A COMPARISON OF LEAST SQUARES DUMMY VARIABLE (LSDV) AND THE POOLED ESTIMATOR IN FIXED EFFECT MODEL

[1]O. M. Olayiwola, [2]Adedayo. A. Adepoju, [2]J. T. Olajide
[1]Department of Physical Sciences, Ajayi Crowther University, Oyo, Oyo State, Nigeria.
[2]Department of Statistics, University of Ibadan, Oyo State, Nigeria.
[3]Department of Mathematics / Statistics, The Polytechnic, Ibadan, Oyo State, Nigeria.

ABSTRACT

This paper examines a comparison of Least Squares Dummy Variable and a pooled estimator in a fixed effect model. The aims of the research are: to estimate the individuals firms parameters by using least squares dummy variables. To estimate parameter of the pooled observations using ordinary least squares.To estimate the behavioral relationship between individuals variables and to test for the significance difference across the groups. The framework was based on a fixed effect model. The analysis of a panel model was carried out using the Ordinary Least Squares (OLS) and Least Squares Dummy Variable (LSDV) methods. Various tests were carried out to determine which of the methods to use when dealing with a panel data. The results of the analysis showed significant difference across the different groups effect. F is also significant at 95% level by using either the fixed effect or pooled model in a panel data. Also, a measure of fit of the model carried out showed that the fixed effect model significantly explained the variation in the dependent variable while pooling the model explained a very small proportion of the total variation in the dependent variable. In the light of the above, it will be appropriate to use a fixed effect least squares dummy variable rather than pooling the data in the analysis of a panel data.

Keywords: Panel data, Ordinary least squares, Dummy variable, Pooled model, Fixed effect model,

INTRODUCTION

The term "panel data" refers to data sets where there is data on the same individual over several periods of time. The main advantage with having panel data as compared to a single cross-section or series of cross-sections with non overlapping cross-section units is that it allows to test and relax the assumptions that are implicit in cross-sectional analysis. Most economic data sets usually combine time series with cross sections data. Availability of data on a large number of individuals, but on each individual only over a very short period of time, has become increasingly common in a number of different fields in economics. Since only a few observations are available over time, but a great many observations are available for different individuals at a point in time, it is exceptionally important to make the efficient use of the data across individuals to estimate that part of the behavioral relationship containing variables that differ substantially from one individual to another, in order that the lesser amount of information over time can be used to best advantage in estimation of the relationship studied.

Aims and objectives of this paper are as follows:

(i) to estimate the individuals firms parameters by using least squares dummy variables;
(ii) to estimate parameter of the pooled observations using ordinary least squares;
(iii) to estimate the behavioral relationship between individuals variables;
(iv) to test for the significance difference across the groups.

Many recent studies have analyzed panel, or longitudinal data sets; two very famous ones are the National Longitudinal Survey of Labor Market Experience (NLS) and the Michigan Panel study of Income Dynamics (PSID). In these data sets very large cross sections, consisting of thousands of micro units, are followed through time, but the number of periods is often quite small. The PSID, for example, is a study of roughly 6,000 families and 15,000 individuals who have been interviewed periodically from 1968 to the present. Another group of intensively studied panel data sets were those from the negative income tax experiments of the early 1970s in which thousands of families were followed for 8 or13 quarters. Constructing long, evenly spaced, time series in contexts such as these data are typically used, it is unnecessary. Time effects are often viewed as "transitions" or discrete changes of state. They are typically modeled as specific to the period in which they occur and are not carried across periods within a cross-section unit. Panel data sets are more oriented toward cross-section analyses; they are wide but typically short. The fundamental advantage of a panel data set over a cross section is that it allows the researcher great flexibility differences in behavior across individuals.

The Model

The basic framework for this discussion is a regression model of the form

$$y_{it} = \alpha_i + \beta x_{it} + \varepsilon_{it} \qquad i = 1,\dots,n \text{ and } t = 1,\dots,T \qquad \dots (1)$$

where y_{it} is the value of dependent variable for cross section unit at time t.

x_{it} is the value of the explanatory variable for unit i at time t, and

α_i the individual effect is taken to be constant over time t and specific to the individual cross-sectional unit i.

