

(34) $j = 7, \quad t = 4$

$$0.0X_{174} + 0.0X_{175} - 0.0Y_{175} - 0.1158X_{274} + 1.1452X_{275} - 1.1452Y_{275} - 0.1873X_{374} + 1.1869X_{375} - 1.1864Y_{375} - 0.2207X_{474} + 1.4286X_{475} - 1.4286Y_{475} + 1.1034X_{575} - 1.1034Y_{575} + n^a_{74} - p^a_{74} = 111$$

(35) $j = 7, \quad t = 5$

$$0.0X_{175} + 0.0X_{176} - 0.0Y_{176} - 0.1158X_{275} + 1.1452X_{276} - 1.1452Y_{276} - 0.1873X_{375} + 1.1864X_{376} - 1.1864Y_{376} - 0.2207X_{475} + 1.4286X_{476} - 1.4286Y_{476} + 1.1034X_{576} - 1.1034Y_{576} + n^a_{75} - p^a_{75} = 122$$

(36) $j = 8, \quad t = 1$

$$- 0.1885X_{181} + 1.2000X_{182} - 1.2000Y_{182} - 0.1024X_{281} + 1.1305X_{282} - 1.1305Y_{282} - 0.0606X_{381} + 1.0000X_{382} - 1.0000Y_{382} + 0.0X_{481} + 1.0000X_{482} - 1.0000Y_{482} + 1.0000X_{582} - 1.0000Y_{582} + n^{a}_{81} - p^{a}_{81} = 31$$

(37) $j = 8, \quad t = 2$

$$- 0.1885X_{182} + 1.2000X_{183} - 1.2000Y_{183} - 0.1024X_{282} + 1.1305X_{283} - 1.1305Y_{283} - 0.0606X_{382} + 1.0000X_{383} - 1.0000Y_{383} + 0.0X_{482} + 1.0000X_{483} - 1.0000Y_{483} + 1.0000X_{583} - 1.0000Y_{583} + n^{a}_{82} - p^{a}_{82} = 37$$

(38) $j = 8, \quad t = 3$

$$- 0.1885X_{183} + 1.2000X_{184} - 1.2000Y_{184} - 0.1024X_{283} + 1.1305X_{284} - 1.1305Y_{284} - 0.0606X_{383} + 1.0000X_{384} - 1.0000Y_{384} + 0.0X_{483} + 1.0000X_{484} - 1.0000Y_{484} + 1.0000X_{584} - 1.0000Y_{584} + n^{a}_{83} - p^{a}_{83} = 44$$

(39) $j = 8, \quad t = 4$

$$- 0.1885X_{184} + 1.2000X_{185} - 1.2000Y_{185} - 0.1024X_{284} + 1.1305X_{285} - 1.1305Y_{285} - 0.0606X_{384} + 1.0000X_{385} - 1.0000Y_{385} + 0.0X_{484} + 1.0000X_{485} - 1.0000Y_{485} + 1.0000X_{585} - 1.0000Y_{585} + n^{a}_{84} - p^{a}_{84} = 50$$

(40) $j = 8, \quad t = 5$

$$- 0.1885X_{185} + 1.2000X_{186} - 1.2000Y_{186} - 0.1024X_{285} + 1.1305X_{286} - 1.1305Y_{286} - 0.0606X_{385} + 1.0000X_{386} - 1.0000Y_{386} + 0.0X_{485} + 1.0000X_{486} - 1.0000Y_{486} + 1.0000X_{586} - 1.0000Y_{586} + n^{a}_{85} - p^{a}_{85} = 59$$

<u>(ii) Student Enrolment Goals (40)</u>

(41) $j = 1, \quad t = 1$

$$- 0.2592X_{111} + 1.3125X_{112} - 1.3125Y_{112} - 0.1302X_{211} + 1.2096X_{212} - 1.2096Y_{212} - 0.0846X_{311} + 1.0966X_{312} - 1.0966Y_{312} - 0.1014X_{411} + 1.1538X_{412} - 1.1538Y_{412} + 1.0144X_{512} - 1.0144Y_{512} + n^{b}_{11} - p^{b}_{11} = 235$$

(42) $j = 1, \quad t = 2$

$$- 0.2592X_{112} + 1.3125X_{113} - 1.3125Y_{113} - 0.1302X_{212} + 1.2096X_{213} - 1.2096Y_{213} - 0.0846X_{312} + 1.0966X_{313} - 1.0966Y_{313} - 0.1014X_{412} + 1.1538X_{413} - 1.1538Y_{413} + 1.0144X_{513} - 1.0144Y_{513} + n^{b}_{12} - p^{b}_{12} = 245$$

(43) $j = 1, \quad t = 3$

$$- 0.2592X_{113} + 1.3125X_{114} - 1.3125Y_{114} - 0.1302X_{213} + 1.2096X_{214} - 1.2096Y_{214} - 0.0846X_{313} + 1.0966X_{314} - 1.0966Y_{314} - 0.1014X_{413} + 1.1538X_{414} - 1.1538Y_{414} + 1.0144X_{514} - 0.0144Y_{514} + n^{b}_{13} - p^{b}_{13} = 256$$

(44) j = 1, t = 4

$$- 0.2592X_{114} + 1.3125X_{115} - 1.3125Y_{115} - 0.1302X_{214}$$
$$+ 1.2096X_{215} - 1.2096Y_{215} - 0.0846X_{314} + 1.0966X_{315}$$
$$- 1.0966Y_{315} - 0.1014X_{414} + 1.1538X_{415} - 1.1538Y_{415}$$
$$+ 1.0144X_{515} - 1.0144Y_{515} + n^{b}_{14} - p^{b}_{14} = 264$$

(45) j = 1, t = 5

$$- 0.2592X_{115} + 1.3125X_{116} - 1.3125Y_{116} - 0.1302X_{215}$$
$$+ 1.2096X_{216} - 1.2096Y_{216} - 0.0846X_{315} + 1.0966X_{316}$$
$$- 1.0966Y_{316} - 0.1014X_{415} + 1.1538X_{416} - 1.1538Y_{416}$$
$$+ 1.0144X_{516} - 1.0144Y_{516} + n^{b}_{15} - p^{b}_{15} = 267$$

(46) j = 2, t = 1

$$- 0.2079X_{121} + 1.2728X_{122} - 1.2728Y_{122} - 0.0585X_{221}$$
$$+ 1.0912X_{222} - 1.0912Y_{222} - 0.0666X_{321} + 1.0867X_{322}$$
$$- 1.0867Y_{322} - 0.0348X_{421} + 1.3043X_{422} - 1.3043Y_{422}$$
$$+ 1.0438X_{522} - 1.0438Y_{522} + n^{b}_{21} - p^{b}_{21} = 115$$

(47) j = 2, t = 2

$$- 0.2079X_{122} + 1.2728X_{123} - 1.2728Y_{123} - 0.0585X_{222}$$
$$+ 1.0912X_{223} - 1.0912Y_{223} - 0.0666X_{322} + 1.0867X_{323}$$
$$- 1.0867Y_{323} - 0.0348X_{422} + 1.3043X_{423} - 1.3043Y_{423}$$
$$+ 1.0438X_{523} - 1.0438Y_{523} + n^{b}_{22} - p^{b}_{22} = 125$$

(48) $j = 2, \quad t = 3$

$$- 0.2079X_{123} + 1.2728X_{124} - 1.2728Y_{124} - 0.0585X_{223}$$
$$+ 1.0912X_{224} - 1.0912Y_{224} - 0.0666X_{323} + 1.0867X_{324}$$
$$- 1.0867Y_{324} - 0.0348X_{423} + 1.3043X_{424} - 1.3043Y_{424}$$
$$+ 1.0438X_{524} - 1.0438Y_{524} + n^b_{23} - p^b_{23} = 135$$

(49) $j = 2, \quad t = 4$

$$- 0.2079X_{124} + 1.2728X_{125} - 1.2728Y_{125} - 0.0585X_{224}$$
$$+ 1.0912X_{225} - 1.0912Y_{225} - 0.0666X_{324} + 1.0867X_{325}$$
$$- 1.0867Y_{325} - 0.0348X_{425} + 1.3043X_{425} - 1.3043Y_{425}$$
$$+ 1.0438X_{525} - 1.0438Y_{525} + n^b_{24} - p^b_{24} = 143$$

(50) $j = 2, \quad t = 5$

$$- 0.2079X_{125} + 1.2728X_{126} - 1.2728Y_{126} - 0.0585X_{225}$$
$$+ 1.0912X_{226} - 1.0912Y_{226} - 0.0666X_{325} + 1.0867X_{326}$$
$$- 1.0867Y_{326} - 0.0348X_{425} + 1.3043X_{426} - 1.3043Y_{426}$$
$$+ 1.0438X_{526} - 1.0438Y_{526} + n^b_{25} - p^b_{25} = 150$$

(51) $j = 3, \quad t = 1$

$$- 0.1377X_{131} + 1.2821X_{132} - 1.2821Y_{132} - 0.0826X_{231}$$
$$+ 1.1474X_{232} - 1.1474Y_{232} - 0.0739X_{331} + 1.1008X_{332}$$
$$- 1.1008Y_{332} - 0.0453X_{431} + 1.1072X_{432} - 1.1072Y_{432}$$
$$+ 1.0537X_{532} - 1.0537Y_{532} + n^b_{31} - p^b_{31} = 160$$

(52) j = 3, t = 2

$$- 0.1377X_{132} + 1.2821X_{133} - 1.2821Y_{133} - 0.0826X_{232} + 1.1474X_{233} - 1.1474Y_{233} - 0.0739X_{332} + 1.1008X_{333} - 1.1008Y_{333} - 0.0453X_{432} + 1.1072X_{433} - 1.1072Y_{433} + 1.0537X_{533} - 1.0537Y_{533} + n^b_{32} - p^b_{32} = 181$$

(53) j = 3, t = 3

$$- 0.1377X_{133} + 1.2821X_{134} - 1.2821Y_{134} - 0.0826X_{233} + 1.1474X_{234} - 1.1474Y_{234} - 0.0739X_{333} + 1.1008X_{334} - 1.1008Y_{334} - 0.0453X_{433} + 1.1072X_{434} - 1.1072Y_{434} + 1.0537X_{534} - 1.0537Y_{534} + n^b_{33} - p^b_{33} = 201$$

(54) j = 3, t = 4

$$- 0.1377X_{134} + 1.2821X_{135} - 1.2821Y_{135} - 0.0826X_{234} + 1.1474X_{235} - 1.1474Y_{235} - 0.0739X_{334} + 1.1008X_{335} - 1.1008Y_{335} - 0.0453X_{434} + 1.1072X_{435} - 1.1072Y_{435} + 1.0537X_{535} - 1.0537Y_{535} + n^b_{34} - p^b_{34} = 222$$

(55) j = 3, t = 5

$$- 0.1377X_{135} + 1.2821X_{136} - 1.2821Y_{136} - 0.0826X_{235} + 1.1474X_{236} - 1.1474Y_{236} - 0.0739X_{335} + 1.1008X_{336} - 1.1008Y_{336} - 0.0453X_{435} + 1.1072X_{436} - 1.1072Y_{436} + 1.0537X_{536} - 1.0537Y_{536} + n^b_{35} - p^b_{35} = 226$$

(56) $j = 4, \quad t = 1$

$$- 0.1608X_{141} + 1.4667X_{142} - 1.4667Y_{142} - 0.1446X_{241}$$
$$+ 1.1791X_{242} - 1.1791Y_{242} - 0.1344X_{341} + 1.1812X_{342}$$
$$- 1.1812Y_{342} - 1.0721X_{441} + 1.1447X_{442} - 1.1447Y_{442}$$
$$+ 1.0444X_{542} - 1.0444Y_{542} + n^{b}_{41} - p^{b}_{41} = 122$$

(57) $j = 4, \quad t = 2$

$$- 0.1608X_{142} + 1.4667X_{143} - 1.4667Y_{143} - 0.1446X_{242}$$
$$+ 1.1791X_{243} - 1.1791Y_{243} - 0.1344X_{342} + 1.1812X_{343}$$
$$- 1.1812Y_{343} - 0.0721X_{442} + 1.1447X_{443} - 1.1447Y_{443}$$
$$+ 1.0444X_{543} - 1.0444Y_{543} + n^{b}_{42} - p^{b}_{42} = 143$$

(58) $j = 4, \quad t = 3$

$$- 0.1608X_{143} + 1.4667X_{144} - 1.4667Y_{144} - 0.1446X_{243}$$
$$+ 1.1791X_{244} - 1.1791Y_{244} - 0.1344X_{343} + 1.1812X_{344}$$
$$- 1.1812Y_{344} - 0.0721X_{443} + 1.1447X_{444} - 1.1447Y_{444}$$
$$+ 1.0444X_{544} - 1.0444Y_{544} + n^{b}_{43} - p^{b}_{43} = 162$$

(59) $j = 4, \quad t = 4$

$$- 0.1608X_{144} + 1.4667X_{145} - 1.4667Y_{145} - 0.1446X_{244}$$
$$+ 1.1791X_{245} - 1.1791Y_{245} - 0.1344X_{344} + 1.1812X_{345}$$
$$- 1.1812Y_{345} - 0.0721X_{444} + 1.1447X_{445} - 1.1447Y_{445}$$
$$+ 1.0444X_{545} - 1.0444Y_{545} + n^{b}_{44} - p^{b}_{44} = 184$$

(60) $j = 4, \quad t = 5$

$$- 0.1608X_{145} + 1.4667X_{146} - 1.4667Y_{146} - 0.1446X_{245} + 1.1791X_{246} - 1.1791Y_{246} - 0.1344X_{345} + 1.1812X_{346} - 1.1812Y_{346} - 0.0721X_{445} + 1.1447X_{446} - 1.1447Y_{446} + 1.0444X_{546} - 1.0444Y_{546} + n^b_{45} - p^b_{45} = 187$$

(61) $j = 5, \quad t = 1$

$$- 0.2062X_{151} + 1.3235X_{152} - 1.3235Y_{152} - 0.1143X_{251} + 1.1600X_{252} - 1.1600Y_{252} - 0.0811X_{351} + 1.1039X_{352} - 1.1039Y_{352} - 0.1689X_{451} + 1.3572X_{452} - 1.3572Y_{452} + 1.0697X_{552} - 1.0697Y_{552} + n^b_{51} - p^b_{51} = 64$$

