Sensitivity of Estimators to Three Levels of Correlation between Error Terms.

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ABSTRACT

A Monte Carlo simulation is employed to investigate the sensitivity of simultaneous equation techniques under different levels of correlation between random deviates. Three arbitrary levels of correlation between pairs of random deviates were assumed. Three small sample sizes were used in this experiment, N = 15, N=25 and N = 40 each replicated 100 times. A number of factors should be taken into account in choosing an estimation method. Although system methods are asymptotically most efficient in the absence of violation of mutual independence of random errors, system methods are more sensitive to any kind of error than single equation methods.

In practice, models are never perfectly specified nor are they completely free of correlated random deviates. It is a matter of judgment whether the correlation is strong enough to warrant avoidance of system methods.

As sample size increases, the TAB for all the estimators decreased consistently except for FIML. OLS, 2SLS, LIML and FIML are remarkably insensitive to the choice of triangular matrices (P1 and P2) when using TAB to judge their performances. Best RSS estimates of 2SLS, LIML, and 3SLS are found in the feebly correlated region.

(Keywords: limited-information estimators, full-information estimators, sensitivity, finite-sample, Monte Carlo experiment, correlation coefficient)

INTRODUCTION

Beginning with the method developed by [15] for solving the problem of single equation bias, econometricians have devoted considerable effort to developing additional methods for estimating the structural parameters of simultaneous equation models [28], [16], [25], and [27]. While it has been fairly easy to develop the asymptotic properties of these estimators, a distinguishing characteristic of econometric models is that they are invariably based upon small samples of data and thus, the asymptotic properties of the various estimators are not necessarily the best guide in selecting the appropriate estimating procedure. One approach to this problem has been the derivation of the exact finite-sample properties of some estimators by [30], [31], [32], and [19].

Relatively little is known about the finite sample distributions of the various estimators. The exact finite sample distributions of limited-information maximum likelihood estimates and two-stage least squares estimates have been derived by Basmann in certain special cases ([30] and [32]). He found that these distributions do not always possess finite moments of low order; in certain cases even the mean does not exist. An alternative approach to uncovering the small sample properties of various structural equation estimators has been to conduct sampling experiments with the aid of more or less artificial models. The most notable among these have been studies by [11], [19], and [29]. Several small models are examined in these studies from various points of view; the general conclusions emerging from them are excellently summarized by [9].

Another approach, which is generally applicable to all estimators, has been to conduct sampling experiments with different simultaneous equation models using small samples of data which have been artificially generated [34], [30], [6], [33], [20], [23], [10], and [11]. More recent work has been done by ([1]; [2]; [4]), [5], [8], and [7]. The net result of all of these studies has been to show that there exist no clear guide-lines for the choice of an estimator for econometric models.

The general consensus of opinion, however, is that, thus far, two-stage least squares is the cheapest, easiest, and most efficient estimator in most situations [24]. A different approach to the simultaneous equation bias problem is the full information maximum likelihood (FIML) estimation method [3].

It has been shown by [13] that the full-information maximum likelihood method of estimating the coefficients of structural equations is a generalization of the least squares principles. These estimates are consistent and efficient. Nevertheless, the properties of other types of estimator continue to be of interest because of the computational difficulty of obtaining full-information estimates ([12] and [14]).

Noteworthy among alternative methods are limited-information maximum likelihood, indirect least squares, two-stage least squares, direct least squares (the last two being special cases of the general *k-class* of estimators), three-stage least squares, linearized and several others ([17], [18], [21], and [26]). With the exception of direct least squares these methods also possess the properties of consistency although they yield biased estimates in finite samples [22].

Compared to the instrumental variables methods (2SLS and 3SLS), the FIML method has these advantages and disadvantages:

1. FIML does not require instrumental variables.

2. FIML requires that the model include the full equation system, with as many equations as there are endogenous variables. With 2SLS or 3SLS you can estimate some of the equations without specifying the complete system.

3. FIML assumes that the equations errors have a multivariate normal distribution. If the errors are not normally distributed, the FIML method may produce poor results. 2SLS and 3SLS do not assume a specific distribution for the errors.

