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Bayesian Optimal Filtering in Dynamic Linear Models: An Empirical Study of Economic Time Series Data

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Abstract

This paper reviews a recursive Bayesian methodology for optimal data cleaning and filtering of economic time series data with the aim of using the Kalman filter to estimate the parameters of a specified state space model which describes an economic phenomena under study. The Kalman filter, being a recursive algorithm, is ideal for usage on time-dependent data. As an example, the yearly measurements of eight key economic time series data of the Nigerian economy is used to demonstrate that the integrated random walk model is suitable for modeling time series with no clear trend or seasonal variation. We find that the Kalman filter is both predictive and adaptive, as it looks forward with an estimate of the variance and mean of the time series one step into the future and it does not require stationarity of the time series data considered.

Keywords: Bayesian inference; Kalman Filter; economic data; dynamic linear model.
Mathematics Subject Classification: 53C25; 83C05; 57N16

1 Introduction

State space models originated in engineering in the early sixties [1, 2]. Their use have since been extended to a large number of applied fields such as econometrics, time series analysis, genetics,

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spatio-temporal analysis, hydrology, climatology, speech recognition, signal processing, population dynamics etc. A state space model allows the researcher to model an observed (multiple) time series y_t, $t = 1, 2, ..., T$ which is driven by a stochastic process. Many econometric models (time series models, in particular) can be written in state space form [3, 4, 5]. The obvious flexibility of the state space approach has contributed to their immense popularity in econometric time series analysis in recent years [6, 7]. Dynamic linear models (DLMs) are a particular class of state space models that allow many of the relevant inferences to be carried out exactly using the Kalman

filter, especially in the case of a completely specified model like the one considered in this paper. They provide a very rich class of models for the analysis and forecasting of time series data [5].

Economic indicators are usually complex high frequency data that can be easily analyzed using state space models [8]. In this paper, we specify an integrated random walk state space model with application to some economic time series data of the Nigerian economy to see how well filtered the series will be.Basically, the objectives of this paper is twofold: (i)To investigate the performance of the Kalman filter technique on a specified local level state space model (ii)To detect the suitability of Integrated Random Walk (IRW) in describing Nigerian economic time series data. The rest of the paper is organized as follows: In section 2, we present a brief review of literature on application of state space models to time series and econometric analysis. Section 3 is on the model and Kalman filter method used in this paper,while section 4 borders on an application to the Nigerian economy, and section 5 concludes the paper.

2 Application of State Space Models in Time Series Econometrics

In recent years, there has been an increasing interest in the application of state- space model and its variants in econometrics and time series analysis, partly because of its flexibility and widely because of the development of modern Monte Carlo methods. Economic theory and practice is often concerned with latent or unobserved dynamic processes,hence the state space form of a dynamic system with unobserved components is a very powerful and flexible instrument because it builds on the dependence structure of a Markov chain to define more complex models for the dynamic observations [5].

A state space model is in principle, any model that includes an observation process y_t and a state process θ_t. It consists of a state equation and an observation equation which can be used for modeling univariate and multivariate time series in the presence of non-stationarity, structural changes, irregular patterns.The linear econometric state space model encompasses two models which are widely used in time series analysis and economics. First, the observation equation resembles a linear regression model with the distinction that the coefficients are stochastic dynamic processes rather than fixed parameters. Second, the state vector follows a first order vector autoregressive model, which in turn is a generalisation of the scalar Autoregressive Moving Average (ARMA) models. All univariate ARMA models can be written as a linear state space model [9]. We are motivated to apply this method to modeling economic indicators because, in many real real life applications, the driving force behind the evolution of economic variables are not always observable. Usually, it is assumed that there is an unobservable Markov chain θ_t called the 'state' and that y_t is an imprecise measurement of θ_t.

In econometric applications, we think of the state θ_t as an auxilliary time series that is assumed to follow a markov process which facilitates the task of specifying the probability distribution of the observable time series y_t. State-space models consider a time series as the output of a dynamic system perturbed by random disturbances. They have an elegant and powerful probabilistic structure, offering a flexible framework for a very wide range of applications. Computations can be implemented by recursive algorithms [10, 11]. While the state equation formulates the dynamics of the state variables, the observation equation relates the observed variables to the unobserved state vector. The state vector can contain trend, seasonal, cycle and regression components plus an error term, and has to be estimated from the data,hence the maximum likelihood estimates of the parameters can be obtained by applying the Kalman filter.