(62) $j = 5, \quad t = 2$

$$- 0.2062X_{152} + 1.3235X_{153} - 1.3235Y_{153} - 0.1143X_{252} + 1.1600X_{253} - 1.1600Y_{253} - 0.0811X_{352} + 1.1039X_{353} - 1.1039Y_{353} - 0.1689X_{452} + 1.3572X_{453} - 1.3572Y_{453} + 1.0697X_{553} - 1.0697Y_{553} + n^b_{52} - p^b_{52} = 68$$

(63) $j = 5, \quad t = 3$

$$- 0.2062X_{153} + 1.3235X_{154} - 1.3235Y_{154} - 0.1143X_{253} + 1.1600X_{254} - 1.1600Y_{254} - 0.0811X_{353} + 1.1039X_{354} - 1.1039Y_{354} - 0.1689X_{453} + 1.3572X_{454} - 1.3572Y_{454} + 1.0697X_{554} - 1.0697Y_{554} + n^b_{53} - p^b_{53} = 72$$

(64) $j = 5, \quad t = 4$

$$- 0.2062X_{154} + 1.3235X_{155} - 1.3235Y_{155} - 0.1143X_{254} + 1.1600X_{255} - 1.1600Y_{255} - 0.0811X_{354} + 1.1039X_{355} - 1.1039Y_{355} - 0.1689X_{454} + 1.3572X_{455} - 1.3572Y_{455} + 1.0697X_{555} - 1.0697Y_{555} + n^{b}_{54} - p^{b}_{54} = 78$$

(65) $j = 5, \quad t = 5$

$$- 0.2062X_{155} + 1.3235X_{156} - 1.3235Y_{156} - 0.1143X_{255} + 1.1600X_{256} - 1.1600Y_{256} - 0.0811X_{355} + 1.1039X_{356} - 1.1039Y_{356} - 0.1689X_{455} + 1.3572X_{456} - 1.3592Y_{456} + 1.0697Y_{556} - 1.0697Y_{556} + n^{b}_{55} - p^{b}_{55} = 78$$

(66) $j = 6, \quad t = 1$

$$0.0X_{161} + 1.4286X_{162} - 1.4286Y_{162} - 0.1329X_{261} + 1.1597X_{262} - 1.1597Y_{262} + 0.1022X_{361} + 1.1087X_{362} - 1.1087Y_{362} - 0.1191X_{461} + 1.2143X_{462} - 1.2143Y_{462} + 1.0115X_{562} - 1.0115Y_{562} + n^{b}_{61} - p^{b}_{61} = 93$$

(67) $j = 6, \quad t = 2$

$$0.0X_{162} + 1.4286X_{163} - 1.4286Y_{163} - 0.1329X_{262} + 1.1597X_{263} - 1.1597Y_{263} + 0.1022X_{362} + 1.1087X_{363} - 1.1087Y_{363} - 0.1191X_{462} + 1.2143X_{463} - 1.2143Y_{463} + 1.0115X_{563} - 1.0115Y_{563} + n^{b}_{62} - p^{b}_{62} = 114$$

(68) $j = 6, \quad t = 3$

$$0.0X_{163} - 1.4286X_{164} - 1.4286Y_{164} - 0.1329X_{263}$$
$$+ 1.1597X_{264} - 1.1597Y_{264} + 0.1022X_{363} + 1.1087X_{364}$$
$$- 1.1087Y_{364} - 0.1191X_{463} + 1.2143X_{464} - 1.2143Y_{464}$$
$$+ 1.0115X_{564} - 1.0115Y_{564} + n^{b}_{63} - p^{b}_{63} = 135$$

(69) $j = 6, \quad t = 4$

$$0.0X_{164} + 1.4286X_{165} - 1.4286Y_{165} - 0.1329X_{264}$$
$$+ 1.1597X_{265} - 1.1597Y_{265} + 0.1022X_{364} + 1.1087X_{365}$$
$$- 1.1087Y_{365} - 0.1191X_{464} + 1.2143X_{465} - 1.2143Y_{465}$$
$$+ 1.0115X_{565} - 1.0115Y_{565} + n^{b}_{64} - p^{b}_{64} = 156$$

(70) $j = 6, \quad t = 5$

$$0.0X_{165} + 1.4286X_{166} - 1.4286Y_{166} - 0.1329X_{265}$$
$$+ 1.1597X_{266} - 1.1597Y_{266} + 0.1022X_{365} + 1.1087X_{366}$$
$$- 1.1087Y_{366} - 0.1191X_{465} + 1.2143X_{466} - 1.2143Y_{466}$$
$$+ 1.0115X_{566} - 1.0115Y_{566} + n^{b}_{65} - p^{b}_{65} = 157$$

(71) $j = 7, \quad t = 1$

$$0.0X_{171} + 0.0X_{172} - 0.0Y_{172} - 0.1158X_{271}$$
$$+ 1.1452X_{272} - 1.1452Y_{272} - 0.1873X_{371} + 1.1864X_{372}$$
$$- 1.1864Y_{372} - 0.2207X_{471} + 1.4286X_{472} - 1.4286Y_{472}$$
$$+ 1.1034X_{572} - 1.1034Y_{572} + n^{b}_{71} - p^{b}_{71} = 45$$

(72) $j = 7, \quad t = 2$

$$0.0X_{172} + 0.0X_{173} - 0.0Y_{173} - 0.1158X_{272}$$
$$+ 1.1452X_{273} - 1.1452Y_{273} - 0.1873X_{372} + 1.1864X_{373}$$
$$- 1.1864Y_{373} - 0.2207X_{472} + 1.4286X_{473} - 1.4286Y_{473}$$
$$+ 1.1034X_{573} - 1.1034Y_{573} + n^{b}_{72} - p^{b}_{72} = 48$$

(73) $j = 7, \quad t = 3$

$$0.0X_{173} + 0.0X_{174} - 0.0Y_{174} - 0.1158X_{273}$$
$$+ 1.1452X_{274} - 1.1452Y_{274} - 0.1873X_{373} + 1.1864X_{374}$$
$$- 1.1864Y_{374} - 0.2207X_{473} + 1.4286X_{474} - 1.4286Y_{474}$$
$$+ 1.1034X_{574} - 1.1034Y_{574} + n^{b}_{73} - p^{b}_{73} = 51$$

(74) $j = 7, \quad t = 4$

$$0.0X_{174} + 0.0X_{175} - 0.0X_{175} - 0.1158X_{274}$$
$$+ 1.1452X_{275} - 1.1452Y_{275} - 0.1873X_{374} + 1.1864X_{375}$$
$$- 1.1864Y_{375} - 0.2207X_{474} + 1.4286X_{475} - 1.4286Y_{475}$$
$$+ 1.1034X_{575} - 1.1034Y_{575} + n^{b}_{74} - p^{b}_{74} = 76$$

(75) $j = 7, \quad t = 5$

$$0.0X_{175} + 0.0X_{176} - 0.0Y_{176} - 0.1158X_{275}$$
$$+ 1.1452X_{276} - 1.1452Y_{276} - 0.1873X_{375} + 1.1864X_{376}$$
$$- 1.1864Y_{376} - 0.2207X_{475} + 1.4286X_{476} - 1.4286Y_{476}$$
$$+ 1.1034X_{576} - 1.1034Y_{576} + n^{b}_{75} - p^{b}_{75} = 76$$

(76) $j = 8, \quad t = 1$

$$- 0.1885X_{181} + 1.2000X_{182} - 1.2000Y_{182} + 0.1024X_{281} + 1.1305X_{282} - 1.1305Y_{282} - 0.0606X_{381} + 1.0000X_{382} - 1.0000Y_{382} + 0.0X_{481} + 1.0000X_{482} - 1.0000Y_{482} + 1.0000X_{582} - 1.0000Y_{582} + n^b_{81} - p^b_{81} = 45$$

(77) $j = 8, \quad t = 2$

$$- 0.1885X_{182} + 1.2000X_{183} - 1.2000Y_{183} - 0.1024X_{282} + 1.1305X_{283} - 1.1305Y_{283} - 0.0606X_{382} + 1.0000X_{383} - 1.0000Y_{383} + 0.0X_{482} + 1.0000X_{483} - 1.0000Y_{483} + 1.0000X_{583} - 1.0000Y_{583} + n^b_{82} - p^b_{82} = 60$$

(78) $j = 8, \quad t = 3$

$$- 0.1885X_{183} + 1.2000X_{184} - 1.2000Y_{184} - 0.1024X_{283} + 1.1305X_{284} - 1.1305Y_{284} - 0.0606X_{383} + 1.0000X_{384} - 1.0000X_{384} + 0.0X_{483} + 1.0000X_{484} - 1.000Y_{484} + 1.0000X_{584} - 1.0000Y_{584} + n^b_{83} - p^b_{83} = 71$$

(79) $j = 8, \quad t = 4$

$$- 0.1885X_{184} + 1.2000X_{185} - 1.2000Y_{185} - 0.1024X_{284} + 1.1305X_{285} - 1.1305Y_{285} - 0.0606X_{384} + 1.0000X_{385} - 1.0000Y_{385} + 0.0X_{484} + 1.0000X_{485} - 1.0000Y_{485} + 1.0000Y_{585} - 1.0000Y_{585} + n^b_{84} - p^b_{84} = 82$$

(80) $j = 8, \quad t = 5$

$$-0.1885X_{185} + 1.2000X_{186} - 1.2000Y_{186} - 0.1024Y_{285}$$
$$+ 1.1305X_{286} - 1.1305Y_{286} - 0.0606X_{385} + 1.0000X_{386}$$
$$- 1.0000Y_{386} + 0.0X_{485} + 1.0000X_{486} - 1.0000Y_{486}$$
$$+ 1.0000Y_{586} - 1.0000Y_{586} + n^{b}_{85} - p^{b}_{85} = 83$$

(iii) Academic Rank Distribution Goals (120)

(81) $i = A = \{ 1,2 \}, \quad j = 1, \quad t = 1$

$$- 0.1814X_{111} + 0.9188X_{112} - 0.9188Y_{112} + 0.0391X_{211}$$
$$+ 0.8467X_{212} - 0.8467Y_{212} + 0.0254X_{311} - 0.3290X_{312}$$
$$+ 0.3290Y_{312} + 0.0304X_{411} - 0.3461X_{412} + 0.3461Y_{412}$$
$$- 0.3043X_{512} + 0.3043Y_{512} + n^{c}_{A11} - p^{c}_{A11} = 0$$

(82) $i = A = \{1,2\}, \quad j = 1, \quad t = 2$

$$- 0.1814X_{112} + 0.9188X_{113} - 0.9188Y_{113} + 0.0391X_{212}$$
$$+ 0.8467X_{213} - 0.8467Y_{213} + 0.0254X_{312} - 0.3290X_{313}$$
$$+ 0.3290Y_{313} + 0.0304X_{412} - 0.3461X_{413} + 0.3461Y_{413}$$
$$- 0.3043X_{513} + 0.3043Y_{513} + n^{c}_{A12} - p^{c}_{A12} = 0$$

(83) $i = A = \{1,2\}, \quad j = 1, \quad t = 3$

$$- 0.1814X_{113} + 0.9188X_{114} - 0.9188Y_{114} + 0.0391X_{213}$$
$$+ 0.8467X_{214} - 0.8667Y_{214} + 0.0254X_{313} - 0.3290X_{314}$$
$$+ 0.3290Y_{314} + 0.0304X_{413} - 0.3461X_{414} + 0.3461Y_{414}$$
$$- 0.3043X_{514} + 0.3043Y_{514} + n^{c}_{A13} - p^{c}_{A13} = 0$$

(84) $i = A = \{1,2\}, \; j = 1, \; t = 4$

$$- 0.1814X_{114} + 0.9188X_{115} - 0.9188Y_{115} + 0.0391X_{214} + 0.8467X_{215} - 0.8667Y_{215} + 0.0254X_{314} - 0.3290X_{315} + 0.3290Y_{315} + 0.0304X_{414} - 0.3461X_{415} + 0.3461Y_{415} - 0.3043X_{515} + 0.3043Y_{515} + n^c_{A14} - p^c_{A14} = 0$$

(85) $i = A = \{1,2\}, \; j = 1, \; t = 5$

$$- 0.1814X_{115} + 0.9188X_{116} - 0.9188Y_{116} + 0.0391X_{215} + 0.8467X_{216} - 0.8667Y_{216} + 0.0254X_{315} - 0.3290X_{316} + 0.3290Y_{316} + 0.0304X_{415} - 0.3461X_{416} + 0.3461Y_{416} - 0.3043X_{516} + 0.3043Y_{516} + n^c_{A15} - p^c_{A15} = 0$$

(86) $i = A = \{1,2\}, \; j = 2, \; t = 1$

$$- 0.1455X_{121} + 0.8910X_{122} - 0.8910Y_{122} + 0.0176X_{221} + 0.7634X_{222} - 0.7634Y_{222} + 0.0200X_{321} - 0.3260X_{322} + 0.3260Y_{322} + 0.0104X_{421} - 0.3913X_{422} + 0.3913Y_{422} - 0.3131X_{522} + 0.3131Y_{522} + n^c_{A21} - p^c_{A21} = 0$$

(87) $i = A = \{1,2\}, \; j = 2, \; t = 2$

$$- 0.1455X_{122} + 0.8910X_{123} - 0.8910Y_{123} + 0.0176X_{222} + 0.7634X_{223} - 0.7634Y_{223} + 0.0200X_{322} - 0.3260X_{323} + 0.3260Y_{323} + 0.0104X_{422} - 0.3913X_{423} + 0.3913Y_{423} - 0.3131X_{523} + 0.3131Y_{523} + n^c_{A22} - p^c_{A22} = 0$$

(88) $i = A = \{1,2\}$, $j = 2$, $t = 3$

$$- 0.1455X_{123} + 0.8910X_{124} - 0.8910Y_{124} + 0.0176X_{223} + 0.7634X_{224} - 0.7634Y_{224} + 0.0200X_{323} - 0.3260X_{324} + 0.3260Y_{324} + 0.0104X_{423} - 0.3913X_{424} + 0.3913Y_{424} - 0.3131X_{524} + 0.3131Y_{524} + n^{c}_{A23} - p^{c}_{A23} = 0$$