4. The FIML method is computationally expensive.

The random deviates on which the selection of error terms in Monte Carlo studies is based are usually assumed to be pair wise uncorrelated. This is not always true although the correlation coefficients are usually small. Since random deviates will lose the quality of randomness if they are forced to be orthogonal, the objective of this paper is focused on investigating the sensitivity of estimators of a two-equation model in the presence of three levels of unintended correlation between pairs of normal deviates used in the Monte Carlo experiment.

THE MODEL

Numerous methods have been developed for estimating the coefficients of a system of simultaneous linear structural equation of the form:

$$By + \Gamma z = u \quad \textbf{(1)}$$

It is assumed that *z* is a vector of exogenous variables (assumed to be identical in repeated samples and not to contain lagged values of endogenous variables), *u* is a vector of jointly normally distributed error terms with mean zero and covariance matrix Σ, *y* is a vector of endogenous variables, and B (nonsingular) and Γ are matrices of coefficients.

Assume the following two-equation model:

$$Y_{1t} = \beta_{12} Y_{2t} + \gamma_{11} X_{1t} + U_{1t}$$

$$Y_{2t} = \beta_{21} Y_{1t} + \gamma_{22} X_{2t} + \gamma_{23} X_{3t} + U_{2t} \quad \textbf{(2)}$$

where the Y's are the endogenous variables, X's are the predetermined variables and U's are the random disturbance terms, $\beta' s$ and $\gamma' s$ are the parameters.

The first equation is over-identified while the second equation is a just identified equation. The error terms were not independent ([2] and [4]).

The reduced form equation of the above equation (2) is given as:

$$By = \Gamma x + u \quad \textbf{(3)}$$

$$y = B^{-1}\Gamma x + B^{-1}u$$
$$= \Pi x + v$$

where,

$$\Pi = -B^{-1}\Gamma$$

$$= \frac{1}{1-\beta_{12}\beta_{21}}\begin{bmatrix} \gamma_{11} & \beta_{21}\gamma_{22} & \beta_{21}\gamma_{23} \\ \beta_{12}\gamma_{11} & \gamma_{22} & \gamma_{23} \end{bmatrix}$$

$$B^{-1}\Gamma x = \frac{1}{1-\beta_{12}\beta_{21}}\begin{bmatrix} \gamma_{11} & \beta_{21}\gamma_{22} & \beta_{21}\gamma_{23} \\ \beta_{12}\gamma_{11} & \gamma_{22} & \gamma_{23} \end{bmatrix}\begin{bmatrix} X_{1t} \\ X_{2t} \\ X_{3t} \end{bmatrix}$$

$$= \frac{1}{1-\beta_{12}\beta_{21}}\begin{bmatrix} \gamma_{11}X_{1t} + \beta_{21}\gamma_{22}X_{2t} + \beta_{21}\gamma_{23}X_{3t} \\ \beta_{12}\gamma_{11}X_{1t} + \gamma_{22}X_{2t} + \gamma_{23}X_{3t} \end{bmatrix}$$

But, $v = B^{-1}u$

$$= \frac{1}{1-\beta_{12}\beta_{21}}\begin{bmatrix} 1 & \beta_{21} \\ \beta_{12} & 1 \end{bmatrix}\begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

$$= \frac{1}{1-\beta_{12}\beta_{21}}\begin{bmatrix} u_{1t} + \beta_{21}u_{2t} \\ \beta_{12}u_{1t} + u_{2t} \end{bmatrix}$$

$$y = \frac{1}{1-\beta_{12}\beta_{21}}\begin{bmatrix} \gamma_{11}X_{1t} + \beta_{21}\gamma_{22}X_{2t} + \beta_{21}\gamma_{23}X_{3t} \\ \beta_{12}\gamma_{11}X_{1t} + \gamma_{22}X_{2t} + \gamma_{23}X_{3t} \end{bmatrix} + \frac{1}{1-\beta_{12}\beta_{21}}\begin{bmatrix} u_{1t} & \beta_{21}u_{2t} \\ \beta_{12}u_{1t} & u_{2t} \end{bmatrix}$$