In the Box and Jenkins methodology, observations are first made stationary [12, 7]. This is accomplished by using various data transformations like taking logarithms and time differences before fitting ARMA models to the transformed data. Wide availability of computer software and a well developed mathematical theory has contributed to the enduring popularity of this approach.The state space methodology, however, offers a number of important advantages over the popular class of ARMA models in so many ways. First is the fact that the state space method does not require the data to be stationary, which eliminates a problem which the Box-Jenkins theory offers minimum guidance. Another fact is that state space methods can accommodate a multivariate framework and they are able to deal with irregular patterns, interventions, sudden jumps and regime shifts in data. The problems of estimation and forecasting in state space models are solved by recursively computing the conditional distribution of the quantities of interest, given the available information [13, 14], in this sense, they are quite naturally treated from a Bayesian approach [15]. The series data re well filtered if the filtered forcasts are close to the real data. This paper tends to contribute to the theoretical and empirical literature on the topic by demonstrating the use of Kalman filter for estimating a completely specified random walk model .

3 Model Specification and Methodology

3.1 Linear- Gaussian State Space Model

Consider a linear-Gaussian state space model for an $m \times 1$-dimensional time series y_t consisting of a measurement equation relating the observed time series to a $p \times 1$-dimensional unobserved state vector θ_t and a Markovian transition equation that describes the evolution of the state vector over time [16]. The model takes the following general form.

$$y_t = X_t\theta_t + v_t \qquad v_t \sim N(0, V_t) \tag{3.1}$$

$$\theta_t = G_t\theta_{t-1} + w_t \qquad w_t \sim N_p(0, W_t) \tag{3.2}$$

$$\theta_0 \sim N_p(m_0, C_0) \tag{3.3}$$

where y_t is a vector of dimension $m \times 1$

Equation (1) is known as the observation equation while equation (2) is a first order Markov process called the evolution equation. G_t, X_t, are known matrices of order $p \times p$ and $m \times p$ respectively that determine how the observation and state equations evolve in time [17].
The matrices F_t, V_t, G_t and W_t are known as the system matrices and contain non-random elements. If they do not depend deterministically on t, the state space system is time invariant, otherwise they are time varying.
v_t and w_t are assumed to be independent both within and between, and independent of θ_0 The initial state vector θ_0 is assumed to be normally distributed with parameters m_0 and C_0 as shown in (3) where $E(v_t\theta_0') = 0$, and $E(w_t\theta_0') = 0$ for $t = 1, ..., T$. If some or all the elements of θ_t are assumed to be covariance stationary, the corresponding elements of m_0 and C_0 can be solved analytically from the elements of the system matrices. However, for non- stationary elements of θ_t, it is customary to set the corresponding elements of C_0 to a very large positive number [17].

3.2 Optimal State Estimation and Prediction with Kalman Filter

In a state space model, the unobserved state vector θ is the signal and the measurement error w_t is the noise. The objective of filtering is to track the dynamic evolution of unobservable state variables using (typically) noisy measurements of observables [18]. This requires the computation of integrals over the unobserved states, which in general must be approximated numerically. Given observed data $y_1, ..., y_2$, the goals of state space estimation using the Kalman filter is twofold:
(1) Optimal signal extraction (estimation) and
(2) Optimal h-step ahead prediction of states and data.
Basically, there are two types of state estimation, namely (a)Filtering and
(b)Smoothing. Filtering implies getting optimal estimates of θ_t given information available at time t, $D_t = (y_1, ..., y_t)$, so that $E(\theta_t|D_t)$ is the filtered estimate of θ. On the other hand, smoothing implies getting optimal estimates of θ_t given information available at time T, $D_t = (y_1, ..., y_T)$, so that $E(\theta_t|D_T)$ is the smoothed estimate of θ.

A filter is a device which separates entities into their constituting components.The process of finding the best estimate from noisy data is known as filtering out the noise. It does not just clean up the data measurements but also projects these measurements into the state estimate.The Kalman filter was developed in 1960 by Rudolf E. Kalman [1] as an algorithm used for aerospace guidance applications and to solve state space models in the linear case. It is an optimal estimator which infers parameters of interest from indirect, and sometimes inaccurate and uncertain observations. It is an optimal process because if all noise is gaussian, the Kalman filter minimizes the mean square error of the estimated parameters. The Kalman filter calculates the mean and variance of the unobserved state θ_t, given the observations. It provides an exact and complete solution to the Bayesian filtering problem which is very economical in computer calculation and storage requirements. It is a recursive algorithm i.e the current best estimate is updated whenever a new observation is obtained.It is an optimal process because if all noise is Gaussian, the Kalman filter minimizes the mean square error of the estimated parameters. The mean and variance of the measurements needs to be known before implementing a Kalman filter.