(89) $i = A = \{1,2\}$, $j = 2$, $t = 4$

$$- 0.1455X_{124} + 0.8910X_{125} - 0.8910Y_{125} + 0.0176X_{224} + 0.7634X_{225} - 0.7634Y_{225} + 0.0200X_{324} - 0.3260X_{325} + 0.3260Y_{325} - 0.0104X_{424} - 0.3913X_{425} + 0.3913Y_{425} - 0.3131Y_{525} + 0.3131Y_{525} + n^{c}_{A24} - p^{c}_{A24} = 0$$

(90) $i = A = \{1,2\}$, $j = 2$, $t = 5$

$$- 0.1455X_{125} + 0.8910X_{126} - 0.8910Y_{126} + 0.0176X_{225} + 0.7634X_{226} - 0.7634Y_{226} + 0.0200X_{325} - 0.3260X_{326} + 0.3260Y_{326} - 0.0104X_{425} - 0.3913X_{426} + 0.3913Y_{426} - 0.3131X_{526} + 0.3131Y_{526} + n^{c}_{A25} - p^{c}_{A25} = 0$$

(91) $i = 2 = \{1,2\}$, $j = 3$, $t = 1$

$$- 0.0964X_{131} + 0.8975X_{132} - 0.8975Y_{132} + 0.0248X_{231} + 0.8032X_{232} - 0.8022Y_{232} + 0.0222X_{331} - 0.3302X_{332} + 0.3302Y_{332} + 0.0136X_{431} - 0.3322X_{432} + 0.3322Y_{432} - 0.3161X_{532} + 0.3161Y_{532} + n^{c}_{A31} - p^{c}_{A31} = 0$$

(92) $i = A = \{1,2\}$, $j = 3$, $t = 2$

$$- 0.0964X_{132} + 0.8975X_{133} - 0.8975Y_{133} + 0.0248X_{232} + 0.8032X_{233} - 0.8032Y_{233} + 0.0222X_{332} - 0.3302X_{333} + 0.3302Y_{333} + 0.0136X_{432} - 0.3322X_{433} + 0.3322Y_{433} - 0.3161X_{533} + 0.3161Y_{533} + n^c_{A32} - p^c_{A32} = 0$$

(93) $i = A = \{1,2\}$, $j = 3$, $t = 3$

$$- 0.0964X_{133} + 0.8975X_{134} - 0.8975Y_{134} + 0.0248X_{233} + 0.8032X_{234} - 0.8032Y_{234} + 0.0222X_{333} - 0.3302X_{334} + 0.3302Y_{334} + 0.0136X_{433} - 0.3322X_{434} + 0.3322Y_{434} - 0.3161X_{534} + 0.3161Y_{534} + n^c_{A33} - p^c_{A33} = 0$$

(94) $i = A = \{1,2\}$, $j = 3$, $t = 4$

$$- 0.0964X_{134} + 0.8975X_{135} - 0.8975Y_{135} + 0.0248X_{234} + 0.8032X_{235} - 0.8032Y_{235} + 0.0222X_{334} - 0.3302X_{335} + 0.3302Y_{335} + 0.0136X_{434} - 0.3322X_{435} + 0.3322Y_{435} - 0.3161X_{535} + 0.3161Y_{535} + n^c_{A34} - p^c_{A34} = 0$$

(95) $i = A = \{1,2\}$, $j = 3$, $t = 5$

$$- 0.0964X_{135} + 0.8975X_{136} - 0.8975Y_{136} + 0.0248X_{235} + 0.8032X_{236} - 0.8032Y_{236} + 0.0222X_{335} - 0.3302X_{336} + 0.3302Y_{336} + 0.0136X_{435} - 0.3322X_{436} + 0.3322Y_{436} - 0.3161X_{536} + 0.3161Y_{536} + n^c_{A35} - p^c_{A35} = 0$$

(96) $i = A = \{1,2\}$, $j = 4$, $t = 1$

$$- 0.1126X_{141} + 1.0267X_{142} - 1.0267Y_{142} + 0.0434X_{241} + 0.8254X_{242} - 0.8254Y_{242} + 0.0403X_{341} - 0.3544X_{342} + 0.3544Y_{342} + 0.0216X_{441} - 0.3434X_{442} + 0.3434Y_{442} - 0.3133X_{542} + 0.3133Y_{542} + n^c_{A41} - p^c_{A41} = 0$$

(97) $i = A = \{1,2\}$, $j = 4$, $t = 2$

$$- 0.1126X_{142} + 1.0267X_{143} - 1.0267Y_{143} + 0.0434X_{242} + 0.8254X_{243} - 0.8254Y_{243} + 0.0403X_{342} - 0.3544X_{343} + 0.3544Y_{343} + 0.0216X_{442} - 0.3434X_{443} + 0.3434Y_{443} - 0.3133X_{543} + 0.3133Y_{543} + n^c_{A42} - p^c_{A42} = 0$$

(98) $i = A = \{1,2\}$, $j = 4$, $t = 3$

$$- 0.1126X_{143} + 1.0267X_{144} - 1.0267Y_{144} + 0.0434X_{243} + 0.8254X_{244} - 0.8254Y_{244} + 0.0403X_{343} - 0.3544X_{344} + 0.3544Y_{344} + 0.0216X_{443} - 0.3434X_{444} + 0.3434Y_{444} - 0.3133X_{544} + 0.3133Y_{544} + n^c_{A23} - p^c_{A23} = 0$$

(99) $i = A = \{1,2\}$, $j = 4$, $t = 4$

$$- 0.1126X_{144} + 1.0267X_{145} - 1.0267Y_{145} + 0.0434X_{244} + 0.8254X_{245} - 0.8254Y_{245} + 0.0403X_{344} - 0.3544X_{345} + 0.3544Y_{345} + 0.0216X_{444} - 0.3434X_{445} + 0.3434Y_{445} - 0.3133X_{545} + 0.3133Y_{545} + n^c_{A24} - p^c_{A24} = 0$$

(100) $i = A = \{1,2\}$, $j = 4$, $t = 5$

$$- 0.1126X_{145} + 1.0267X_{146} - 1.0267Y_{146} + 0.0434X_{245} + 0.8254X_{246} - 0.8254Y_{246} + 0.0403X_{345} - 0.3544X_{346} + 0.3544Y_{346} + 0.0216X_{445} - 0.3434X_{446} + 0.3434Y_{446} - 0.3133X_{546} + 0.3133Y_{546} + n^c_{A25} - p^c_{A25} = 0$$

(101) $i = A = \{1,2\}$, $j = 5$, $t = 1$

$$- 0.1443X_{151} + 0.9265X_{152} - 0.9265Y_{152} + 0.0343X_{251} + 0.8120X_{252} - 0.8120Y_{252} + 0.0243X_{351} - 0.3312X_{352} + 0.3312Y_{352} + 0.0507X_{451} - 0.4072X_{452} + 0.4072Y_{452} - 0.3209X_{552} + 0.3209Y_{552} + n^c_{A51} - p^c_{A51} = 0$$

(102) $i = A = \{1,2\}$, $j = 5$, $t = 2$

$$- 0.1443X_{152} + 0.9265X_{153} - 0.9265Y_{153} + 0.0343X_{252} + 0.8120X_{253} - 0.8120Y_{253} + 0.0243X_{352} - 0.3312X_{353} + 0.3312Y_{353} + 0.0507X_{452} - 0.4072X_{453} + 0.4072Y_{453} - 0.3209X_{553} + 0.3209Y_{553} + n^c_{A52} - p^c_{A52} = 0$$

(103) $i = A = \{1,2\}$, $j = 5$, $t = 3$

$$- 0.1443X_{153} + 0.9265X_{154} - 0.9265Y_{154} + 0.0343X_{253} + 0.8120X_{254} - 0.8120Y_{254} + 0.0243X_{353} - 0.3312X_{354} + 0.3312Y_{354} + 0.0507X_{453} - 0.4072X_{454} + 0.4072Y_{454} - 0.3209X_{554} + 0.3209Y_{554} + n^c_{A53} - p^c_{A53} = 0$$

(104) $i = A = \{1,2\}, \ j = 5, \ t = 4$

$$- 0.1443X_{154} + 0.9265X_{155} - 0.9265Y_{155} + 0.0343X_{254} + 0.8120X_{255} - 0.8120Y_{255} + 0.0243X_{354} - 0.3312X_{355} + 0.3312Y_{355} + 0.0507X_{454} - 0.4072X_{455} + 0.4072Y_{455} - 0.3209X_{555} + 0.3209Y_{555} + n^{c}_{A54} - p^{c}_{A54} = 0$$

(105) $i = A = \{1,2\}, \ j = 5, \ t = 5$

$$- 0.1443X_{155} + 0.9265X_{156} - 0.9265Y_{156} + 0.0343X_{255} + 0.8120X_{256} - 0.8120Y_{256} + 0.0243X_{355} - 0.3312X_{356} + 0.3312Y_{356} + 0.0507X_{455} - 0.4072X_{456} + 0.4072Y_{456} - 0.3209X_{556} + 0.3209Y_{556} + n^{c}_{A55} - p^{c}_{A55} = 0$$

(106) $i = A = \{1,2\}, \ j = 6, \ t = 1$

$$0.0X_{161} + 1.0000X_{162} - 1.0000Y_{162} + 0.0399X_{261} + 0.8118X_{262} - 0.08118Y_{262} + 0.0307X_{361} - 0.3327X_{362} + 0.3327Y_{362} + 0.0357X_{461} - 0.3643X_{462} + 0.3643Y_{462} - 0.3035X_{562} + 0.3035Y_{562} + n^{c}_{A61} - p^{c}_{A61} = 0$$

(107) $i = A = \{1,2\}, \ j = 6, \ t = 2$

$$0.0X_{162} + 1.0000X_{163} - 1.0000Y_{163} + 0.0399X_{262} + 0.8118X_{263} - 0.8118Y_{263} + 0.0307X_{362} - 0.3327X_{363} + 0.3327Y_{363} + 0.0357X_{462} - 0.3643X_{463} + 0.3643Y_{463} - 0.3035X_{563} + 0.3035Y_{563} + n^{c}_{A62} - p^{c}_{A62} = 0$$

(108) $i = A = \{1,2\}$, $j = 6$, $t = 3$

$$0.0X_{163} + 1.0000X_{164} - 1.0000Y_{164} + 0.0399X_{263}$$
$$+ 0.8118X_{264} - 0.8118Y_{264} + 0.0307X_{363} - 0.3327X_{364}$$
$$+ 0.3327Y_{364} + 0.0357X_{463} - 0.3643X_{464} + 0.3643Y_{464}$$
$$- 0.3035X_{564} + 0.3035Y_{564} + n^{c}_{A63} - p^{c}_{A63} = 0$$

(109) $i = A = \{1,2\}$, $j = 6$, $t = 4$

$$0.0X_{164} + 1.0000X_{165} - 1.0000Y_{165} + 0.0399X_{264}$$
$$+ 0.8118X_{265} - 0.8118Y_{265} + 0.0307X_{364} - 0.3327X_{365}$$
$$+ 0.3327Y_{365} + 0.0357X_{464} - 0.3643X_{465} + 0.3643Y_{465}$$
$$- 0.3035X_{565} + 0.3035Y_{565} + n^{c}_{A64} - p^{c}_{A64} = 0$$

(110) $i = A = \{1,2\}$, $j = 6$, $t = 5$

$$0.0X_{165} + 1.000X_{166} - 1.0000Y_{166} + 0.0399X_{265}$$
$$+ 0.8118X_{266} - 0.8118Y_{266} + 0.0307X_{365} - 0.3327Y_{366}$$
$$+ 0.3327Y_{366} + 0.0357X_{465} - 0.3643X_{466} + 0.3643Y_{466}$$
$$- 0.3035X_{566} + 0.3035Y_{566} + n^{c}_{A65} - p^{c}_{A65} = 0$$

(111) $i = A = \{1,2\}$, $j = 7$, $t = 1$

$$0.0X_{171} + 0.0X_{172} - 0.0Y_{172} + 0.0347X_{271}$$
$$+ 0.8016X_{272} - 0.8016Y_{272} + 0.0562X_{371} - 0.3559X_{372}$$
$$+ 0.3559Y_{372} + 0.0662X_{471} - 0.4286X_{472} + 0.4286Y_{472}$$
$$- 0.3310X_{572} + 0.3310Y_{572} + n^{c}_{A71} - p^{c}_{A71} = 0$$

(112) $i = A = \{1,2\}$, $j = 7$, $t = 2$

$$0.0X_{172} + 0.0X_{173} - 0.0Y_{173} + 0.0347X_{272}$$
$$+ 0.8016X_{273} - 0.8016Y_{273} + 0.0562X_{372} - 0.3559X_{373}$$
$$+ 0.3559Y_{373} + 0.0662X_{472} - 0.4286X_{473} + 0.4286Y_{473}$$
$$- 0.3310X_{573} + 0.3310Y_{573} + n^{c}_{A72} - p^{c}_{A72} = 0$$

(113) $i = A = \{1,2\}$, $j = 7$, $t = 3$

$$0.0X_{173} + 0.0X_{174} - 0.0Y_{174} + 0.0347X_{273}$$
$$+ 0.8016X_{274} - 0.8016Y_{274} + 0.0562X_{373} - 0.3559Y_{374}$$
$$+ 0.3559Y_{374} + 0.0662X_{473} - 0.4286X_{474} + 0.4286Y_{474}$$
$$- 0.3310X_{574} + 0.3310Y_{574} + n^{c}_{A73} - p^{c}_{A73} = 0$$

(114) $i = A = \{1,2\}$, $j = 7$, $t = 4$

$$0.0X_{174} + 0.0X_{175} - 0.0Y_{175} + 0.0347X_{274}$$
$$+ 0.8016X_{275} - 0.8016Y_{275} + 0.0562X_{374} - 0.3559X_{375}$$
$$+ 0.3559Y_{375} + 0.0662X_{474} - 0.4286X_{475} + 0.4286Y_{475}$$
$$- 0.3310X_{575} + 0.3310Y_{575} + n^{c}_{A74} - p^{c}_{A74} = 0$$

(115) $i = A = \{1,2\}$, $j = 7$, $t = 5$

$$0.0X_{175} + 0.0X_{176} - 0.0Y_{176} + 0.0347X_{275}$$
$$+ 0.8016X_{276} - 0.8016Y_{276} + 0.0562X_{375} - 0.3559X_{376}$$
$$+ 0.3559X_{376} + 0.0662X_{475} - 0.4286X_{476} + 0.4286Y_{476}$$
$$- 0.3310X_{576} + 0.3310Y_{576} + n^{c}_{A75} - p^{c}_{A75} = 0$$