This can be written as:

$$y_{1t} = \frac{\gamma_{11}}{1-\beta_{12}\beta_{21}}X_{1t} + \frac{\beta_{21}\gamma_{22}}{1-\beta_{12}\beta_{21}}X_{2t} + \frac{\beta_{21}\gamma_{23}}{1-\beta_{12}\beta_{21}}X_{3t} + \frac{1}{1-\beta_{12}\beta_{21}}u_{1t} + \frac{1}{1-\beta_{12}\beta_{21}}u_{2t} \quad \textbf{(4)}$$

$$y_{2t} = \frac{\beta_{12}\gamma_{11}}{1-\beta_{12}\beta_{21}}X_{1t} + \frac{\gamma_{22}}{1-\beta_{12}\beta_{21}}X_{2t} + \frac{\gamma_{23}}{1-\beta_{12}\beta_{21}}X_{3t} + \frac{\beta_{12}}{1-\beta_{12}\beta_{21}}u_{1t} + \frac{1}{1-\beta_{12}\beta_{21}}u_{2t} \quad \textbf{(5)}$$

The reduced form of Equations (4) and (5) are:

$$y_{1t} = \Pi_{11}X_{1t} + \Pi_{12}X_{2t} + \Pi_{13}X_{3t} + V_{1t} \quad \textbf{(6)}$$

$$y_{2t} = \Pi_{21}X_{1t} + \Pi_{22}X_{2t} + \Pi_{23}X_{3t} + V_{2t} \quad \textbf{(7)}$$

DESIGN OF EXPERIMENTS

Three arbitrary levels of correlation between pairs of random deviates are assumed. These three scenarios of correlation are then used to generate pairs of normal deviates of sizes N = 15, 25, and 40, with 100 replications. Each set of normal deviates with the different sample sizes are then transformed using the upper (P_1) triangular matrix. The procedure was repeated using the lower triangular matrix (P_1'), such that in each case, $\Omega = P_1P_1'$.

To generate the data, the structural Equations (2) were transformed to the reduced form, error terms for sample sizes of fifteen, twenty-five and forty were produced by a random normal deviate generator and values for the endogenous variables were calculated. For each sample size, hundred sets of data were generated, with the vectors of exogenous variables remaining the same for each set of data.

Five estimators are used in this experiment; they are Ordinary Least Squares (OLS), Two Stage Least Squares (2SLS), Limited Information Maximum Likelihood (LIML), Three Stage Least Squares (3SLS) and Full Information Maximum Likelihood (FIML).

In assessing the performance for the various estimators, an examination of the means and standard deviations of the estimates of structural parameters was made and from this some summary statistics were prepared. These permitted evaluations on the basis of two criteria, smallest bias and smallest standard deviation.

A combined or scalar measure of these two criteria could be Root Mean Square Error (MSE) or Mean Absolute Error (MAE). One investigator has stated that on a priori grounds it is hard to choose between these measures [10, p12]; therefore, a summary statistics using two measures; total absolute bias and sum of squared residuals are included for this study.

SIMULATION RESULTS

Table 1: Summary of Total Absolute Bias R=100, P_1

Level of correlation	OLS			2SLS			LIML		
	N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25	N=40
r<-0.05	4.967447	4.948403	4.874522	4.902149	3.897816	3.881116	4.384517	4.600761	3.574813
-0.05<r<0.05	4.884578	4.88579	4.733118	4.635532	3.492337	3.616084	3.393374	3.043991	2.933429
r>0.05	4.84921	4.828668	4.423479	5.10576	4.186388	3.698753	4.947764	3.40555	3.146334

Level of correlation	3SLS			FIML		
	N=15	N=25	N=40	N=15	N=25	N=40
r<-0.05	3.996025	2.280115	2.760661	11.514893	23.234947	9.408441
-0.05<r<0.05	4.027558	2.899392	2.803212	12.582233	16.593795	12.561232
r>0.05	4.996303	4.00182	3.257218	14.484919	11.052439	9.298833