Equation (1) is a classical regression model. If α_i is to be the same across all units, then ordinary least squares provides consistent and efficient estimates of α and β. The various cases considered are:

Pooled Regression: If α_i contains only a constant term, then ordinary least square provides consistent and efficient estimates of the common α and the slope vector β.

2 **Fixed Effects:** If α_i is unobserved, but correlated with x_{it}, the least squares estimator of β is biased and inconsistent as a consequence of an omitted variable. However, in this instance, the model $y_{it} = \alpha_i + \beta x_{it} + \varepsilon_{it}$.

Where α_i, embodies all the observable effects and specifies an estimable conditional mean. This fixed effects approach takes α_i to be a group-specific constant term in the regression model. It should be noted that the term "fixed" as used here indicates that the term does not vary over time, not that it is non-stochastic, which need not be the case.

Random Effects: In the random effects model, the α_i are treated as random variables, rather than fixed constants. The α_i 's are assumed to be independent of the error ε_{it} and also mutually independent. This model is also known as the variance component model. It became popular in econometrics following the paper by Balestra and Nerlove on the demand for natural gas.

We shall assume that

$$\alpha_i \sim IID(0, \sigma\alpha^2)$$
$$\varepsilon_{it} \sim IID(0, \sigma\varepsilon^2)$$

and that α_i and ε_{it} are independent.

For the sake of simplicity we shall use only one explanatory variable. The model is the same as equation (1) except that α_i are random variables

The Pooled Estimator

Our starting point is by considering the simplest estimation method which proceeds by essentially ignoring the panel structure of the data.

Most times, the data are stacked to form

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_n \end{bmatrix}, \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix}, \qquad \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_n \end{bmatrix}$$

where y is an nTx1, x is an nTxK and ε is an nTx1.
The standard linear model can then be expressed in a more compact form as follows

$$y = X\beta + \varepsilon \qquad \text{.... (2)}$$

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \cdot \\ \cdot \\ \beta_k \end{bmatrix}$$

Stack the data as described above and let the model be given by:

$$y = X\beta + \varepsilon \qquad \text{..... (3)}$$

We now assume that $\varepsilon_{it} \sim iid(0, \sigma^2)$ for all i and t. That is, for a given individual, observations are serially uncorrelated; and across individuals and time, the errors are homoscedastic.
Estimation of this model is straightforward. The assumptions correspond to the classical linear model. Efficient estimation proceeds by stacking the data as already shown and using OLS (Ordinary Least Squares). By assuming each observation is iid, however, we have essentially ignored the panel structure of the data. Although, this estimation is the easiest, it is often appropriate for reasons that we now pursue.

Fixed Effects
This formulation of the model assumes that differences across unit can be captured in differences in the constant term.

Simulation Process
Data can be classified into two, they are: **Real** and **Simulated** data. "**Real**" data consist of actual measurements on some physical phenomenon such as level of activity of an economy or the behavior of real consumers. The alternative is **simulated** data, produced by the analyst with a random number generator, usually for the purpose of studying the behavior of econometric estimates for which the statistical properties are unknown or impossible to derive. This study employs simulated data with a random number generator. The exogenous variables were generated by choosing a random variable x_{it} (for i = 1, 2, ...,10 and t = 1, 2, ... , 5) uniformly distributed on the interval [0, 1]. In the experiments N =10 and T=5. (Where t is the period, I is the number of individual over period t and n is the number of individuals).
The next step in each repetition was the generation of random variable U_{it}, which is normally distributed with mean zero and unit variance. The parameter α was taken to have values 1,2...10. For each such value, β=1

Parameter values
As indicated above, in all experiments reported here T=5 and n = 10. The parameter α was taken to have values 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10. Throughout the experiment β was taken to be unity. (Where n is the number of individual effects and T is the period). For y_{it}'s we substitute for the various values of ε_{it}, α_i's, β

and x_{it} for each cells in the data to form a panel data model.

DATA GENERATION PROCEDURE

As earlier stated, simulated data were used for this research work by the used of computer software. In generating the data, Microsoft Excel 2003 was used while both Excel and SPSS were used for analysis.
Data were generated on the model

$$y_{it} = \alpha_i + \beta x_{it} + \varepsilon_{it}$$

Random Number Generation

The error term ε_{it} was generated to possess a normal distribution with mean zero and unit variance i.e. $\varepsilon_{it} \sim N(0,1)$. Then we set the number of variables, n to be ten and number of period (T) is five. (Where n is the number of individuals and T is the periods of time). Setting the random seed and update it seeds by seed, for this data we set random seed number as 10, 20, 30... 1000. For independent variables x_{it}, it was generated from a uniform distribution U (0, 1) in the range of 10 to 20. Then we use the same set of variable x_{it} for the whole data generated while only the error terms, ε_{it} and constant variables α_i's were varied.