(116) i = A = {1,2} , j = 8, t = 1

$- 0.1320X_{181} + 0.8400X_{182} - 0.8400Y_{182} + 0.0307X_{281}$
$+ 0.7914X_{282} - 0.7914Y_{282} + 0.0182X_{381} - 0.3000X_{382}$
$+ 0.3000Y_{382} + 0.0X_{481} - 0.3000X_{482} + 0.3000Y_{482}$
$- 0.3000X_{582} + 0.3000Y_{582} + n^{c}_{A81} - P^{c}_{A81} = 0$

(117) i = A = {1,2} , j = 8, t = 2

$- 0.1320X_{182} + 0.8400X_{183} - 0.8400Y_{183} + 0.0307X_{282}$
$+ 0.7914X_{283} - 0.7914Y_{283} + 0.0182X_{382} - 0.3000X_{383}$
$+ 0.3000Y_{383} + 0.0X_{482} - 0.3000X_{483} + 0.3000Y_{483}$
$- 0.3000X_{583} + 0.3000Y_{583} + n^{c}_{A82} - P^{c}_{A82} = 0$

(118) i = A = {1,2} , j = 8, t = 3

$- 0.1320X_{183} + 0.8400X_{184} - 0.8400Y_{184} + 0.0307X_{283}$
$+ 0.7914X_{284} - 0.7914Y_{284} + 0.0182X_{283} - 0.300X_{384}$
$+ 0.3000Y_{384} + 0.0X_{483} - 0.3000X_{484} + 0.3000Y_{484}$
$- 0.3000X_{584} + 0.3000Y_{584} + n^{c}_{A83} - P^{c}_{A83} = 0$

(119) i = A = {1,2} , j = 8, t = 4

$- 0.1320X_{184} + 0.8400X_{185} - 0.8400Y_{185} + 0.0307X_{284}$
$+ 0.7914X_{285} - 0.7914Y_{285} + 0.0182X_{284} - 0.3000X_{385}$
$+ 0.3000Y_{385} + 0.0X_{484} - 0.3000X_{485} + 0.3000Y_{485}$
$- 0.3000X_{585} + 0.3000Y_{585} + n^{c}_{A84} - P^{c}_{A84} = 0$

(120) i = A ={ 1,2} , j = 8, t = 5

$$- 0.1320X_{185} + 0.8400X_{186} - 0.8400Y_{186} + 0.0307X_{285} + 0.7914X_{286} - 0.7914Y_{286} + 0.0182X_{285} - 0.3000X_{386} + 0.3000X_{386} + 0.0X_{485} - 0.3000X_{486} + 0.3000Y_{486} - 0.3000X_{586} + 0.3000Y_{586} + n^{c}_{A85} - p^{c}_{A85} = 0$$

(121) i = B = {3} , j = 1, t = 1

$$0.1037X_{111} - 0.5250X_{112} + 0.5250Y_{112} - 0.0781X_{211} - 0.4838X_{212} + 0.4838Y_{212} + 0.0338X_{311} - 0.4386X_{312} + 0.4386Y_{312} + 0.0406X_{411} - 0.4615X_{412} + 0.4615Y_{412} - 0.4058X_{512} + 0.4058Y_{512} + n^{c}_{B11} - p^{c}_{B11} = 0$$

(122) i = B = {3} , j = 1, t = 2

$$0.1037X_{112} - 0.5250X_{113} + 0.5250Y_{113} - 0.0781X_{212} - 0.4838X_{213} + 0.4838Y_{213} + 0.0338X_{312} - 0.4386X_{313} + 0.4386Y_{313} + 0.0406X_{412} - 0.4615X_{413} + 0.4615Y_{413} - 0.4058X_{513} + 0.4058Y_{513} + n^{c}_{B12} - p^{c}_{B12} = 0$$

(123) i = B = {3} , j = 1, t = 3

$$0.1037X_{113} - 0.5250X_{114} + 0.5250Y_{114} - 0.0781X_{213} - 0.4838X_{214} + 0.4838Y_{214} + 0.0338X_{313} - 0.4386X_{314} + 0.4386Y_{314} + 0.0406X_{413} - 0.4615X_{414} + 0.4615Y_{414} - 0.4058X_{514} + 0.4058Y_{514} + n^{c}_{B13} - p^{c}_{B13} = 0$$

(124) $i = B = \{3\}$, $j = 1$, $t = 4$

$$0.1037X_{114} - 0.5250X_{115} + 0.5250Y_{115} - 0.0781X_{214}$$
$$- 0.4838X_{215} + 0.4838Y_{215} + 0.0338X_{314} - 0.4386X_{315}$$
$$+ 0.4386Y_{315} + 0.0406X_{414} - 0.4615X_{415} + 0.4615Y_{415}$$
$$- 0.4058X_{515} + 0.4058Y_{515} + n^{c}_{B14} - p^{c}_{B14} = 0$$

(125) $i = B = \{3\}$, $j = 1$, $t = 5$

$$0.1037X_{115} - 0.5250X_{116} + 0.5250Y_{116} - 0.0781X_{215}$$
$$- 0.4838X_{216} + 0.4838Y_{215} + 0.0338X_{315} - 0.4386X_{316}$$
$$+ 0.4386Y_{316} + 0.0406X_{415} - 0.4615X_{416} + 0.4615Y_{416}$$
$$- 0.4058X_{516} + 0.4058Y_{516} + n^{c}_{B15} - p^{c}_{B15} = 0$$

(126) $i = B = \{3\}$, $j = 2$, $t = 1$

$$0.0832X_{121} - 0.5091X_{122} + 0.5091Y_{122} - 0.0351X_{221}$$
$$- 0.4365X_{222} + 0.4365Y_{222} + 0.0266X_{321} - 0.4347X_{322}$$
$$+ 0.4347Y_{322} + 0.0139X_{421} - 0.5217X_{422} + 0.5217Y_{422}$$
$$- 0.4175X_{522} + 0.4175Y_{522} + n^{c}_{B21} - p^{c}_{B21} = 0$$

(127) $i = B = \{3\}$, $j = 2$, $t = 2$

$$0.0832X_{122} - 0.5091X_{123} + 0.5091Y_{123} - 0.0351X_{222}$$
$$- 0.4365X_{223} + 0.4365Y_{223} + 0.0266X_{322} - 0.4347X_{323}$$
$$+ 0.4347Y_{323} + 0.0139X_{422} - 0.5217X_{423} + 0.5217Y_{423}$$
$$- 0.4175X_{523} + 0.4175Y_{523} + n^{c}_{B22} - p^{c}_{B22} = 0$$

(128) i = B = {3} , j = 2, t = 3

$$0.0832X_{123} - 0.5091X_{124} + 0.5091Y_{124} - 0.0351X_{223} - 0.4365X_{224} + 0.4365Y_{224} + 0.0266X_{323} - 0.4347X_{324} + 0.4347Y_{324} + 0.0139X_{423} - 0.5217X_{424} + 0.5217Y_{424} - 0.4175X_{524} + 0.4175Y_{524} + n^{c}_{B23} - p^{c}_{B23} = 0$$

(129) i = B = {3} , j = 2, t = 4

$$0.0832X_{124} - 0.5091X_{125} + 0.5091Y_{125} - 0.0351X_{224} - 0.4365X_{225} + 0.4365Y_{225} + 0.0266X_{324} - 0.4347X_{325} + 0.4347Y_{325} + 0.0139X_{424} - 0.5217X_{425} + 0.5217Y_{425} - 0.4175X_{525} + 0.4175Y_{525} + n^{c}_{B24} - p^{c}_{B24} = 0$$

(130) i = B = {3} , j = 2, t = 5

$$0.0832X_{125} - 0.0591X_{126} + 0.5091Y_{126} - 0.0351X_{225} - 0.4365X_{226} + 0.4365Y_{226} + 0.0266X_{325} - 0.4347X_{326} + 0.4347Y_{326} + 0.0139X_{425} - 0.5217X_{426} + 0.5217Y_{426} - 0.4175X_{526} + 0.4175Y_{526} + n^{c}_{B25} - p^{c}_{B25} = 0$$

(131) i = B = {3} , j = 3, t = 1

$$0.0551X_{131} - 0.5128X_{132} + 0.5128Y_{132} - 0.0496X_{231} - 0.4590X_{232} + 0.4590Y_{232} + 0.0296X_{331} - 0.4403X_{332} + 0.4403Y_{332} + 0.0181X_{431} - 0.4429X_{432} + 0.4429Y_{432} - 0.4215X_{532} + 0.4215Y_{532} + n^{c}_{B31} - p^{c}_{B31} = 0$$

(132) $i = B = \{3\},\ j = 3,\ t = 2$

$$0.0551X_{132} - 0.5128X_{133} + 0.5128Y_{133} - 0.0496X_{232}$$
$$- 0.4590X_{233} + 0.4590Y_{233} + 0.0296X_{332} - 0.4403X_{333}$$
$$+ 0.4403Y_{333} + 0.0181X_{432} - 0.4429X_{433} + 0.4429Y_{433}$$
$$- 0.4215X_{533} + 0.4215Y_{533} + n^{c}_{B32} - p^{c}_{B32} = 0$$

(133) $i = B = \{3\},\ j = 3,\ t = 3$

$$0.0551X_{133} - 0.5128X_{134} + 0.5128Y_{134} - 0.0496X_{233}$$
$$- 0.4590X_{234} + 0.4590Y_{234} + 0.0296X_{333} - 0.4401X_{334}$$
$$+ 0.4403Y_{334} + 0.0181X_{433} - 0.4429X_{434} + 0.4429Y_{434}$$
$$- 0.4215X_{534} + 0.4215Y_{534} + n^{c}_{B33} - p^{c}_{B33} = 0$$

(134) $i = B = \{3\},\ j = 3,\ t = 4$

$$0.0551X_{134} - 0.5128X_{135} + 0.5128Y_{135} - 0.0496X_{234}$$
$$- 0.4590X_{235} + 0.4590Y_{235} + 0.0296X_{334} - 0.4403X_{335}$$
$$+ 0.4403Y_{335} + 0.0181X_{434} - 0.4429X_{435} + 0.4429Y_{435}$$
$$- 0.4215X_{535} + 0.4215Y_{535} + n^{c}_{B34} - p^{c}_{B34} = 0$$

(135) $i = B = \{3\},\ j = 3,\ t = 5$

$$0.0551X_{135} - 0.5128X_{136} + 0.5128Y_{136} - 0.0496X_{235}$$
$$- 0.4590X_{236} + 0.4590Y_{236} + 0.0296X_{335} - 0.4403X_{336}$$
$$+ 0.4403Y_{336} + 0.0181X_{435} - 0.4429X_{436} + 0.4429Y_{436}$$
$$- 0.4215X_{536} + 0.4215Y_{536} + n^{c}_{B35} - p^{c}_{B35} = 0$$

(136) $i = B = \{3\}$, $j = 4$, $t = 1$

$$0.0643X_{141} - 0.5867X_{142} + 0.5867Y_{142} - 0.0868X_{241} - 0.4716X_{242} + 0.4716Y_{242} + 0.0538X_{341} - 0.4725X_{342} + 0.4725Y_{342} + 0.0288X_{441} - 0.4579X_{442} + 0.4579Y_{442} - 0.4178X_{542} + 0.4178Y_{542} + n^c_{B41} - p^c_{B41} = 0$$

(137) $i = B = \{3\}$, $j = 4$, $t = 2$

$$0.0643X_{142} - 0.5867X_{143} + 0.5867Y_{143} - 0.0868X_{242} - 0.4716X_{243} + 0.4716Y_{243} + 0.0538X_{342} - 0.4725X_{343} + 0.4725Y_{343} + 0.0288X_{442} - 0.4579X_{443} + 0.4579Y_{443} - 0.4178X_{543} + 0.4178Y_{543} + n^c_{B42} - p^c_{B42} = 0$$

(138) $i = B = \{3\}$, $j = 4$, $t = 3$

$$0.0643X_{143} - 0.5867X_{144} + 0.5867Y_{144} - 0.0868X_{243} - 0.4716X_{244} + 0.4716Y_{244} + 0.0538X_{343} - 0.4725X_{344} + 0.4725Y_{344} + 0.0288X_{443} - 0.4579X_{444} + 0.4579Y_{444} - 0.4178X_{544} + 0.4178Y_{544} + n^c_{B43} - p^c_{B43} = 0$$

(139) $i = B = \{3\}$, $j = 4$, $t = 4$

$$0.0643X_{144} - 0.5867X_{145} + 0.5867Y_{145} - 0.0868X_{244} - 0.4716X_{245} + 0.4716Y_{245} + 0.0538X_{344} - 0.4725X_{345} + 0.4725Y_{345} + 0.0288X_{444} - 0.4579X_{445} + 0.4579Y_{445} - 0.4178X_{545} + 0.4178Y_{545} + n^c_{B44} - p^c_{B44} = 0$$

(140) i = B = {3} , j = 4, t = 5

$$0.0643X_{145} - 0.5867X_{146} + 0.5867Y_{146} - 0.0868X_{245} - 0.4716X_{246} + 0.4716Y_{246} + 0.0538X_{345} - 0.4725X_{346} + 0.4725Y_{346} + 0.0288X_{445} - 0.4579X_{446} + 0.4579Y_{446} - 0.4178X_{546} + 0.4178Y_{546} + n^c_{B45} - p^c_{B45} = 0$$

(141) i = B = {3} , j = 5, t = 1

$$0.0825X_{151} - 0.5294X_{152} + 0.5294Y_{152} - 0.0686X_{251} - 0.4640X_{252} + 0.4640Y_{252} + 0.0324X_{351} - 0.4416X_{352} + 0.4416Y_{352} + 0.0676X_{451} - 0.5429X_{452} + 0.5429Y_{452} - 0.4279X_{552} + 0.4279Y_{552} + n^c_{B51} - p^c_{B51} = 0$$

(142) i = B = {3} , j = 5, t = 2

$$0.0825X_{152} - 0.5294X_{153} + 0.5294Y_{153} - 0.0686X_{252} - 0.4640X_{253} + 0.4640Y_{253} + 0.0324X_{352} - 0.4416X_{353} + 0.4416Y_{353} + 0.0676X_{452} - 0.5429X_{453} + 0.5429Y_{453} - 0.4279X_{553} + 0.4279Y_{553} + n^c_{B52} - p^c_{B52} = 0$$