Table 2: Summary of Total Absolute Bias R=100, P_2

Level of correlation	OLS			2SLS			LIML		
	N=15	N=25	N=40	N=15	N=25	N=40	N=15	N=25	N=40
r<-0.05	4.888746	4.890336	5.044038	4.096107	4.339463	4.412196	3.785076	4.293223	4.867604
-0.05<r<0.05	4.85784	4.865581	5.015919	4.715642	3.671604	3.555401	5.078825	3.403852	2.982053
r>0.05	4.851891	4.877528	4.933268	3.947009	4.117722	3.6645	4.066545	3.736673	3.103178

Level of correlation	3SLS			FIML		
	N=15	N=25	N=40	N=15	N=25	N=40
r<-0.05	2.761579	3.095991	4.088159	15.690135	18.081122	11.662543
-0.05<r<0.05	3.142008	1.725094	2.910107	21.060479	27.288149	11.39745
r>0.05	4.554659	3.070647	3.73892	23.081417	11.108718	9.666032

Tables 1 and 2 contain summaries of the performance of estimators using total absolute bias (TAB) of estimates. To reduce the dimension of the results displayed in Tables 1 and 2, the total absolute biases are summed across correlation levels for each estimator; this will facilitate a study of the asymptotic behavior of TAB for each estimator, computation of the average bias for each estimator and its dispersion over sample sizes, all of which will also help in ranking the estimators under P_1 and P_2 in increasing order of average of total absolute bias. Tables 1 and 2 are used to generate Table 3.

The entries in the rows of Table 3 for P_1 show that the sums of total absolute bias decrease as the sample size increases for OLS, 2SLS, LIML, and 3SLS, the sums do not reveal any such asymptotic behavior for FIML where the sample size 25 appears to be a turning point (maximum bias for FIML). For P_2 the row entries reveal asymptotic behavior for 2SLS, LIML and FIML while 3SLS has sample size 25 as a convex turning point and the sums increase as the sample size increases for OLS.

This result shows that estimates of absolute bias are sensitive to changes in the sample sizes. It is also of interest to rank the estimators on the basis of the magnitude of total absolute bias and to examine the dispersion of the estimates using the coefficient of variation. These averages and the coefficients of variation of the 3 estimates for each estimator are displayed in table 4 for P_1 and P_2.

Using the Average Total Absolute Bias (ATAB) and its Coefficient of Variation (CV) presented in Table 4, the five estimators are ranked as shown in Table 5 in increasing order of bias and coefficient of variation under P_1 and P_2.

It is noteworthy in respect of average absolute bias that the five estimators rank uniformly under P_1 and P_2. This finding clearly shows that the ranking of the estimators in terms of the magnitude of the average total absolute bias is invariant to the choice of the upper (P_1) or lower (P_2) triangular matrix.

It is also remarkable that whereas the average absolute biases of the other four estimators range between 9 and 15, those of FIML maintain a very distant fifth position with 40 and about 50 for P_1 and P_2, respectively.

The poor ranking of FIML in this situation of correlated disturbances and over-identified equation may be attributed to the fact that it uses more information as an estimator than any of the other four estimators. The only remarkable uniformity in the ranking of estimators on the dispersion of the total absolute bias is the fact that the 3SLS and FIML are in the fourth and fifth positions, respectively, under P_1 and P_2.

Finally, a decision on the best estimator for this model cannot be taken on the basis of our findings on total absolute bias alone. This is because the yardstick is the total absolute bias of two equations, which differ in their identifiability status. In estimating multi-equation models, the choice of estimator is equation specific. Hence, the findings here will have to be reconciled with findings elsewhere before a prescription of best estimator of each equation can be suggested.

To further study the asymptotic behavior as well as the sensitivity of each estimator to changes in TAB of estimates over replication, Tables 1 and 2 are used to chart the behavior of estimators over correlation coefficients and sample sizes and these are presented in Table 6 for both P_1 and P_2, respectively.

Table 3: Sums of Total Absolute Bias over Correlation Levels, Replication Numbers or Sample Sizes.