METHODOLOGY

The ordinary Least Squares (OLS) method

Among all the various econometric methods that can be used to derive estimates of the parameters of econometric relationship from data, the ordinary least squares (OLS) or the Classical Least Squares (CLS) stand on the top of the priority list. The method seeks the minimization of the sum of the squares of the deviation of the actual observations on a variable from the values that would be obtained based on the regression equation. The coefficients of the regression are therefore, estimated by minimizing the residual sum of squares. To complete the specification of the simple model we need some assumptions about the random variable U. The variable U is a generated random variable, it has zero mean, constant variance, the value of each Ui are normally distributed, non autocorrelation or serial independence of the U's, the relationship being studied is identified and there is correct specification of the model.
Using matrix notation, the standard regression may be written as:

$Y = X\beta + U$

Where Y, X, β and U are defined as

β is a k-element vector of regression coefficients (parameters)

U is a vector of n error terms or disturbances.

Y is a vector of n sample observations on the dependent variable.

X is an n x k matrix of observations on k independent (explanatory) variables.

The sum of Squares residual is given as $U = Y - X\beta$.

The Least Square Dummy Variable or Fixed Effect Model

The model

$$y_{it} = \alpha_i + \beta x_{it} + U_{it} \qquad i = 1,2,...n, t = 1,2,...T$$

$$U_{it} \sim \text{IND}(0, \sigma^2)$$

Define

$$\bar{x}_i = 1/T\sum x_{it}, \bar{y}_{i.} = 1/T\sum y_{it}$$

$$W_{xx} = \sum(x_{it} - \bar{x}_i)^2 = \sum \bar{x}_{it}^2 - n\bar{x}^2$$

$$W_{xy_i} = \sum(x_{it} - \bar{x}_i)(y_{it} - y_i) = \sum x_i y_i - n\overline{xy}$$

$$W_{yy} = \sum(y_{it} - \bar{y})^2 = \sum y_{it}^2 - n\bar{y}^2.$$

Since $y_{it} = \alpha_i + \beta x_{it} + U_{it}$

$$U_{it} = y_{it} - \alpha_i - \beta x_{it} \qquad \text{-------}(3.1)$$

Let

$Q = \sum (y_{it} - \alpha_i - \beta x_{it})^2$ (sum of square of error term)

Coefficient of determination, R^2

R^2 is a measure of fit of the model. Two equivalent ways to compute R^2 are

(1) The proportion of the total variation in y that is accounted for by variation in the regressors.

$$R^2 = 1 - \frac{e'e}{y'Ay} = 1 - \frac{RSS}{TSS} = \frac{ESS}{TSS} \quad \ldots(3.4)$$

(2) The squared correlation between the observed values of y and the predictions produced by estimated regression equation.

$$R^2 = \frac{\sum (y_i - \bar{y})(y_{it} - \hat{y})}{\left[\sum (y_i - \bar{y})^2 \left[\sum (\bar{y}_i - \hat{y})^2\right]\right]} \quad \ldots (3.5)$$

If X matrix contains a constant term R^2 is constrained to lie between 0 and 1.
R^2 will never decrease when another variable is added to a regression equation.

Testing the significance of the groups effects

The t ratio for ai can be used for a test of the hypothesis that αi equals to zero. This hypothesis about one specific group, however, is typically not useful for testing in this regression context. If we are interested in differences across groups, then we can test the hypothesis that the constant terms are all equal with an F test. Under the null hypothesis of equality, the estimator is pooled least squares. The F ratio used for this test is

$$F(n-1, nT-n-k) = \frac{(R^2LSDV - R^2pooled)/(n-1)}{(1 - R^2LSDV)/(nT-n-k)}, \quad \ldots(3.6)$$

Where LSDV indicates the dummy variable model and pooled indicates the pooled or restricted model with only a single overall constant term. Alternatively, the model may have been estimated with an overall constant and n-1 dummy variable instead. All other results (i.e. the least squares slopes, S^2, R^2) will be unchanged, but rather than estimate α_i, each dummy variable coefficient will now be an estimate of $\alpha_i - \alpha 1$ where group "1" is the omitted group. The F test that the coefficient on these n-1 dummy variables are zero identical to one above. It is important to keep in mind, however, that although the statistical results are the same, the interpretation of the dummy variable coefficients in the two formulations is different.