Given the initial state and covariance, we have sufficient information to find the optimal state estimate using the Kalman filter equations. The Kalman filter has originally been applied by engineers and physicists to estimate the state of a noisy system. The classic Kalman filter application is the example of tracking an orbiting satellites whose exact position and speed, which are not directly measurable at any point of time, but can be estimated using available data . A discussion of engineering-type applications applications to non-linear models is provided in [19, 20].

As mentioned earlier, the Kalman filter is a set of recursion equations for determining the optimal estimates of the state vector θ_t, given information available at time t, D_t. The filter consists of two sets of equations:
(i)Prediction Equations (ii)Updating Equations
The filter prediction and update steps require a few basic matrix calculations of which only the conditional means and variances of the filtering and prediction density need to be stored in each step of the iteration. To describe the filtering process, we let

$$m_t = E(\theta_t|D_t)$$

be the optimal estimator of θ_t based on D_t and let

$$C_t = E((\theta_t - m_t)(\theta_t - m_t)^T|D_t)$$

be the mean square error matrix of m_t.

Prediction Equations

Let $\theta_{t-1}|y_{1:t-1} \sim N(m_{t-1}, C_{t-1})$, where $y_{1:t-1}$ denote all observations up to time $t-1$. Then the one-step-ahead predictive density $\theta_t|y_{1:t-1}$ is Gaussian with parameters:

$$E(\theta_t|y_{1:t-1}) = m_{t-1} \equiv A_t$$
$$Var(\theta_t|y_{1:t-1}) = C_{t-1} + W_t \equiv R_t$$

The one-step-ahead predictive density of $Y_t|y_{1:t-1}$ is Gaussian with parameters:

$$f_t = E(Y_t|y_{1:t-1}) = X_t A_t$$
$$Q_t = Var(Y_t|y_{1:t-1}) = X_t R_t X_t^{'} + V$$

The filtering density of θ_t given $y_{1:t}$ is Gaussian with parameters:

$$m_t = E(\theta_t|y_{1:t}) = A_t + R_t X_t^{'} Q_t^{-1} e_t$$
$$C_t = Var(\theta_t|y_{1:t}) = R_t - R_t X_t^{'} Q_t^{-1} X_t R_t$$

where $e_t = Y_t - f_t$ is the forecast error.

Given m_{t-1} and C_{t-1} at time $t-1$, the optimal predictor of θ_t and its associated MSE matrix are

$$m_{t/t-1} = E(\theta_t|D_{t-1}) = G_t m_{t-1}$$
$$C_{t/t-1} = E((\theta_t - m_{t-1})(\theta_t - m_{t-1})'|D_{t-1})$$
$$= G_t C_{t-1} G_t' + W_t$$

.

Basically, in the Kalman recursions for prediction, the linear estimation of θ_t in terms of:
$y_0..., y_{t-1}$ defines the prediction problem,
$y_0, ..., y_t$ defines the filterring problem, while
$y_0, ..., y_T, T > t$, defines the smoothing problem.

Updating Equations

When new observations y_t become available, the optimal predictor $m_{t|t-1}$ and its MSE matrix are updated using

$$m_t = m_{t|t-1} + C_{t|t-1} F_t' Q_t^{-1}(y_t - F_t m_{t|t-1})$$
$$= m_{t|t-1} + C_{t|t-1} F_t' Q_t^{-1} v_t$$
$$C_t = C_{t|t-1} - C_{t|t-1} F_t' Q_t^{-1} F_t' C_{t|t-1}$$
$$= C_{t|t-1} - K_t v_t$$

where

$$Q_t = F_t C_{t|t-1} F_t' + V_t$$

and

$$K_t = C_{t|t-1} F_t' Q_t^{-1}$$

is the Kalman gain matrix which gives the weight on new information $e_t = y_t - F_t m_{t|t-1}$ in the updating equation for m_t.

Proofs of this algorithm and detailed procedures can be found in [6, 17], and recent applications to mixed-measurement time series can be found in [21].