Triangular Matrix	**Replications**	**OLS**			**2SLS**			**LIML**			**3SLS**			**FIML**		
		SAMPLE SIZES														
		15	**25**	**40**	**15**	**25**	**40**	**15**	**25**	**40**	**15**	**25**	**40**	**15**	**25**	**40**
P_1	**100**	14.70	14.66	14.03	14.64	11.58	11.19	12.72	11.05	9.65	13.02	9.18	8.82	38.58	50.88	31.27
P_2	**100**	14.60	14.63	14.99	12.76	12.13	11.63	12.93	11.43	10.95	10.46	7.89	10.74	59.83	56.48	32.73

Table 4: Average Total Absolute Bias and their Coefficient of Variation (P_1 and P_2).

Triangular Matrix		**OLS**	**2SLS**	**LIML**	**3SLS**	**FIML**
P_1	Mean	14.46	12.47	11.14	10.34	40.24
	C.V	0.0260	0.1515	0.1380	0.2251	0.2463
P_2	Mean	14.74	12.17	11.77	9.70	49.68
	C.V	0.0147	0.0465	0.0878	0.1620	0.2974

Table 5: Ranking of Estimators under P_1 and P_2 on ATAB and CV.

ATAB		**CV**	
P_1	**P_2**	**P_1**	**P_2**
3SLS	3SLS	OLS	OLS
LIML	LIML	2SLS	2SLS
2SLS	2SLS	LIML	LIML
OLS	OLS	3SLS	3SLS
FIML	FIML	FIML	FIML

Table 6: Trends of Total Absolute Bias as Error Correlation changes from High Negative through Small (negative and positive) to High Positive Values, R=100.

Estimator	P_1			P_2		
	Sample size (N)			Sample size (N)		
	15	25	40	15	25	40
OLS	\	\	\	\	\	\
2SLS	V	V	V	Λ	V	V
LIML	V	V	V	Λ	V	V
3SLS	/	/	/	/	V	V
FIML	/	Λ	\	/	Λ	\

The entries show that under P_1, for OLS, the model absolute bias decreased consistently as correlation changes over the three ranges rose consistently for 3SLS and attained a minimum (**V**) as correlation changes from high negative value through low negative or positive values to high positive values for 2SLS and LIML. The behavior is inconclusive FIML. Under P_2, the findings are generally less conclusive, however, model absolute bias is downward sloping for OLS (similar to behavior in P_1) and has a convex behavior with the turning point at the middle

These tables are arranged to facilitate the study of the asymptotic distribution of the sum of squared residuals. They reveal changes in the estimates of RSS as N increases at different levels of error correlation.

For OLS, LIML and FIML, the RSS obtained in equation two, the just identified equation, are smaller at all levels of error correlation than those obtained in equation one, the over-identified equation. For 2SLS and 3SLS, the estimates obtained in equation one are smaller than those obtained in two.

An overview of these tables reveals that RSS for OLS follow a consistent pattern column-wise, i.e. for the two equations and at all levels of correlation coefficient, RSS increase as sample size increases for both P_1 and P_2.

As expected the RSS displayed in these tables (7 and 9) for P_1 and P_2 are fairly uniform row-wise for all estimators except the FIML where estimated RSS vary sample sizes. Also the RSS for FIML are remarkably higher than for the other four estimators.

As before, to gain some insight into the behavior of the estimated RSS as correlation of the error

interval for 2SLS, LIML and 3SLS at N=25 and N=40, respectively.

Theoretically, one expects the "**V**" trend to be the most frequent since that would imply that total absolute bias is a minimum when correlation of the error term is smallest (negative or positive). This is reflected to a large extent by estimates of 2SLS and LIML based on P_1 and 2SLS, LIML, and 3SLS based on P_2. The sum of squared residuals of each equation for all the five estimators are displayed in Tables 7 and 9.

term changes from $r<-0.05$, through $-0.05<r<0.05$ to $r>0.05$; the relevant charts are displayed in Tables 8 and 10 (using the results of the sum of squared residuals of estimates displayed in Tables 7 and 9) for the three sample sizes given 100 replications for P_1 and P_2. For example, in Table 7 for EQ1, N=15 RSS fell from 8.469256 to 7.697955 and fell further to 7. 222594 across the three levels of correlation coefficient, this is represented by the trend "****".