ANALYSIS AND RESULTS

Pooled estimation output

From the data, which contain 50 observations (for which we run regression on one hundred groups), the regression statistics on the model as per the table on the appendix shows that F is significant at 95% level. Also considering the coefficient of determination, R^2 pooled of the model which is generally low meaning that the model explained a very small proportion of the total variation in the dependent variable. See the details in the appendix.

Testing the significance of the groups effect.

The table on the appendix contains the estimated parameters of individual effects. .
The F statistic for testing the joint significance of the fixed effects is

$(R^2LSDV - R^2pooled)/(n-1)$

$F(n-1, nT-n-k) = (1 - R^2LSDV)/(nT-n-k)$ as given in equation (3.6)

$$F(9, 30) = \frac{(0.993364 - 0.570688)/9}{(1 - 0.993364)/30} = 212.3146$$

The same procedure was used for others in the group as shown in the appendix.

The critical value from the F table is 2.21. so the evidence is strongly in favour of a group specific effect in the data. Thus, on this basis, there appear to be significant difference across the different groups in the model. On the other hand, the coefficient of determination, R^2LSDV of the fixed effect model is high and this implies that the model significantly explain the variation in the dependent variable.

CONCLUSION

The results of the analysis showed significant difference across the different groups effect. F is also significant at 95% level by using either the fixed effect or pooled model in a panel data. Also, a measure of fit of the model carried out showed that the fixed effect model significantly explained the variation in the dependent variable while pooling the model explained a very small proportion of the total variation in the dependent variable. In the light of the above, it will be appropriate to use a fixed effect least squares dummy variable rather than pooling the data in the analysis of a panel data.

REFERENCES

Maddala, G.S. Introduction to Econometrics (3rd edition). John Wiley and Son Ltd. New York.

Maddala, G.S.(1971). "The use of Variance Components Model in pooling Cross-section and Time-series Data." Econometrica Vol.39 1971 pp.341-358

Marc Nerlove (1971). "Further Evidence on the Estimation of Dynamic Economic Relations From a Time-series of cross-sections." Econometrica, vol 39, 1971 pp.359-382

Ogunsanya, B.G. (2003). " A panel study of income Dynamics of some states in Nigeria'.

Panel Data: The Michigan Panel Study of Income Dynamics', The Journal of Human Resources, vol. 33, no. 2 (spring), pp. 251-299.

Spieß, M. and Kroh, M (2004) 'Documentation of Sample Sizes and Panel Attrition in the German Socio Economic Panel (GSOEP)', Research Note No. 28a, German Institute for Economic Research (DIW), Berlin.27 Wadsworth, M.E.J., Butterworth, S.L., Hardy, R.J., Kuh, D.J., Richards, M.,Langenberg, C.

Willian H. Greene (2003). Econometric Analysis (5th edition). Pearson Education International. New York University. Prentice Hall.

INTERNATIONAL JOURNAL OF PURE AND APPLIED SCIENCE

Volume 4, Number 1, 2011

APPENDIX.