4 Empirical Illustration

4.1 Dynamic Linear Modeling of Some Nigerian Economic Data

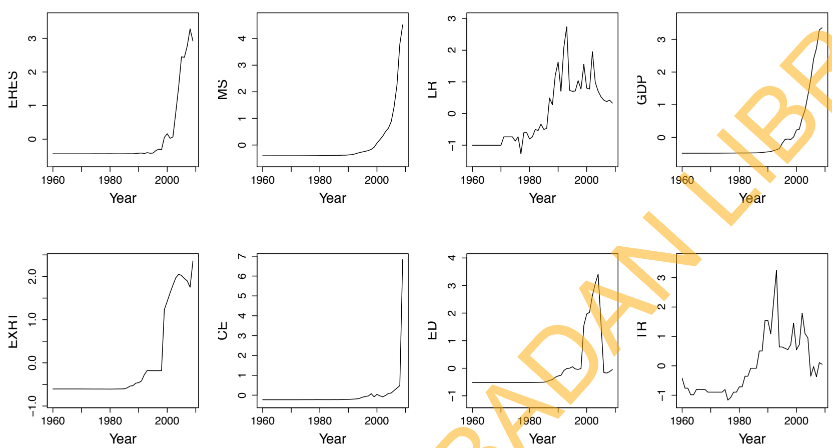

Figure 1: Annual time-series data on External Reserve (ERES), Money Supply (MS), Lending Rate (LR), Gross Domestic Product (GDP), Exchange Rate (EXRT), Capital Expenditure (CE), External Debt (ED) and Treasury bill Rate (TR). These data were standardized prior to analysis.

By way of illustration, in this section, we fit a linear-Gaussian univariate Dynamic Linear Model (DLM) to eight economic indicators of the Nigerian economy (shown in Figure 1). The data used in this research are Nigerian economic indicators sourced from websites of the Central Bank of Nigeria (CBN)(http://www.cenbank.org/economic-indicators) and World Bank (http://data.worldbank.org/country/nigeria).

We consider a first order integrated random walk model which is a variant of the DLM presented in equations (1) and (2) where:

$$y_t = \theta_t + v_t \qquad v_t \sim N(0, \sigma_v^2) \tag{4.1}$$

$$\theta_t = \theta_{t-1} + w_t \qquad w_t \sim N(0, \sigma_w^2) \tag{4.2}$$

where y_t is the value of the time series we are trying to model with respect to θ_t and θ can be thought of as a time-varying slope parameter. Due to the Markovian structure of the states θ_t, we estimate the model by the method of Kalman filter [1, 6, 21] by computing the predictive and filtering distributions of θ_t inductively starting from $\theta_0 \sim N(m_0, C_0)$. The Kalman filter calculates the mean and variance of the unobserved state θ_t, given the observations. It is a recursive algorithm i.e the current best estimate is updated whenever a new observation is obtained. This model is fully specified with $C_0 = 1000$, $m_0 = 0$, $\sigma_w^2 = 0.2$ and $\sigma_v^2 = 3$. We compute the filtering and predictive densities of the eight economic time series using the recursive Kalman filter algorithm, starting from a prior specification of θ_t as $\theta_0|D_0 \sim N(m_0, C_0)$, then computing $p(\theta_1|D_1)$ and proceeding recursively to detect the state of the system with respect to time.The recursive Bayesian estimation is also known as Bayes filter. Given that the true state of θ is assumed to be an unobserved Markov process and the measurement of time series y_t are the observed states of an Integrated Random Walk model given by equations 4 and 5 above. Due to the Markovian assumption, the probability of the current true state given the immediate previous one is conditionally independent of the other earlier states. The following picture represents the specified Markovian state space model.

$$\boxed{\begin{array}{ccccccccccccc} \theta_0 & \rightarrow & \theta_1 & \rightarrow & \theta_2 & \rightarrow \cdots & \theta_{t-1} & \rightarrow & \theta_t & \rightarrow & \theta_{t+1} & \rightarrow \cdots \\ & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & \\ & & Y_1 & & Y_2 & & Y_{t-1} & & Y_t & & Y_{t+1} & \end{array}}$$

Figure 2: Markovian Structure of State Space Models.

$$p(\theta_t|\theta_{t-1}, \theta_{t-2}, ..., \theta_0) = p(\theta_t|\theta_{t-1}) \tag{4.3}$$