At N=25, for the same equation RSS maintained the downward trend "****". This is repeated for each parameter to obtain the different trends shown in Tables 8 and 10.

In Tables 8 and 10 for the two equations under P_1, the downward sloping trend is most frequent for OLS, which implies that, the RSS decrease consistently as correlation coefficient changes from highly negative, through feeble to highly positive range. For FIML, identical results are obtained for the two equations and triangular matrices.

It is also worth mentioning that, the trends under 2SLS, LIML and 3SLS are similar for the two equations when both P_1 and P_2 are considered.

Table 7: Summary of Sum of Squared Residuals for Three Correlation Levels R=100, P_1.

Estimator	Level of correlation	EQ1			EQ2		
		N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	8.469256	14.53042	23.15642	5.633654	9.655884	16.66429
	-0.05<r<0.05	7.697955	13.86687	21.90421	5.196804	9.082502	15.82505
	r>0.05	7.222594	11.99807	18.88565	4.765228	7.69349	14.32214
2SLS	r<-0.05	52.04317	90.07682	109.1199	45.97432	115.9413	201.2111
	-0.05<r<0.05	26.93059	81.35832	140.1711	51.94936	76.6478	115.6186
	r>0.05	37.66998	85.28595	99.51579	73.40602	192.3063	151.4921
LIML	r<-0.05	184.874	1105.567	458.6458	45.97432	115.9413	201.2111
	-0.05<r<0.05	134.3037	483.5175	758.5876	51.94936	76.6478	115.6186
	r>0.05	1121.227	1104.505	542.7323	73.40602	192.3063	151.4921
3SLS	r<-0.05	52.04317	90.07682	109.1199	83.76712	909.1305	600.1468
	-0.05<r<0.05	26.93059	81.35832	140.1711	276.3245	106.6174	248.7424
	r>0.05	37.66998	85.28595	99.51579	485.2463	458.2371	2334.198
FIML	r<-0.05	1399.557	17482.48	1659.51	882.1201	11472.43	841.4681
	-0.05<r<0.05	3899.056	11258.74	8061.006	3494.945	7905.957	5532.736
	r>0.05	5747.848	3371.886	2351.284	4886.049	2625.551	1729.908

Table 8: Charts of the Behavior of RSS of Estimators over Correlation Coefficients for each Sample Size R=100; P_1.

Estimator	EQ1			EQ2		
	Sample size			Sample size		
	15	25	40	15	25	40
OLS	\	\	\	\	\	\
2SLS	V	V	Λ	/	V	V
LIML	V	V	Λ	/	V	V
3SLS	V	V	Λ	/	V	V
FIML	/	\	Λ	/	\	Λ

Table 9: Summary of Sum of Squared Residuals for Three Correlation Levels R=100, P_2.

Estimator	Level of correlation	EQ1			EQ2		
		N=15	N=25	N=40	N=15	N=25	N=40
OLS	r<-0.05	8.025363	14.39986	22.30994	5.93094	10.02278	17.42814
	-0.05<r<0.05	7.672212	13.98346	22.12155	5.257079	9.217827	16.65403
	r>0.05	7.518787	11.88107	19.20711	4.756709	7.553808	13.31428
2SLS	r<-0.05	61.16581	69.22513	268.809	88.53055	231.2958	125.553
	-0.05<r<0.05	36.15682	84.48975	108.098	195.3029	105.2122	107.1845
	r>0.05	94.27346	108.824	112.8809	226.2144	334.4002	221.0916
LIML	r<-0.05	247.6202	349.9958	10355.51	88.53055	231.2958	125.553
	-0.05<r<0.05	1165.518	417.5886	361.349	195.3029	105.2122	107.1845
	r>0.05	2263.259	604.6652	436.645	226.2144	334.4002	221.0916
3SLS	r<-0.05	61.16581	69.22513	268.809	632.2091	1106.055	182.4555
	-0.05<r<0.05	36.15682	84.48975	108.098	274.0143	259.8756	127.3117
	r>0.05	94.27346	108.824	112.8809	3144.892	1141.369	402.9815
FIML	r<-0.05	3810.179	13611.22	5834.852	2999.988	9414.314	3631.625
	-0.05<r<0.05	16084.06	37128.56	4677.572	12926.5	28985.17	2890.633
	r>0.05	20454.36	2092.687	3030.157	15550.94	1522.433	2260.823