(143) i = B = {3} , j = 5, t = 3

$$0.0825X_{153} - 0.5294X_{154} + 0.5294Y_{154} - 0.0686X_{253} - 0.4640X_{254} + 0.4640Y_{254} + 0.0324X_{353} - 0.4416X_{354} + 0.4416Y_{354} + 0.0676X_{452} - 0.5429X_{454} + 0.5429Y_{454} - 0.4279X_{554} + 0.4279Y_{554} + n^c_{B53} - p^c_{B53} = 0$$

(144) $i = B = \{3\}$, $j = 5,\ t = 4$

$$0.0825X_{154} - 0.5294X_{155} + 0.5294Y_{155} - 0.0686X_{254}$$
$$- 0.4640X_{255} + 0.4640Y_{255} + 0.0324X_{354} - 0.4416X_{355}$$
$$+ 0.4416Y_{355} + 0.0676X_{453} - 0.5429X_{455} + 0.5429Y_{455}$$
$$- 0.4279X_{555} + 0.4279Y_{555} + n^{c}_{B54} - p^{c}_{B54} = 0$$

(145) $i = B = \{3\}$, $j = 5,\ t = 5$

$$0.0825X_{155} - 0.5294X_{156} + 0.5294Y_{156} - 0.0686X_{255}$$
$$- 0.4640X_{256} + 0.4640Y_{256} + 0.0324X_{355} - 0.4416X_{356}$$
$$+ 0.4416Y_{356} + 0.0676X_{454} - 0.5429X_{456} + 0.5429Y_{456}$$
$$- 0.4279X_{556} + 0.4279Y_{556} + n^{c}_{B55} - p^{c}_{B55} = 0$$

(146) $i = B = \{3\}$, $j = 6,\ t = 1$

$$0.0X_{161} - 0.5714X_{162} + 0.5714Y_{162} - 0.0797X_{261}$$
$$- 0.4639X_{262} + 0.4639Y_{262} + 0.0409X_{361} - 0.4435X_{362}$$
$$+ 0.4435Y_{362} + 0.0476X_{461} - 0.4857X_{462} + 0.4857Y_{462}$$
$$- 0.4046X_{562} + 0.4046Y_{562} + n^{c}_{B61} - p^{c}_{B61} = 0$$

(147) $i = B = \{3\}$, $j = 6,\ t = 2$

$$0.0X_{162} - 0.5714X_{163} + 0.5714Y_{163} - 0.0797X_{262}$$
$$- 0.4639X_{263} + 0.4639Y_{263} + 0.0409X_{362} - 0.4435Y_{363}$$
$$+ 0.4435Y_{363} + 0.0476X_{462} - 0.4857X_{463} + 0.4857Y_{463}$$
$$- 0.4046X_{563} + 0.4046Y_{563} + n^{c}_{B62} - p^{c}_{B62} = 0$$

(148) $i = B = \{3\}$, $j = 6$, $t = 3$

$$0.0X_{163} - 0.5714X_{164} + 0.5714Y_{164} - 0.0797X_{263} - 0.4639X_{264} + 0.4639Y_{264} + 0.0409X_{363} - 0.4435X_{364} + 0.4435Y_{364} + 0.0476X_{463} - 0.4857X_{464} + 0.4857Y_{464} - 0.4046X_{564} + 0.4046Y_{564} + n^{c}_{B63} - p^{c}_{B63} = 0$$

(149) $i = B = \{3\}$, $j = 6$, $t = 4$

$$0.0X_{164} - 0.5714X_{165} + 0.5714Y_{165} - 0.0797X_{264} - 0.4639X_{265} + 0.4639Y_{265} + 0.0409X_{364} - 0.4435X_{365} + 0.4435Y_{365} + 0.0476X_{464} - 0.4857X_{465} + 0.4857Y_{465} - 0.4046X_{565} + 0.4046Y_{565} + n^{c}_{B64} - p^{c}_{B64} = 0$$

(150) $i = B = \{3\}$, $j = 6$, $t = 5$

$$0.0X_{165} - 0.5714X_{166} + 0.5714Y_{166} - 0.0797X_{265} - 0.4639X_{266} + 0.4639Y_{266} + 0.0409X_{365} - 0.4435X_{366} + 0.4435Y_{366} + 0.0476X_{465} - 0.4857X_{466} + 0.4857Y_{466} - 0.4046X_{566} + 0.4046Y_{566} + n^{c}_{B65} - p^{c}_{B65} = 0$$

(151) $i = B = \{3\}$, $j = 7$, $t = 1$

$$0.0X_{171} + 0.0X_{172} - 0.0Y_{172} - 0.0695X_{271} - 0.4581X_{272} + 0.4581Y_{272} + 0.0749X_{371} - 0.4746X_{372} + 0.4746Y_{372} + 0.0883X_{471} - 0.5714X_{472} + 0.5714Y_{472} - 0.4414X_{572} + 0.4414Y_{572} + n^{c}_{B71} - p^{c}_{B71} = 0$$

(152) $i = B = \{3\}$, $j = 7$, $t = 2$

$$0.0X_{172} + 0.0X_{173} - 0.0Y_{173} - 0.0695X_{272} - 0.4581X_{273} + 0.4581Y_{273} + 0.0749X_{372} - 0.4746X_{373} + 0.4746Y_{373} + 0.0883X_{472} - 0.5714X_{473} + 0.5714Y_{473} - 0.4414X_{573} + 0.4414Y_{573} + n^{c}_{B72} - p^{c}_{B72} = 0$$

(153) $i = B = \{3\}$, $j = 7$, $t = 3$

$$0.0X_{173} + 0.0X_{174} - 0.0Y_{174} - 0.0695X_{273} - 0.4581X_{274} + 0.4581Y_{274} + 0.0749X_{373} - 0.4746X_{374} + 0.4746Y_{374} + 0.0883X_{473} - 0.5714X_{474} + 0.5714Y_{474} - 0.4414X_{574} + 0.4414Y_{574} + n^{c}_{B73} - p^{c}_{B73} = 0$$

(154) $i = B = \{3\}$, $j = 7$, $t = 4$

$$0.0X_{174} + 0.0X_{175} - 0.0Y_{175} - 0.0695X_{274} - 0.4581X_{275} + 0.4581Y_{275} + 0.0749X_{374} - 0.4746X_{375} + 0.4746Y_{375} + 0.0883X_{474} - 0.5714X_{475} + 0.5714Y_{475} - 0.4414X_{575} + 0.4414Y_{575} + n^{c}_{B74} - p^{c}_{B74} = 0$$

(155) $i = B = \{3\}$, $j = 7$, $t = 5$

$$0.0X_{175} + 0.0X_{176} - 0.0Y_{176} - 0.0695X_{275} - 0.4581X_{276} + 0.4581Y_{276} + 0.0749X_{375} - 0.4746X_{376} + 0.4746Y_{376} + 0.0883X_{475} - 0.5714X_{476} + 0.5714Y_{476} - 0.4414X_{576} + 0.4414Y_{576} + n^{c}_{B75} - p^{c}_{B75} = 0$$

(156) $i = B = \{3\}$, $j = 8$, $t = 1$

$$0.0754X_{181} - 0.4800X_{182} + 0.4800Y_{182} - 0.0614X_{281}$$
$$- 0.4522X_{282} + 0.4522Y_{282} + 0.0242X_{381} - 0.4000X_{382}$$
$$+ 0.4000Y_{382} + 0.0X_{481} - 0.4000X_{482} + 0.4000Y_{482}$$
$$- 0.4000X_{582} + 0.4000Y_{582} + n^{c}_{B81} - p^{c}_{B81} = 0$$

(157) $i = B = \{3\}$, $j = 8$, $t = 2$

$$0.0754X_{182} - 0.4800X_{183} + 0.4800Y_{183} - 0.0614X_{282}$$
$$- 0.4522X_{283} + 0.4522Y_{283} + 0.0242X_{382} - 0.400X_{383}$$
$$+ 0.4000Y_{383} + 0.0X_{482} - 0.4000X_{483} + 0.4000Y_{483}$$
$$- 0.4000X_{583} + 0.4000Y_{583} + n^{c}_{B82} - p^{c}_{B82} = 0$$

(158) $i = B = \{3\}$, $j = 8$, $t = 3$

$$0.0754X_{183} - 0.4800X_{184} + 0.4800Y_{184} - 0.0614X_{283}$$
$$- 0.4522X_{284} + 0.4522Y_{284} + 0.0242X_{383} - 0.400X_{384}$$
$$+ 0.4000Y_{384} + 0.0X_{483} - 0.4000X_{484} + 0.4000Y_{484}$$
$$- 0.4000X_{584} + 0.4000Y_{584} + n^{c}_{B83} - p^{c}_{B83} = 0$$

(159) $i = B = \{3\}$, $j = 8$, $t = 4$

$$0.0754X_{184} - 0.4800X_{185} + 0.4800Y_{185} - 0.0614X_{284}$$
$$- 0.4522X_{285} + 0.4522Y_{285} + 0.0242X_{384} - 0.400X_{385}$$
$$+ 0.4000Y_{385} + 0.0X_{484} - 0.4000X_{485} + 0.400Y_{485}$$
$$- 0.4000X_{585} + 0.4000Y_{585} + n^{c}_{B84} - p^{c}_{B84} = 0$$

(160) $i = B = \{3\}$, $j = 8$, $t = 5$

$$0.0754X_{185} - 0.4800X_{186} + 0.4800Y_{186} - 0.0614Y_{285} - 0.4522X_{286} + 0.4522Y_{286} + 0.0242X_{385} - 0.400X_{386} + 0.4000Y_{386} + 0.0X_{485} - 0.4000X_{486} + 0.4000Y_{486} - 0.4000X_{586} + 0.4000Y_{586} + n^{c}_{B85} - p^{c}_{B85} = 0$$

(161) $i = C = \{4,5\}$, $j = 1$, $t = 1$

$$0.0778X_{111} - 0.3938X_{112} + 0.3938Y_{112} + 0.0391X_{211} - 0.3629X_{212} + 0.3629Y_{212} - 0.0592X_{311} - 0.3290X_{312} + 0.3290Y_{312} - 0.0729X_{411} + 0.8077X_{412} - 0.8077Y_{412} + 0.7100X_{512} - 0.7100Y_{512} + n^{c}_{C11} - p^{c}_{C11} = 0$$

(162) $i = C = \{4,5\}$, $j = 1$, $t = 2$

$$0.0778X_{112} - 0.3938X_{113} + 0.3938Y_{113} + 0.0391X_{212} - 0.3629X_{213} + 0.3629Y_{213} - 0.0592X_{312} - 0.3290X_{313} + 0.3290Y_{313} - 0.0729X_{412} + 0.8077X_{413} - 0.8077Y_{413} + 0.7100X_{513} - 0.7100Y_{513} + n^{c}_{C12} - p^{c}_{C12} = 0$$

(163) $i = C = \{4,5\}$, $j = 1$, $t = 3$

$$0.0778X_{113} - 0.3938X_{114} + 0.3938Y_{114} + 0.0391X_{213} - 0.3629X_{214} + 0.3629Y_{214} - 0.0592X_{313} - 0.3290X_{314} + 0.3290Y_{314} - 0.0729X_{413} + 0.8077X_{414} - 0.8077Y_{414} + 0.7100X_{514} - 0.7100Y_{514} + n^{c}_{C13} - p^{c}_{C13} = 0$$

(164) $i = C = \{4,5\}$, $j = 1$, $t = 4$

$$0.0778X_{114} - 0.3938X_{115} + 0.3938Y_{115} + 0.0391X_{214} - 0.3629X_{215} + 0.3629Y_{215} - 0.0592X_{314} - 0.3290X_{315} + 0.3290Y_{315} - 0.0729X_{414} + 0.8077X_{415} - 0.8077Y_{415} + 0.7100X_{515} - 0.7100Y_{515} + n^c_{C14} - p^c_{C14} = 0$$

(165) $i = C = \{4,5\}$, $j = 1$, $t = 5$

$$0.0778X_{115} - 0.3938X_{116} + 0.3938Y_{116} + 0.0391X_{215} - 0.3629X_{216} + 0.3629Y_{216} - 0.0592X_{315} - 0.3290X_{316} + 0.3290Y_{316} - 0.0729X_{415} + 0.8077X_{416} - 0.8077Y_{416} + 0.7100X_{516} - 0.7100Y_{516} + n^c_{C15} - p^c_{C15} = 0$$

(166) $i = C = \{4,5\}$, $j = 2$, $t = 1$

$$0.0624X_{121} - 0.3818X_{122} + 0.3818Y_{122} + 0.0176X_{221} - 0.3274X_{222} + 0.3274Y_{222} - 0.0466X_{321} - 0.3260X_{322} + 0.3260Y_{322} - 0.0244X_{421} + 0.9130X_{422} - 0.9130Y_{422} + 0.7307X_{522} - 0.7307Y_{522} + n^c_{C21} - p^c_{C21} = 0$$

(167) $i = C = \{4,5\}$, $j = 2$, $t = 2$

$$0.0624X_{122} - 0.3818X_{123} + 0.3818Y_{123} + 0.0176X_{222} - 0.3274X_{223} + 0.3274Y_{223} - 0.0466X_{322} - 0.3260X_{323} + 0.3260Y_{323} - 0.0244X_{422} + 0.9130X_{423} - 0.9130Y_{423} + 0.7307X_{523} - 0.7307Y_{523} + n^c_{C22} - p^c_{C22} = 0$$

(168) $i = C = \{4,5\}$, $j = 2$, $t = 3$

$$0.0624X_{123} - 0.3818X_{124} + 0.3818Y_{124} + 0.0176X_{223} - 0.3274X_{224} + 0.3274Y_{224} - 0.0466X_{323} - 0.3260X_{324} + 0.3260Y_{324} - 0.0244X_{423} + 0.9130X_{424} - 0.9130Y_{424} + 0.7307X_{524} - 0.7307Y_{524} + n^{c}_{C23} - p^{c}_{C23} = 0$$

(169) $i = C = \{4,5\}$, $j = 2$, $t = 4$

$$0.0624X_{124} - 0.3818X_{125} + 0.3818Y_{125} + 0.0176X_{224} - 0.3274X_{225} + 0.3274Y_{225} - 0.0466X_{324} - 0.3260X_{325} + 0.3260Y_{325} - 0.0244X_{424} + 0.9130X_{425} - 0.9130Y_{425} + 0.7307X_{525} - 0.7307Y_{525} + n^{c}_{C24} - p^{c}_{C24} = 0$$