N	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	R^2_{LSDV}	α	β	R^2_{POOLED}	F	F_{POOLED}	F_{LSDV}
	-2.42	-1.67	-0.73	0.26	1.18	2.29	3.24	4.17	5.29	6.18	0.99	2.30	1.211	0.571	212.31	63.81	530.74
	-0.79	1.24	2.09	1.97	3.38	5.07	6.10	6.22	7.73	8.59	0.96	4.44	1.068	0.483	40.44	44.77	36.44
	-1.70	0.04	0.96	1.87	4.05	3.99	5.13	6.51	7.22	8.55	0.99	4.08	1.082	0.437	830.64	37.31	1573.23
	1.34	2.68	2.68	4.08	5.30	5.19	6.51	7.19	9.10	9.22	0.99	5.55	0.995	0.477	573.69	43.73	1573.23
	-0.46	0.65	2.23	3.05	3.65	5.22	5.57	6.71	8.10	9.04	0.99	4.75	1.056	0.453	794.23	39.79	1548.01
	-0.32	1.13	1.64	2.60	3.77	4.37	6.59	6.54	7.69	9.11	0.99	5.00	1.033	0.457	1378.27	40.34	2700.38
	0.11	1.55	3.35	3.84	4.45	6.46	7.05	8.20	8.29	10.30	0.99	5.98	0.958	0.394	1112.21	31.20	1954.61
	-0.43	1.05	0.72	3.26	4.10	4.75	6.46	7.24	7.86	9.05	0.99	5.03	1.053	0.440	1303.67	37.79	2480.25
	1.04	2.23	3.27	3.31	5.01	5.16	7.20	8.51	8.77	9.82	0.92	6.17	0.958	0.414	23.13	33.95	44.51
0	0.02	1.74	2.33	3.10	4.42	5.27	7.17	6.60	8.73	9.05	0.95	5.24	1.003	0.443	37.39	38.13	74.18
1	1.53	2.92	3.76	5.36	5.63	6.70	7.82	8.68	10.51	10.63	0.94	6.73	0.9016	0.383	35.03	29.79	62.59
2	0.45	1.44	1.95	3.71	4.85	6.55	6.88	7.50	9.60	9.54	0.95	5.46	0.989	0.400	38.60	31.95	70.73
3	0.99	1.54	2.37	4.10	4.83	5.96	7.47	7.61	8.89	9.89	0.94	6.09	.978	0.419	33.85	34.55	64.47
4	0.77	1.21	3.44	4.62	5.13	6.27	7.80	8.61	10.36	10.57	0.95	6.66	0.943	0.354	47.50	26.34	80.20
5	1.01	2.41	2.95	4.41	5.00	6.67	6.78	7.83	9.32	10.21	0.94	5.85	0.968	0.427	34.23	35.78	66.19
6	1.53	2.76	4.36	4.29	6.63	6.31	8.02	8.89	9.40	10.78	0.92	6.93	0.905	0.385	23.22	30.09	42.39
7	0.62	1.66	3.12	4.35	5.04	5.53	6.58	7.82	8.72	9.77	0.93	5.87	0.981	0.433	25.92	36.67	51.34
8	0.19	0.56	3.07	4.30	5.50	6.09	6.61	7.63	8.16	10.10	0.95	5.94	0.965	0.399	39.37	31.84	72.01
9	0.15	1.60	3.01	4.31	3.95	5.14	6.76	7.39	8.41	9.53	0.95	5.70	0.972	0.435	38.04	36.96	74.34
0	0.40	1.44	2.49	4.91	5.06	5.70	7.14	7.59	8.23	9.05	0.94	5.88	0.962	0.435	30.88	37.02	60.91
1	-1.04	-0.36	1.63	1.87	3.10	4.38	5.08	6.68	7.34	8.31	0.96	4.31	1.074	0.455	44.03	40.09	88.90
2	-0.67	1.70	2.66	2.90	3.34	5.69	5.97	7.32	7.28	9.22	0.96	4.86	1.037	0.469	52.64	42.33	108.49
3	-0.73	0.69	1.67	2.98	4.09	5.46	6.06	7.12	8.18	9.34	0.94	4.89	1.029	0.411	33.50	33.49	63.26