Table 10: Charts of the Behavior of RSS of Estimators over Correlation Coefficients for each Sample Size R=100; P_2.

Estimator	EQ1			EQ2		
	Sample size			Sample size		
	15	25	40	15	25	40
OLS	\	\	\	\	\	\
2SLS	V	V	\	V	V	V
LIML	V	V	\	V	V	V
3SLS	V	V	\	V	V	V
FIML	/	Λ	\	/	Λ	\

Table 11: Summary of Frequencies of Correlation-based Charts of Behavior of TAB and RSS.

Attribute	Table	\		/		Λ		V	
		P_1	P_2	P_1	P_2	P_1	P_2	P_1	P_2
TAB	6	27	27	27	13	7	20	40	40
RSS	8 &10	27	37	17	7	17	7	40	50

The most popular chart in respect of the two equations under both P_1 and P_2 is '**V**' followed by the downward trend "****". These tables also reveal that the results obtained for OLS under P_1 are similar to those obtained under P_2.

On the behavior of RSS as correlation coefficient changes through the three cardinal levels, OLS estimator shows the most stable pattern of declining RSS i.e. the downward sloping ("****") trend (6/6 for both equations P_1 and P_2). The 2SLS, LIML and 3SLS estimators also have a concave ("**V**") trend predominantly for sample sizes 15 and 25 for equation 1, 25 and 40 for equation 2 under P_1. This pattern is repeated for these estimators under P_2 except for sample size 40 of the first equation.

The frequencies of the four trends (****, /, Λ and **V**) are relatively more uniform under P_1 than under P_2. This suggests that the identifiability status of the two equations affects the behavior of RSS under P_1 than P_2 in some respects.

The marginal totals of three tables (6, 8 and 10) of frequencies of four correlation-based charts (****, **/**, Λ **,** **V**) of behavior of the two attributes are displayed in Table 10. These percentages show the frequencies of these charts for both equations.

There is a remarkable uniformity in the column-wise comparison of the entries in Table 10 for both criteria of P_1 where the frequencies are similar for the two charts (****, **V**). The upward sloping chart (representing increasing values of TAS or RSS across the three correlation levels) and the convex chart (representing maximum values of TAS or RSS at the middle interval) are less frequent than the other two charts (****, **V**) which have relatively high frequencies for both equations in P_1 and P_2.

CONCLUSION

The sensitivity of the simultaneous equation techniques to violation of mutual independence of random deviates in a two-equation model has been investigated.

Based on TAB, it can be concluded that since the 3SLS estimator has the minimum ATAB for both P_1 and P_2 is the best followed by LIML which is also closely followed by 2SLS. The OLS is however, on top of the group when comparing the performances of the estimators using coefficient of Variation followed by 2SLS and LIML.

To further examine the sensitivity of each estimator to changes in TAB of estimates over 100 replications, a detailed table presentation of the behavior of estimators over correlation coefficients and sample sizes are charted and presented in table 6 for both P_1 and P_2. The model absolute bias for 2SLS and LIML attained a minimum at the feebly correlated region while OLS performed poorly with an increasing TAB as the correlation changes over the three cardinal points. The behavior of FIML revealed no reasonable pattern.

Using the RSS, the performances of 2SLS, LIML and 3SLS are similar for both equations and triangular matrices.

Best RSS estimates of 2SLS, LIML, and 3SLS are found in the feebly correlated region which is consistent with the theory. That is, the "**V**" trend is expected to be the most frequent since that would imply that residual sum of squares is a minimum when correlation of the error term is smallest (negative or positive).

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