(170) $i = C = \{4,5\}$, $j = 2$, $t = 5$

$$0.0624X_{125} - 0.3818X_{126} + 0.3818Y_{126} + 0.0176X_{225} - 0.3274X_{226} + 0.3274Y_{226} - 0.0466X_{325} - 0.3260X_{326} + 0.3260Y_{326} - 0.0244X_{425} + 0.9130X_{426} - 0.9130Y_{426} + 0.7307X_{526} - 0.7307Y_{526} + n^{c}_{C25} - p^{c}_{C25} = 0$$

(171) $i = C = \{4,5\}$, $j = 3$, $t = 1$

$$0.0413X_{131} - 0.3846X_{132} + 0.3846Y_{132} + 0.0248X_{231} - 0.3442X_{232} + 0.3442Y_{232} - 0.0517X_{331} - 0.3302X_{332} + 0.3302Y_{332} - 0.0317X_{431} + 0.7750X_{432} - 0.7750Y_{432} + 0.7376X_{532} - 0.7376Y_{532} + n^{c}_{C31} - p^{c}_{C31} = 0$$

(172) $i = C = \{4,5\}$, $j = 3$, $t = 2$

$$0.0413X_{132} - 0.3846X_{133} + 0.3846Y_{133} + 0.0248X_{232} - 0.3442X_{233} + 0.3442Y_{233} - 0.0517X_{332} - 0.3302X_{333} + 0.3302Y_{333} - 0.0317X_{432} + 0.7750X_{433} - 0.7750Y_{433} + 0.7376X_{533} - 0.7376Y_{533} + n^{c}_{C32} - p^{c}_{C32} = 0$$

(173) $i = C = \{4,5\}$, $j = 3$, $t = 3$

$$0.0413X_{133} - 0.3846X_{134} + 0.3846Y_{134} + 0.0248X_{233} - 0.3442X_{234} + 0.3442Y_{234} - 0.0517X_{333} - 0.3302X_{334} + 0.3302Y_{334} - 0.0317X_{433} + 0.7750X_{434} - 0.7750Y_{434} + 0.7376X_{534} - 0.7376Y_{534} + n^{c}_{C33} - p^{c}_{C33} = 0$$

(174) $i = C = \{4,5\}$, $j = 3$, $t = 4$

$$0.0413X_{134} - 0.3846X_{135} + 0.3846Y_{135} + 0.0248X_{234} - 0.3442X_{235} + 0.3442Y_{235} - 0.0517X_{334} - 0.3302X_{335} + 0.3302Y_{335} - 0.0317X_{434} + 0.7750X_{435} - 0.7750Y_{435} + 0.7376X_{535} - 0.7376Y_{535} + n^{c}_{C34} - p^{c}_{C34} = 0$$

(175) $i = C =\{4,5\}$, $j = 3$, $t = 5$

$$0.0413X_{135} - 0.3846X_{136} + 0.3846Y_{136} + 0.0248X_{235} - 0.3442X_{236} + 0.034442Y_{236} - 0.0517X_{335} - 0.3302X_{336} + 0.3302Y_{336} - 0.0317X_{435} + 0.7750X_{436} - 0.7750Y_{436} + 0.7376X_{536} - 0.7376Y_{536} + n^{c}_{C35} - p^{c}_{C35} = 0$$

(176) $i = C = \{4,5\}$, $j = 4,$ $t = 1$

$$0.0482X_{141} - 0.4400X_{142} + 0.4400Y_{142} + 0.0434X_{241} - 0.3537X_{242} + 0.3537Y_{242} - 0.0941X_{341} - 0.3544X_{342} + 0.3544Y_{342} - 0.0505X_{441} + 0.8013X_{442} - 0.8013Y_{442} + 0.7311X_{542} - 0.7311Y_{542} + n^{c}_{C41} - p^{c}_{C41} = 0$$

(177) $i = C = \{4,5\}$, $j = 4,$ $t = 2$

$$0.0482X_{142} - 0.4400X_{143} + 0.4400Y_{143} + 0.0434X_{242} - 0.3537X_{243} + 0.3537Y_{243} - 0.0941X_{342} - 0.3544X_{343} + 0.3544Y_{343} - 0.0505X_{442} + 0.8013X_{443} - 0.8013Y_{443} + 0.7311X_{543} - 0.7311Y_{543} + n^{c}_{C42} - p^{c}_{C42} = 0$$

(178) $i = C = \{4,5\}$, $j = 4,$ $t = 3$

$$0.0482X_{143} - 0.4400X_{144} + 0.4400Y_{144} + 0.0434X_{243} - 0.3537X_{244} + 0.3537Y_{244} - 0.0941X_{343} - 0.3544X_{344} + 0.3544Y_{344} - 0.0505X_{443} + 0.8013X_{444} - 0.8013Y_{444} + 0.7311X_{544} - 0.7311Y_{544} + n^{c}_{C43} - p^{c}_{C43} = 0$$

(179) $i = C = \{4,5\}$, $j = 4,$ $t = 4$

$$0.0482X_{144} - 0.4400X_{145} + 0.4400Y_{145} + 0.0434X_{244} - 0.3537X_{245} + 0.3537Y_{245} - 0.0941X_{344} - 0.3544X_{345} + 0.3544Y_{345} - 0.0505X_{444} + 0.8013X_{445} - 0.8013Y_{445} + 0.7311X_{545} - 0.7311Y_{545} + n^{c}_{C44} - p^{c}_{C44} = 0$$

(180) $i = C = \{4,5\}$, $j = 4$, $t = 5$

$$0.0482X_{145} - 0.4400X_{146} + 0.4400Y_{146} + 0.0434X_{245} - 0.3537X_{246} + 0.3537Y_{246} - 0.0941X_{345} - 0.3544X_{346} + 0.3544Y_{346} - 0.0505X_{445} + 0.8013X_{446} - 0.8013Y_{446} + 0.7311X_{546} - 0.7311Y_{546} + n^{c}_{C45} - p^{c}_{C45} = 0$$

(181) $i = C = \{4,5\}$, $j = 5$, $t = 1$

$$0.0619X_{151} - 0.3971X_{152} + 0.3971Y_{152} + 0.0343X_{251} - 0.3480X_{252} + 0.3480Y_{252} - 0.0568X_{351} - 0.3312X_{352} + 0.3312Y_{352} - 0.1182X_{451} + 0.9500X_{452} - 0.9500Y_{452} + 0.7488X_{552} - 0.7488Y_{552} + n^{c}_{C51} - p^{c}_{C51} = 0$$

(182) $i = C = \{4,5\}$, $j = 5$, $t = 2$

$$0.0619X_{152} - 0.3971X_{153} + 0.3971Y_{153} + 0.0343X_{252} - 0.3480X_{253} + 0.3480Y_{253} - 0.0568X_{352} - 0.3312X_{353} + 0.3312Y_{353} - 0.1182X_{452} + 0.9500X_{453} - 0.9500Y_{453} + 0.7488X_{553} - 0.7488Y_{553} + n^{c}_{C52} - p^{c}_{C52} = 0$$

(183) $i = C = \{4,5\}$, $j = 5$, $t = 3$

$$0.0619X_{153} - 0.3971X_{154} + 0.3971Y_{154} + 0.0343X_{253} - 0.3480X_{254} + 0.3480Y_{254} - 0.0568X_{353} - 0.3312X_{354} + 0.3312Y_{354} - 0.1182X_{453} + 0.9500X_{454} - 0.9500Y_{454} + 0.7488X_{554} - 0.7488Y_{554} + n^{c}_{C53} - p^{c}_{C53} = 0$$

(184) $i = C = \{4,5\}$, $j = 5$, $t = 4$

$$0.0619X_{154} - 0.3971X_{155} + 0.3971Y_{155} + 0.0343X_{254}$$
$$- 0.3480X_{255} + 0.3480Y_{255} - 0.0568X_{354} - 0.3312X_{355}$$
$$+ 0.3312Y_{355} - 0.1182X_{454} + 0.9500X_{455} - 0.9500Y_{455}$$
$$+ 0.7488X_{555} - 0.7488Y_{555} + n^{c}_{C54} - p^{c}_{C54} = 0$$

(185) $i = C = \{4,5\}$, $j = 5$, $t = 5$

$$0.0619X_{155} - 0.3971X_{156} + 0.3971Y_{156} + 0.0343X_{355}$$
$$- 0.3480X_{256} + 0.3480Y_{256} - 0.0568X_{355} - 0.3312X_{356}$$
$$+ 0.3312Y_{356} - 0.1182X_{455} + 0.9500X_{456} - 0.9500Y_{456}$$
$$+ 0.7488X_{556} - 0.7488Y_{556} + n^{c}_{C55} - p^{c}_{C55} = 0$$

(186) $i = C = \{4,5\}$, $j = 6$, $t = 1$

$$0.0X_{161} - 0.4286X_{162} + 0.4286Y_{162} + 0.0399X_{261}$$
$$- 0.3479X_{262} + 0.3479Y_{262} - 0.0715X_{361} - 0.3326X_{362}$$
$$+ 0.3326Y_{362} - 0.0834X_{461} + 0.8500X_{462} - 0.8500Y_{462}$$
$$+ 0.7081X_{562} - 0.7081Y_{562} + n^{c}_{C61} - p^{c}_{C61} = 0$$

(187) $i = C = \{4,5\}$, $j = 6$, $t = 2$

$$0.0X_{162} - 0.4285X_{163} + 0.4286Y_{163} + 0.0399X_{262}$$
$$- 0.3479X_{263} + 0.3479Y_{263} - 0.0715X_{362} - 0.3326X_{363}$$
$$+ 0.3326Y_{363} - 0.0834X_{462} + 0.8500X_{463} - 0.8500Y_{463}$$
$$+ 0.7081X_{563} - 0.7081Y_{563} + n^{c}_{C62} - p^{c}_{C62} = 0$$

(188) $i = C = \{4,5\}$, $j = 6$, $t = 3$

$$0.0X_{163} - 0.4286X_{164} + 0.4286Y_{164} + 0.0399X_{263} - 0.3479X_{264} + 0.3479Y_{264} - 0.0715X_{363} - 0.3326X_{364} + 0.3326Y_{364} - 0.0834X_{463} + 0.8500X_{464} - 0.8500Y_{464} + 0.7081X_{564} - 0.7081Y_{564} + n^{c}_{C63} - P^{c}_{C63} = 0$$

(189) $i = C = \{4,5\}$, $j = 6$, $t = 4$

$$0.0X_{164} - 0.4286X_{165} + 0.4286Y_{165} + 0.0399X_{264} - 0.3479X_{265} + 0.3479Y_{265} - 0.0715X_{364} - 0.3326X_{365} + 0.3326Y_{365} - 0.0834X_{464} + 0.8500X_{465} - 0.8500Y_{465} + 0.7081X_{565} - 0.7081Y_{565} + n^{c}_{C64} - P^{c}_{C64} = 0$$

(190) $i = C = \{4,5\}$, $j = 6$, $t = 5$

$$0.0X_{165} - 0.4286X_{166} + 0.4286Y_{166} + 0.0399X_{265} - 0.3479X_{266} + 0.3479Y_{266} - 0.0715X_{365} - 0.3326X_{366} + 0.3326Y_{365} - 0.0834X_{465} + 0.8500X_{466} - 0.8500Y_{466} + 0.7081X_{566} - 0.7081Y_{566} + n^{c}_{C65} - P^{c}_{C65} = 0$$

(191) $i = C = \{4,5\}$, $j = 7$, $t = 1$

$$0.0X_{171} + 0.0X_{172} + 0.0Y_{172} + 0.0347X_{271} - 0.3436X_{272} + 0.3436Y_{272} - 0.1311X_{371} - 0.3559X_{372} + 0.3559Y_{372} - 0.1545X_{471} + 1.0000X_{472} - 1.0000Y_{472} + 0.7724X_{572} - 0.7724Y_{572} + n^{c}_{C71} - P^{c}_{C71} = 0$$

(192) $i = C = \{4,5\}$, $j = 7$, $t = 2$

$$0.0X_{172} + 0.0X_{173} - 0.0Y_{173} + 0.0347X_{272} - 0.3436X_{273} + 0.3436Y_{273} - 0.1311X_{372} - 0.3559X_{373} + 0.3559Y_{373} - 0.1545X_{472} + 1.000X_{473} - 1.0000Y_{473} + 0.7724X_{573} - 0.7724Y_{573} + n^{c}_{C72} - p^{c}_{C72} = 0$$

(193) $i = C = \{4,5\}$, $j = 7$, $t = 3$

$$0.0X_{173} + 0.0X_{174} - 0.0Y_{174} + 0.0347X_{273} - 0.3436X_{274} + 0.3436Y_{274} - 0.1311X_{373} - 0.3559X_{374} + 0.3559Y_{374} - 0.1545X_{473} + 1.0000X_{474} - 1.0000Y_{474} + 0.7724X_{574} - 0.7724Y_{574} + n^{c}_{C73} - p^{c}_{C73} = 0$$

(194) $i = C = \{4,5\}$, $j = 7$, $t = 4$

$$0.0X_{174} + 0.0X_{175} - 0.0Y_{175} + 0.0347X_{274} - 0.3436X_{275} + 0.3436Y_{275} - 0.1311X_{374} - 0.3559X_{375} + 0.3559Y_{375} - 0.1545X_{474} + 1.0000X_{475} - 1.000Y_{475} + 0.7724X_{575} - 0.7724Y_{575} + n^{c}_{C74} - p^{c}_{C74} = 0$$

(195) $i = C = \{4,5\}$, $j = 7$, $t = 5$

$$0.0X_{175} + 0.0X_{176} - 0.0Y_{176} + 0.0347X_{275} - 0.3436X_{276} + 0.3436Y_{276} - 0.1311X_{375} - 0.3559X_{376} + 0.3559Y_{376} - 0.1545X_{475} + 1.000X_{476} - 1.000Y_{476} + 0.7724X_{576} - 0.7724Y_{576} + n^{c}_{C75} - p^{c}_{C75} = 0$$