4	1.74	0.08	4.33	5.68	7.15	7.19	9.04	10.04	11.06	11.96	0.83	8.31	0.777	0.198	12.71	11.82	17.72
5	-0.35	-0.10	2.15	2.79	5.08	4.52	5.55	7.08	7.56	9.08	0.95	5.06	1.035	0.436	35.44	37.16	69.62
6	-0.44	1.55	2.66	2.78	4.07	4.93	6.53	8.54	8.41	9.24	0.95	5.43	0.998	0.405	37.73	32.66	69.85
7	2.28	3.50	4.64	5.01	6.46	7.44	8.47	9.36	10.09	11.42	0.95	7.38	0.831	0.383	46.68	30.02	82.93
8	2.97	4.02	4.76	4.84	6.90	8.53	9.26	10.90	11.16	11.50	0.83	7.92	1.038	0.284	10.90	19.04	17.59
9	-0.19	0.21	2.03	2.49	3.58	5.35	6.19	7.12	7.72	8.92	0.99	5.08	0.993	0.438	386.17	37.37	70.13
0	0.06	1.38	2.12	2.62	4.83	5.34	7.06	7.79	8.74	9.72	0.96	5.56	0.892	0.404	61.49	32.48	112.05
1	2.12	4.33	4.85	5.73	10.81	7.23	8.59	9.79	11.23	11.03	0.94	7.45	0.953	0.361	36.18	27.12	62.23
2	0.53	1.40	1.87	3.40	5.77	5.72	7.75	8.77	9.05	9.76	0.93	6.20	0.935	0.360	32.12	26.99	55.37
3	1.22	3.07	3.88	4.81	5.01	6.32	7.64	8.62	9.90	10.40	0.94	6.49	0.996	0.410	33.62	33.39	63.11
4	-0.03	2.04	2.04	4.13	4.63	5.85	7.07	7.57	9.38	9.80	0.94	5.52	0.979	0.402	35.18	32.32	65.00
5	-0.23	0.88	2.78	3.45	4.76	5.39	6.90	6.98	7.94	9.72	0.95	5.57	0.988	0.417	44.16	34.28	83.03
6	-0.38	0.57	2.61	3.43	5.33	5.62	7.05	7.51	8.08	9.76	0.94	5.72	1.068	0.392	35.10	30.91	63.66
7	-0.46	0.45	0.89	3.45	3.56	4.88	5.50	7.08	7.21	8.26	0.95	4.66	0.968	0.474	34.14	43.18	72.16
8	0.43	2.11	3.81	4.34	5.81	5.69	7.32	7.21	9.44	9.97	0.94	5.97	0.986	0.423	29.97	35.20	57.85
9	0.36	1.71	3.10	4.32	4.53	6.00	6.50	7.10	9.36	9.89	0.96	5.53	0.946	0.434	52.66	36.04	101.47
0	1.40	2.24	3.24	4.69	6.01	5.28	7.50	7.73	9.11	10.05	0.92	6.42	1.134	0.427	22.93	35.79	45.22
1	-1.62	-0.71	0.01	0.96	2.31	3.06	4.43	5.24	7.61	7.71	0.96	3.32	1.027	0.470	54.77	42.25	113.04
2	0.16	0.78	2.42	2.75	4.29	5.79	5.09	7.30	8.16	9.69	0.93	4.91	0.995	0.429	27.13	36.08	53.21
3	-0.47	0.91	2.50	3.58	4.21	4.94	6.35	7.94	8.64	9.21	0.95	5.41	1.023	0.406	40.42	32.82	74.82
4	0.39	1.910	2.02	3.94	4.22	5.35	5.68	7.22	8.61	9.38	0.94	5.04	0.922	0.462	29.68	41.16	61.68
5	.80	2.81	3.71	4.29	5.06	7.33	8.31	8.69	8.80	11.30	0.90	6.71	0.981	0.352	19.30	26.06	33.60
6	.08	1.72	2.80	3.64	4.18	5.26	6.77	7.81	8.70	9.73	0.96	5.65	0.930	0.420	45.28	34.71	85.55
7	-.13	1.44	3.31	3.73	5.11	6.30	7.64	8.54	8.96	10.48	0.99	6.23	0.941	0.349	1007.22	25.74	1647.67