Similarly, the observed economic time series measurement at the t^{th} time step is dependent only on the current state and is conditionally independent of all other states given the current state:

$$p(y_t|\theta_t, \theta_{t-1}, ..., \theta_0) = p(y_t|\theta_t) \tag{4.4}$$

Hence, we represent the probability distribution over all states of the integrated random walk model as

$$p(\theta_0, ..., \theta_t, y_1, ..., y_t)$$
$$= p(\theta_0) \prod_{i=1}^{t} p(y_i|\theta_i) p(\theta_i|\theta_{i-1})$$

In our Kalman filter estimation of the states θ, the probability distribution of interest is associated with the current states conditioned on the economic time series measurements up to the predict and update steps of the Kalman filter written probabilistically as specified in section 3.3.1 and 3.3.2. The probability distribution of update is proportional to the product of the time series measurement likelihood and the predicted state

$$p(\theta_t|y_{1:t}) = \frac{p(y_t|\theta_t)p(\theta_t|y_{1:t-1})}{p(y_t|y_{1:t-1})} \tag{4.5}$$

$$\propto p(y_t|\theta_t)p(\theta_t|y_{1:t-1})$$

The denominator, $p(y_t|y_{1:t-1})$ is constant relative to θ and was ignored. Our analysis of the eight key economic variables was implemented using the "dlm" package in the R software presented in [5] .

Figure 2 shows that the series are well filtered and depicts how the integrated random walk model performs on the economic data under study. Our Kalman filter analysis indicates that the trend and volatility of the Gross Domestic Product (GDP) and Money Supply (MS) are quite smooth when compared to the more volatile series of lending rate and treasury bill rate which experienced substantial

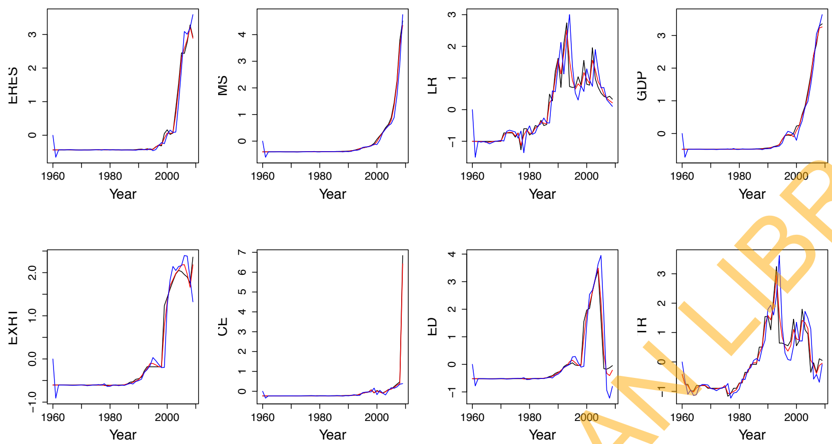

Figure 3: Filtered states, & forecasts. Black line represents the data, red line represents the filtered states, and the blue line represents the one-step-ahead state forecasts.

changes during the recent economic episode of Nigeria. Note the conspicuous spikes at the tail end in the forecasts of exchange rate, external debt, and external reserves. The one-step ahead filtered forcasts of external reserves and external debt suggests that the external reserve increases in the following year while external debt reduces further. Analyzing causes of different responses across variables may require a more structural econometric model of the economy [22]. However, the kind of analysis carried out in this paper could be a starting point for such a project. The case considered here is when we assume that the initial observation and evolution variance are know apriori, thus negating the need for Markov chain Monte Carlo (MCMC) method of estimation.

5 Concluding Remarks

In this paper, the Kalman filter methodology has been successfully tested on a typical macroeconomic dataset. Our method relies solely on recursive Bayesian optimal filtering of the data measurements. We find that the Kalman filter is both predictive and adaptive to the dynamic linear model specified for the economic variables considered. It looks forward with an estimate of the variance and mean of the time series one step into the future without requiring stationarity of the data .Our finding supports earlier works in literature which noted that the integrated random walk model is intuitively suitable for time series showing no clear trends or seasonal variation [5,6,17] .As future work, we propose to use the Kalman filter results (its prediction and corresponding variance) for detection of outliers in high frequency and more volatile economic data especially when the prior observation and evolution variances are unknown in higher order dynamic linear models.

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Competing Interests

The authors declare that no competing interests exist.

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