(196) i = C = {4,5} , j = 8, t = 1

$$0.0566X_{181} - 0.3600X_{182} + 0.3600Y_{182} + 0.0307X_{281} - 0.3392X_{282} + 0.3392Y_{282} - 0.0424X_{381} - 0.3000X_{382} + 0.30000Y_{382} - 0.0X_{481} + 0.7000X_{482} - 0.7000Y_{482} + 0.7000X_{582} - 0.7000Y_{582} + n^{c}_{C81} - p^{c}_{C81} = 0$$

(197) i = C = {4,5} , j = 8, t = 2

$$0.0566X_{182} - 0.3600X_{183} + 0.3600Y_{183} + 0.0307X_{282} - 0.3392X_{283} + 0.3392Y_{283} - 0.0424X_{382} - 0.300X_{383} + 0.3000Y_{383} - 0.0X_{482} + 0.7000X_{483} - 0.7000Y_{483} + 0.7000X_{583} - 0.7000Y_{583} + n^{c}_{C82} - p^{c}_{C82} = 0$$

(198) i = C = {4,5} , j = 8, t = 3

$$0.0566X_{183} - 0.3600X_{184} + 0.3600Y_{184} + 0.0307X_{283} - 0.3392X_{284} + 0.3392Y_{284} - 0.0424X_{383} - 0.3000X_{384} + 0.3000Y_{384} - 0.0X_{483} + 0.7000X_{484} - 0.700Y_{484} + 0.7000X_{584} - 0.7000Y_{584} + n^{c}_{C83} - p^{c}_{C83} = 0$$

(199) i = C = {4,5} , j = 8, t = 4

$$0.0566X_{184} - 0.3600X_{185} + 0.3600Y_{185} + 0.0307X_{284} - 0.3392X_{285} + 0.3392Y_{285} - 0.0424X_{384} - 0.300X_{385} + 0.3000Y_{385} - 0.0X_{484} + 0.7000X_{485} - 0.700Y_{485} + 0.7000X_{585} - 0.7000Y_{585} + n^{c}_{C84} - p^{c}_{C84} = 0$$

(200) $i = C = \{4,5\}$, $j = 8$, $t = 4$

$$0.0566X_{185} - 0.3600X_{186} + 0.3600Y_{186} + 0.0307X_{285} - 0.3392X_{286} + 0.3392Y_{286} - 0.0424X_{385} - 0.3000X_{386} + 0.3000Y_{386} - 0.0X_{485} + 0.7000X_{486} - 0.700Y_{486} + 0.7000X_{586} - 0.7000Y_{586} + n^{c}_{C85} - p^{c}_{C85} = 0$$

Non-goal Constraints

(iv) Maximum Hiring Constraints (40)

(201) $j = 1$, $t = 1$

$$Y_{111} + Y_{211} + Y_{311} + Y_{411} + Y_{511} + n^{d}_{11} - p^{d}_{11} = 35$$

(202) $j = 1$, $t = 2$

$$Y_{112} + Y_{212} + Y_{312} + Y_{412} + Y_{512} + n^{d}_{12} - p^{d}_{12} = 37$$

(203) $j = 1$, $t = 3$

$$Y_{113} + Y_{213} + Y_{313} + Y_{413} + Y_{513} + n^{d}_{13} - p^{d}_{13} = 38$$

(204) $j = 1$, $t = 4$

$$Y_{114} + Y_{214} + Y_{314} + Y_{414} + Y_{514} + n^{d}_{14} - p^{d}_{14} = 40$$

(205) $j = 1$, $t = 5$

$$Y_{115} + Y_{215} + Y_{315} + Y_{415} + Y_{515} + n^{d}_{15} - p^{d}_{15} = 40$$

(206) $j = 2, t = 1$

$$Y_{121} + Y_{221} + Y_{321} + Y_{421} + Y_{521} + n^{d}_{21} - p^{d}_{21} = 17$$

(207) $j = 2, t = 2$

$$Y_{122} + Y_{222} + Y_{322} + Y_{422} + Y_{522} + n^{d}_{22} - p^{d}_{22} = 19$$

(208) $j = 2, t = 3$

$$Y_{123} + Y_{223} + Y_{323} + Y_{423} + Y_{523} + n^{d}_{23} - p^{d}_{23} = 20$$

(209) $j = 2, t = 4$

$$Y_{124} + Y_{224} + Y_{324} + Y_{424} + Y_{524} + n^{d}_{24} - p^{d}_{24} = 22$$

(210) $j = 2, t = 5$

$$Y_{125} + Y_{225} + Y_{325} + Y_{425} + Y_{525} + n^{d}_{25} - p^{d}_{25} = 23$$

(211) $j = 3, t = 1$

$$Y_{131} + Y_{231} + Y_{331} + Y_{431} + Y_{531} + n^{d}_{31} - p^{d}_{31} = 24$$

(212) $j = 3, t = 2$

$$Y_{132} + Y_{232} + Y_{332} + Y_{432} + Y_{532} + n^{d}_{32} - p^{d}_{32} = 27$$

(213) $j = 3, t = 3$

$$Y_{133} + Y_{233} + Y_{333} + Y_{433} + Y_{533} + n^{d}_{33} - p^{d}_{33} = 30$$

(214) $j = 3, t = 4$

$$Y_{134} + Y_{234} + Y_{334} + Y_{434} + Y_{534} + n^{d}_{34} - p^{d}_{34} = 33$$

(215) $j = 3, t = 5$

$$Y_{135} + Y_{235} + Y_{335} + Y_{435} + Y_{535} + n^d_{35} - p^d_{35} = 34$$

(216) $j = 4, t = 1$

$$Y_{141} + Y_{241} + Y_{341} + Y_{441} + Y_{541} + n^d_{41} - p^d_{41} = 18$$

(217) $j = 4, t = 2$

$$Y_{142} + Y_{242} + Y_{342} + Y_{442} + Y_{542} + n^d_{42} - p^d_{42} = 22$$

(218) $j = 4, t = 3$

$$Y_{143} + Y_{243} + Y_{343} + Y_{443} + Y_{543} + n^d_{43} - p^d_{43} = 24$$

(219) $j = 4, t = 4$

$$Y_{144} + Y_{244} + Y_{344} + Y_{444} + Y_{544} + n^d_{44} - p^d_{44} = 28$$

(220) $j = 4, t = 5$

$$Y_{145} + Y_{245} + Y_{345} + Y_{445} + Y_{545} + n^d_{45} - p^d_{45} = 28$$

(221) $j = 5, t = 1$

$$Y_{151} + Y_{251} + Y_{351} + Y_{451} + Y_{551} + n^d_{51} - p^d_{51} = 10$$

(222) $j = 5, t = 2$

$$Y_{152} + Y_{252} + Y_{352} + Y_{452} + Y_{552} + n^d_{52} - p^d_{52} = 10$$

(223) $j = 5, t = 3$

$$Y_{153} + Y_{253} + Y_{353} + Y_{453} + Y_{553} + n^d_{53} - p^d_{53} = 11$$

(224) $j = 5,\ t = 4$

$$Y_{154} + Y_{254} + Y_{354} + Y_{454} + Y_{554} + n^d_{54} - p^d_{54} = 12$$

(225) $j = 5,\ t = 5$

$$Y_{155} + Y_{255} + Y_{355} + Y_{455} + Y_{555} + n^d_{55} - p^d_{55} = 12$$

(226) $j = 6,\ t = 1$

$$Y_{161} + Y_{261} + Y_{361} + Y_{461} + Y_{561} + n^d_{61} - p^d_{61} = 14$$

(227) $j = 6,\ t = 2$

$$Y_{162} + Y_{262} + Y_{362} + Y_{462} + Y_{562} + n^d_{62} - p^d_{62} = 17$$

(228) $j = 6,\ t = 3$

$$Y_{163} + Y_{263} + Y_{363} + Y_{463} + Y_{563} + n^d_{63} - p^d_{63} = 20$$

(229) $j = 6,\ t = 4$

$$Y_{164} + Y_{264} + Y_{364} + Y_{464} + Y_{564} + n^d_{64} - p^d_{64} = 23$$

(230) $j = 6,\ t = 5$

$$Y_{165} + Y_{265} + Y_{365} + Y_{465} + Y_{565} + n^d_{65} - p^d_{65} = 24$$

(231) $j = 7,\ t = 1$

$$Y_{171} + Y_{271} + Y_{371} + Y_{471} + Y_{571} + n^d_{71} - p^d_{71} = 7$$

(232) $j = 7,\ t = 2$

$$Y_{172} + Y_{272} + Y_{372} + Y_{472} + Y_{572} + n^d_{72} - p^d_{72} = 7$$

(233) $j = 7, t = 3$

$$Y_{173} + Y_{273} + Y_{373} + Y_{473} + Y_{573} + n^d_{73} - p^d_{73} = 8$$

(234) $j = 7, t = 4$

$$Y_{174} + Y_{274} + Y_{374} + Y_{474} + Y_{574} + n^d_{74} - p^d_{74} = 11$$

(235) $j = 7, t = 5$

$$Y_{175} + Y_{275} + Y_{375} + Y_{475} + Y_{575} + n^d_{75} - p^d_{75} = 11$$

(236) $j = 8, t = 1$

$$Y_{181} + Y_{281} + Y_{381} + Y_{481} + Y_{581} + n^d_{81} - p^d_{81} = 7$$

(237) $j = 8, t = 2$

$$Y_{182} + Y_{282} + Y_{382} + Y_{482} + Y_{582} + n^d_{82} - p^d_{82} = 9$$

(238) $j = 8, t = 3$

$$Y_{183} + Y_{283} + Y_{383} + Y_{483} + Y_{583} + n^d_{83} - p^d_{83} = 11$$

(239) $j = 8, t = 4$

$$Y_{184} + Y_{284} + Y_{384} + Y_{484} + Y_{584} + n^d_{84} - p^d_{84} = 12$$

(240) $j = 8, t = 5$

$$Y_{185} + Y_{285} + Y_{385} + Y_{485} + Y_{585} + n^d_{85} - p^d_{85} = 13$$

Payroll Budget Constraints

(241) $t = 1$

$- 2244X_{111} + 8915X_{112} - 8915Y_{112} - 1850X_{121}$

$+ 8557X_{122} - 8557Y_{122} - 1257X_{131} + 8164X_{132}$

$- 8164Y_{132} - 1466X_{141} + 9293X_{142} - 9293Y_{142}$

$- 1829X_{151} + 8767X_{152} - 8767Y_{152}$

$- 0.0X_{161} + 9380X_{162} - 9380Y_{162} - 0.0X_{171}$

$+ 0.0X_{172} - 0.0Y_{172} - 1686X_{181} + 8275X_{182}$

$- 8275Y_{182} - 1536X_{211} + 10470X_{212} - 10470Y_{212}$

$- 682X_{221} + 9707X_{222} - 9707Y_{222} - 985X_{231}$

$+ 12934X_{232} - 12934Y_{232} - 1689X_{241} + 10745X_{242}$

$- 10745Y_{242} - 1343X_{251} + 10290X_{252} - 10290Y_{252}$

$- 1559X_{261} + 10191X_{262} - 10191Y_{262} - 1358X_{271}$

$+ 10080X_{272} - 10080Y_{272} - 1215X_{281} + 10114X_{282}$

$- 10114Y_{282} - 1330X_{311} + 12936X_{312} - 12936Y_{312}$

$- 859X_{321} + 12662X_{322} - 12662Y_{322} - 1017X_{331}$

$+ 13122X_{332} - 13122Y_{332} - 1872X_{341} + 13794X_{342}$

$- 13794Y_{342} - 1264X_{351} + 12969X_{352} - 12969Y_{352}$

$- 1458X_{361} + 13016X_{362} - 13016Y_{362} - 2829X_{371}$

$+ 13975X_{372} - 13975Y_{372} - 889X_{381} + 11868X_{382}$

$- 11868Y_{382} - 1584X_{411} + 15832X_{412}$

$- 15832Y_{412} - 540X_{421} + 17898X_{422} - 17898Y_{422}$

$$- 707X_{431} + 15193X_{432} - 15193Y_{432} - 1130X_{441}$$
$$+ 15834X_{442} - 15834Y_{442} - 2594X_{451} + 18176X_{452}$$
$$- 18176Y_{452} - 1859X_{461} + 16529X_{462} - 16529Y_{462}$$
$$- 3416X_{471} + 19446X_{472} - 19446Y_{472} - 0.0X_{481}$$
$$+ 14052X_{482} - 14052Y_{482} + 15850X_{512} - 15850Y_{512}$$
$$+ 16204X_{522} - 16204Y_{522} + 16433X_{532} - 16433Y_{532}$$
$$+ 16374X_{542} - 16374Y_{542} + 16431X_{552} - 16431Y_{552}$$
$$+ 15789X_{562} - 15789Y_{562} + 17081X_{572} - 17081Y_{572}$$
$$+ 15480X_{582} - 15480Y_{582} + n_1^e - p_1^e$$
$$= 11,343,922$$

(242) $t = 2$

$$- 2237X_{112} + 8822X_{113} - 8822Y_{113} - 1721X_{122}$$
$$+ 8535X_{123} - 8535Y_{123} - 1229X_{132} + 8277X_{133}$$
$$- 8277Y_{133} - 1437X_{142} + 9293X_{143} - 9293Y_{143}$$
$$- 1841X_{152} + 8894X_{153} - 8894Y_{153} - 0.0X_{162}$$
$$+ 9380X_{163} - 9380Y_{163} - 0.0X_{172} + 0.0X_{173}$$
$$- 0.0Y_{173} - 1675X_{182} + 8294X_{183} - 8294Y_{183}$$
$$- 1545X_{212} + 10441X_{213} - 10441Y_{213} - 694X_{222}$$
$$+ 9031X_{223} - 9031Y_{223} - 1016X_{232} + 10243X_{233}$$
$$- 10243Y_{233} - 1712X_{242} + 10539X_{243} - 10539Y_{243}$$
$$- 1396X_{252} + 10358X_{253} - 10358Y_{353} - 1644X_{262}$$
$$+ 9780X_{263} - 9780Y_{263} - 1424X_{272} + 10278X_{273}$$