8	.76	2.79	3.07	4.70	4.96	6.29	7.16	8.34	9.49	10.16	0.99	6.04	0.896	0.405	917.07	32.70	1642.52
9	2.07	2.93	3.29	5.03	6.35	6.92	9.00	8.52	9.98	10.58	0.99	7.15	0.922	0.371	611.47	28.27	1035.55
0	1.49	2.06	3.32	3.83	4.85	4.29	6.41	8.51	8.89	10.04	0.99	6.23	1.022	0.382	402.88	29.63	695.23
1	.39	1.68	1.91	3.41	3.68	4.34	6.66	6.96	7.52	8.86	0.99	5.34	0.954	0.485	910.64	45.27	1855.06
2	1.61	2.91	2.78	4.26	5.22	7.08	7.37	8.25	8.88	10.39	0.99	6.21	0.958	0.430	879.27	36.16	1642.25
3	0.82	1.71	3.39	4.11	5.00	6.45	7.27	8.40	9.42	10.13	0.99	6.22	0.982	0.396	940.25	31.53	1659.63
4	0.48	1.09	2.63	3.78	4.36	5.11	6.30	7.91	9.34	9.90	0.99	5.72	0.890	0.404	1298.55	32.59	2321.80
5	2.36	3.94	4.99	5.73	5.85	7.36	8.12	7.54	9.96	11.11	0.99	6.89	0.954	0.430	620.32	37.76	1154.02
6	1.02	2.36	3.17	3.85	4.68	5.46	6.93	8.09	8.88	9.98	0.99,	6.01	0.976	0.430	903.21	36.17	1687.55
7	0.37	1.92	2.94	4.39	5.58	5.83	6.95	7.80	8.86	9.95	0.99	5.38	1.059	0.489	1026.58	35.39	1899.19
8	- 0.70	1.20	2.33	4.39	3.43	4.99	5.36	7.15	7.28	8.63	0.99	4.82	1.025	0.458	911.63	45.92	1900.81
9	- 0.02	1.34	2.10	3.41	4.57	5.40	5.94	7.52	8.02	8.65	0.99	5.04	0.894	0.375	727.45	40.86	1430.96
0	1.66	2.60	4.19	5.57	6.17	7.28	7.08	9.33	10.35	10.90	0.99	6.80	0.983	0.416	1045.49	28.82	1781.62
1	0.42	1.40	2.08	2.65	4.82	6.12	7.29	7.62	8.28	9.39	0.99	5.58	1.020	0.466	1212.47	34.20	2210.93
2	0.21	2.14	3.00	2.61	5.06	5.38	6.29	7.33	8.12	9.85	0.99	5.27	1.034	0.455	1543.18	41.81	3075.17
3	0.04	1.55	2.10	2.98	3.77	5.40	6.17	6.78	8.16	9.96	0.99	5.04	0.994	0.424	1228.76	40.1009	2401.39
4	0.44	1.87	2.57	3.58	4.30	4.88	5.94	8.40	8.72	9.68	0.99	5.54	0.958	0.366	832.21	35.3659	1539.80
5	0.01	1.07	1.90	3.90	4.95	5.57	7.82	8.41	8.60	9.35	0.99	6.09	1.056	0.442	718.68	27.71	1207.77
6	- 039	1.49	2.14	2.81	3.96	6.10	5.93	7.74	8.88	8.11	0.99	4.78	0.964	0.395	1044.12	38.01	1993.04
7	0.37	1.33	3.46	3.02	3.97	6.07	7.28	7.79	9.11	9.44	0.99	5.79	0.957	0.412	794.13	31.31	1397.57
8	0.69	2.24	2.57	4.12	4.54	6.00	7.01	8.05	9.22	9.93	0.99	5.85	1.006	0.411	1151.65	33.58	2084.97
9	- 0.50	2.04	2.58	3.62	4.97	5.68	6.52	8.19	8.52	9.59	0.99	5.37	1.018	0.432	712.67	33.56	1290.63
0	- 056	2.21	2.47	3.71	4.51	5.34	6.58	6.81	8.15	10.28	0.99	5.21	0.896	0.408	1048.20	36.53	1966.12
1	2.59	3.39	4.65	5.26	6.59	7.42	8.13	8.84	10.05	11.15	0.99	7.24	0.989	0.394	709.32	33.14	1278.00
2	0.08	1.13	2.42	3.66	3.84	5.26	6.66	8.05	8.78	9.72	0.99	5.70	0.994	0.456	752.81	31.16	1322.67
3	1.32	2.48	2.71	3.54	4.43	6.18	7.08	7.45	9.19	9.62	0.99	5.72	1.060	0.492	898.36	40.19	1758.60
4	- 042	1.08	2.31	2.56	3.78	3.82	5.55	6.49	7.63	8.49	0.99	4.63	0.980	0.464	799.42	46.51	1677.26