$- 10278Y_{273} - 1258X_{282} + 10043X_{283} - 10043Y_{283}$

$- 1332X_{312} + 13015X_{313} - 13015Y_{313} - 866X_{322}$

$+ 12897X_{323} - 12897Y_{323} - 1048X_{332} + 13539X_{333}$

$- 13539Y_{333} - 1241X_{342} + 13985X_{343} - 13985Y_{343}$

$- 1288X_{352} + 13482X_{353} - 13482Y_{353} - 1459X_{362}$

$+ 13711X_{363} - 13711Y_{363} - 2810X_{372} + 14587X_{373}$

$- 14587Y_{374} - 922X_{382} + 12286X_{383} - 12286Y_{383}$

$- 1575X_{412} + 16023X_{413} - 16023Y_{413} - 545X_{422}$

$+ 18041X_{423} - 18041Y_{423} - 704X_{432} + 16108X_{433}$

$- 16108Y_{433} - 1122X_{442} + 15783X_{443} - 15783Y_{443}$

$- 2655X_{452} + 18474X_{453} - 18474Y_{453} - 1840X_{462}$

$+ 16663X_{463} - 16663Y_{463} - 3385X_{472} + 19446X_{473}$

$- 19446Y_{473} - 0.0X_{482} + 14712X_{483} - 14712Y_{483}$

$+ 15752X_{513} - 15752Y_{513} + 16341X_{523} - 16341Y_{523}$

$+ 16382X_{533} - 16382Y_{533} + 16251X_{543} - 16251Y_{543}$

$+ 16816X_{553} - 16816Y_{553} + 15628X_{563} - 15628Y_{563}$

$+ 16922X_{573} - 16922X_{573} + 15720X_{583} - 15720Y_{583}$

$+ n_2^e - p_2^e \quad = \quad 12,519,538$

(243) t = 3

$$- 2240X_{113} + 8798X_{114} - 8798Y_{114} - 1860X_{123}$$
$$+ 8539X_{124} - 8539Y_{124} - 1231X_{133} + 8452X_{134}$$
$$- 8452Y_{134} - 1441X_{143} + 9434X_{144} - 9434Y_{144}$$
$$- 1837X_{153} + 8871X_{154} - 8871Y_{154} - 0.0X_{163}$$
$$+ 9380X_{164} - 9380Y_{164} - 0.0X_{173} + 0.0X_{174}$$
$$- 0.0Y_{174} - 1665X_{183} + 8156X_{184} - 8156Y_{184}$$
$$- 1545X_{213} + 10455X_{214} - 10455Y_{214} - 723X_{223}$$
$$+ 7764X_{224} - 7764Y_{224} - 1015X_{233} + 10260X_{234}$$
$$- 10260Y_{234} - 1777X_{243} + 10570X_{244} - 10570Y_{244}$$
$$- 1401X_{253} + 10333X_{254} - 10333Y_{254} - 1711X_{263}$$
$$+ 10381X_{264} - 10381Y_{264} - 1421X_{273} + 10281X_{274} - 10281Y_{274}$$
$$- 1260X_{283} + 9990X_{284} - 9990Y_{284} - 1334X_{313}$$
$$+ 13012X_{314} - 13012Y_{314} - 866X_{323} + 13426X_{324}$$
$$- 13426Y_{324} - 1049X_{333} + 14015X_{334} - 14015Y_{334}$$
$$- 1866X_{343} + 13985X_{344} - 13985Y_{344} - 1268X_{353}$$
$$+ 13528X_{354} - 13528Y_{354} - 1461X_{363} + 15635X_{364}$$
$$- 15635Y_{364} - 2819X_{373} + 2298X_{374} - 2298Y_{374}$$
$$- 922X_{383} + 12286X_{384} - 12286Y_{384} - 1579X_{413}$$
$$+ 16023X_{414} - 16023Y_{414} - 545X_{423} + 18041X_{424}$$
$$- 18041Y_{424} - 705X_{433} + 16108X_{434} - 16108Y_{434}$$
$$- 1123X_{443} + 16653X_{444} - 16653Y_{444} - 2593X_{453}$$

$$+ 18127X_{454} - 18127Y_{454} - 1845X_{463} + 16663X_{464}$$
$$- 16663Y_{464} - 3399X_{473} + 19446X_{474} - 19446Y_{476}$$
$$- 0.0X_{483} + 14712X_{484} - 14712Y_{484} + 15791X_{514}$$
$$- 15791Y_{514} + 16249X_{524} - 16249Y_{524} + 16396X_{534}$$
$$- 16396Y_{534} + 16259X_{544} - 16259Y_{544} + 16431X_{554}$$
$$- 16431Y_{554} + 15671X_{564} - 15671Y_{564} + 16992X_{574}$$
$$- 16992Y_{574} + 15720X_{584} - 15720Y_{584} + n_3^e - p_3^e$$
$$= 13,873,535$$

(244) $t = 4$

$$- 2239X_{114} + 8613X_{115} - 8613Y_{115} - 1867X_{124}$$
$$+ 8541X_{125} - 8541Y_{125} - 1233X_{134} + 8436X_{135}$$
$$- 8436Y_{135} - 1439X_{144} + 9575X_{145} - 9575Y_{145}$$
$$- 1835X_{154} + 8856X_{155} - 8856Y_{155} - 0.0X_{164}$$
$$+ 9580X_{165} - 9380Y_{165} - 0.0X_{174} + 0.0X_{175}$$
$$- 0.0Y_{175} - 1679X_{184} + 8218XX_{185} - 8218Y_{185}$$
$$- 1610X_{214} + 10449X_{215} - 10449Y_{215} - 753X_{224}$$
$$+ 9770X_{225} - 9770Y_{225} - 1055X_{234} + 10247X_{235}$$
$$- 10247Y_{235} - 1778X_{244} + 10548X_{245} - 10548Y_{245}$$
$$- 1400X_{254} + 10323X_{255} - 10323Y_{255} - 1700X_{264}$$
$$+ 10422X_{265} - 10422Y_{265} - 1477X_{274} + 10254X_{275}$$
$$- 10254Y_{275} - 1262X_{284} + 10071X_{285} - 10071Y_{285}$$
$$- 1334X_{314} + 13558X_{315} - 13558Y_{315} - 866X_{324}$$
$$+ 13974X_{325} - 13974Y_{325} + 1049X_{334} + 14141X_{335}$$

$- 14141Y_{335} - 1917X_{344} + 14523X_{345} - 14533Y_{345}$
$- 1264X_{354} + 13523X_{355} - 13523Y_{355} - 1462X_{364}$
$+ 14234X_{365} - 14234Y_{365} - 2823X_{374} + 15134X_{375}$
$- 15134Y_{375} - 922X_{384} + 12324X_{385} - 12324Y_{385}$
$- 1578X_{414} + 16023X_{415} - 16023Y_{415} - 545X_{424}$
$+ 18041X_{425} - 18041Y_{425} - 705X_{434} + 16108X_{435}$
$- 16108Y_{435} - 1123X_{444} + 16463X_{445} - 16463Y_{445}$
$- 2607X_{454} + 19519X_{455} - 19519Y_{455} - 1849X_{464}$
$+ 16663X_{465} - 16663Y_{465} - 3406X_{474} - 19446X_{475}$
$- 19446Y_{475} - 0.0X_{484} + 14712X_{485} - 14712Y_{485}$
$+ 15786X_{515} - 15786Y_{515} + 16341X_{525} - 16341Y_{525}$
$+ 16407X_{535} - 16407Y_{535} + 16268X_{545} - 16341Y_{525}$
$+ 16508X_{555} - 16508Y_{555} + 15703X_{565} - 15703Y_{565}$
$+ 17028X_{575} - 17028Y_{575} + 15720X_{585} - 15720Y_{585}$
$+ n_4^c - p_4^e = 15,440,094$

(245) $t = 5$

$- 2239X_{115} + 8820X_{116} - 8820Y_{116} - 1859X_{125}$
$+ 8553X_{126} - 8553Y_{126} - 1232X_{135} + 8370X_{136}$
$- 8370Y_{136} - 1264X_{145} + 9775X_{146} - 9775Y_{146}$
$- 1840X_{155} + 8690X_{156} - 8690Y_{156} - 0.0X_{165}$
$+ 9380X_{166} - 9380Y_{166} - 0.0X_{175} + 0.0X_{176}$

$- 0.0Y_{176} - 1682X_{185} + 8179X_{186} - 8179Y_{186}$

$- 1610X_{215} + 10449X_{216} - 10449Y_{216} - 750X_{225}$

$+ 9758X_{226} - 9758Y_{226} - 1058X_{235} + 10268X_{236}$

$- 10268Y_{236} - 1847X_{245} + 10587X_{246} - 10587Y_{246}$

$- 1452X_{255} + 10350X_{256} - 10350Y_{256} - 1708X_{265}$

$+ 10401X_{266} - 10401Y_{266} - 1472X_{275} + 10272X_{276}$

$- 10272Y_{276} - 1256X_{285} + 10085X_{286} - 10085Y_{286}$

$- 1334X_{315} + 13556X_{316} - 13556Y_{316} - 860X_{325}$

$+ 13927X_{326} - 13927Y_{326} - 1049X_{335} + 14095X_{336}$

$- 14095Y_{336} - 1920X_{345} + 15086X_{346} - 15086Y_{346}$

$- 1265X_{355} + 14022X_{356} - 14022Y_{356} - 1464X_{365}$

$+ 14249X_{366} - 14249Y_{366} - 2826X_{375} + 15084X_{376}$

$- 15084Y_{376} - 922X_{385} + 12269X_{386} - 12269Y_{386}$

$- 1578X_{415} + 16023X_{416} - 16023Y_{416} - 545X_{425}$

$+ 17836X_{426} - 17836Y_{426} - 706X_{435} + 16108X_{436}$

$- 16108Y_{436} - 1124X_{445} + 16492X_{440} - 16492Y_{446}$

$- 2611X_{455} + 18127X_{456} - 18127Y_{456} - 1852X_{465}$

$+ 16663X_{466} - 16663Y_{466} - 3412X_{475} + 19446X_{476}$

$- 19446Y_{476} - 0.0X_{485} + 14712X_{486} - 14712Y_{486}$

$+ 15784X_{516} - 15784Y_{516} + 16341X_{526} - 16341Y_{526}$

$+ 16418X_{536} - 16418Y_{536} + 16275X_{546} - 16275Y_{546}$

$+ 16535X_{556} - 16535Y_{556} + 15726X_{566} - 15726Y_{566}$

$$+ 17056X_{576} - 17056Y_{576} + 15720X_{586} - 15720Y_{586}$$

$$+ n_5^e - p_5^e = 16{,}996{,}873$$

APPENDIX 7

A Short Review of the theory of Postoptimal Sensitivity Analysis in Goal Programming

For our discussion in this section, we introduce the following notation (Ignizio, 1976):

$w_{k,s}$ = the weighting factor of the s^{th} non-basic variable at priority level k.

$U_{i,k}$ = the weighting factor of the i^{th} basic variable at the k^{th} priority level.

$\bar{b}'$ = vector representing the values of the basic variables at any given iteration.

a_k = the level of the achievement of the objective function associated with priority k.

$I_{k,s}$ = the per unit contribution of non-basic variable s toward the achievement of the k^{th} priority level.

$\bar{T}$ = a matrix (called transformation matrix) consisting of the columns in the final simplex tableau associated with the initial basic variables (i.e. associated with the negative deviational variables).

$e_{i,s}$ = coefficient of the i^{th} row associated with nonbasic variable s.

Changes that can be considered in goal programming postoptimal analysis include (Ignizio, 1976):

(i) change in $w_{k,s}$

(ii) change in $U_{i,k}$

(iii) change in the original r.h.s. value of goal i

(iv) change in $c_{i,j}$; the coefficient associated with the j^{th} variable in objective or goal i.

<u>Change in $w_{k,s}$ or $U_{i,k}$</u>

A change in either $w_{k,s}$ or $U_{i,k}$ can affect $I_{k,s}$ or a_k only and these are used to determine whether the GP solution is optimal or not. Denoting all new values with "^" i.e. if the new $w_{k,s} = \hat{w}_{k,s}$ and $\hat{I}_{k,s}$ is the new value of $I_{k,s}$, then

$$\hat{I}_{k,s} = \sum_{i=1}^{m} (e_{i,s} \cdot U_{ik}) - \hat{w}_{k,s} \qquad (1)$$

If $\hat{I}_{k,s}$ is now positive and there is no negative $I_{k,s}$ value at a higher priority level in the same column, the optimal solution mix of the problem will change.

Similarly, a change in the $U_{i,k}$ value will affect <u>both</u>

the unit contribution to the achievement of the objective function by variable s (also called index value) $I_{k,s}$ and the achievement value a_k. The new values are given by

$$\hat{I}_{k,s} = \sum_{i=1}^{m} (e_{i,s} \cdot \hat{U}_{i,k}) - w_{k,s} \quad (2)$$

$$\hat{a}_k = \sum_{i=1}^{m} (b_i \cdot \hat{U}_{i,k}) \quad (3)$$

If a_k was originally zero and is now positive, then the optimal solution mix may change depending on the values of the $\hat{I}_{k,s}$. In particular, if the $\hat{I}_{k,s}$ is positive with no negative index value in its column at a higher priority level, the optimal solution mix will change.

<u>Change in Original r.h.s. Value (b_i) of goal i.</u>

A change in the value of b_i can affect both value of $\bar{b}$ column in the final simplex tableau or the values of the achievement of the objective function at the k^{th} priority level, a_k. If $\bar{b}'$ is the value of the vector reflecting the r.h.s. values of the goals with b_i altered, the new value of the r.h.s. column in the final simplex tableau, $\hat{\bar{b}}$ is given by

$$\hat{\bar{b}} = \bar{T} \cdot \bar{b}' \quad (4)$$

and

$$\hat{a}_k = \sum_{i=1}^{m} \hat{\bar{b}}_i \cdot U_{ik} \quad (5)$$

It is possible that one of the entries of $\hat{\bar{b}}$ can become negative as a result of the operation described by (4) In such a case, the new solution is infeasible. Infeasibility can be resolved by the dual simplex method of goal programming.

Change in $c_{i,j}$

Changes in the $c_{i,j}$ are not as easily analyzed as those described before. However, considering changes associated with non-basic variables only, we have

$$\hat{\bar{e}}_s = \bar{T} \cdot \bar{C}'_s \quad (6)$$

where $\bar{T}$ = transformation matrix, defined previously

$\bar{c}'_s$ = the new vector set of $c_{i,j}$ coefficients under the s^{th} basic variable

$\hat{\bar{e}}'_s$ = is the new vector set of $e_{i,s}$ coefficients in the final simplex tableau under basic variable s. This change will affect the index corresponding to variable s at the appropriate priority level. Whether the change will affect the optimal solution mix depends on the examination of the $I_{k,s}$ values.