5	1.26	1.93	3.13	3.72	4.57	4.98	6.57	7.87	8.36	9.61	0.99	5.94	1.025	0.444	1016.08	41.53	2018.73
6	0.95	2.01	3.56	3.51	4.50	6.25	6.42	7.28	8.75	10.22	0.99	5.66	1.057	0.493	944.81	38.27	1808.07
7	0.51	1.77	2.30	3.60	4.28	5.99	5.89	7.21	7.88	8.77	0.99	5.04	1.046	0.461	794.29	46.62	1668.99
8	- 0.38	0.58	1.71	3.97	4.24	4.71	5.10	6.75	7.65	9.31	0.99	4.36	0.988	0.491	1338.67	42.15	1478.54
9	0.16	0.86	2.33	3.42	4.57	4.80	5.35	6.35	7.71	9.22	0.99	4.90	0.953	0.415	982.59	46.30	2057.04
0	0.45	2.14	2.60	3.79	3.74	5.62	6.45	7.78	8.96	10.19	0.99	5.58	0.987	0.416	909.00	34.10	1636.47
1	1.06	3.22	3.20	3.77	5.38	5.99	7.20	8.23	9.10	11.05	0.99	6.15	1.009	0.417	1411.10	34.15	2571.31
2	0.58	3.20	3.14	3.27	6.08	6.42	6.74	8.74	10.03	9.83	0.99	5.60	0.991	0.447	1192.64	34.27	2176.83
3	1.11	1.00	2.80	2.83	4.29	4.94	6.33	7.56	8.36	8.85	0.99	5.33	0.980	0.419	1053.37	38.87	2030.20
4	1.11	4.06	3.89	5.53	5.73	7.21	7.50	8.58	9.76	10.71	0.99	6.35	0.923	0.389	908.22	34.64	1665.95
5	0.15	-0.15	1.28	3.04	4.20	4.97	6.61	7.13	7.97	9.60	0.99	5.52	0.884	0.381	961.03	30.52	1674.05
6	0.26	1.31	2.54	3.85	5.28	6.04	6.61	7.67	8.90	11.09	0.99	5.85	0.919	0.421	1228.06	29.57	2113.20
7	0.89	1.97	3.26	3.20	4.39	6.40	6.70	8.38	9.17	9.62	0.99	5.76	1.037	0.396	745.38	34.89	1371.93
8	1.15	3.10	3.41	4.31	5.44	5.67	7.50	8.80	9.65	10.66	0.92	6.46	1.037	0.335	22.77	31.46	2143.37
9	1.37	1.84	3.54	4.54	5.42	6.25	8.39	8.99	10.75	10.99	0.99	7.08	0.969	0.355	1944.46	24.18	3112.78
0	0.61	2.02	2.26	5.33	5.50	6.51	7.66	8.51	9.36	10.85	0.99	6.46	0.937	0.450	1158.16	26.38	1911.21
1	- 0.25	0.46	2.99	2.98	4.25	5.31	6.18	6.84	8.58	8.73	0.99	5.08	0.877	0.408	1008.27	39.36	1954.11
2	- 0.83	0.28	1.79	2.86	3.88	4.49	5.82	7.41	8.83	8.96	0.99	4.85	1.046	0.413	1202.83	33.13	2165.37
3	1.07	2.05	3.40	4.13	4.45	6.39	6.84	8.82	9.82	9.39	0.99	6.02	0.948	0.348	823.85	33.86	1496.92
4	0.88	2.36	3.25	4.11	5.01	6.54	7.42	8.41	10.42	11.42	0.99	6.35	0.991	0.389	922.38	25.67	1507.86
5	1.84	3.09	4.94	4.72	5.62	6.27	7.87	8.82	8.73	11.75	0.99	7.23	0.992	0.439	1031.78	30.50	1796.89
6	- 035	-0.46	0.90	1.60	3.37	3.82	5.75	6.24	7.05	8.61	0.99	4.64	0.992	0.428	859.48	37.53	1632.13
7	1.31	1.90	2.90	5.17	5.57	6.22	6.75	7.90	8.72	10.57	0.99	6.28	1.005	0.427	1408.48	35.95	2621.98
8	0.03	1.62	2.83	4.23	4.63	5.06	6.48	7.69	8.22	9.62	0.99	5.64	0.983	0.419	601.30	35.79	1198.48
9	0.73	1.97	2.87	3.72	4.34	5.83	6.95	8.25	8.40	9.47	0.99	5.87	0.004	0.002	256885.2	38.95	2011.78
00	0.24	0.80	2.48	3.66	3.62	5.04	6.16	7.72	7.78	8.68	0.99	5.30	0.32	0.003	324.73	23.65	1123.47