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# The Explicit Formula for the Number of the Distinct Fuzzy Subgroups of the Cartesian Product of the Dihedral Group $2^n$ with a Cyclic Group of Order Eight , where $n > 3$

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***Abstract.*** In this paper, the explicit formulae is given for the number of distinct fuzzy subgroups of the cartesian product of the dihedral group of order $2^n$ with a cyclic group of order 8, where $n > 3$.

***Keywords:*** Fuzzy subgroups, Dihedral Group, Inclusion-exclusion principle, Maximal Subgroups.

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## 1. Introduction

One of the most important problem of fuzzy group theory is to classify the fuzzy subgroup of a finite groups. This topic has enjoyed a rapid development in the last few years.This paper is a follow up from[1,2].

## 2. Methodology

Suppose that $M_1, M_2, \dots, M_t$ are the maximal subgroups of a finite group $G$, and denote $h(G)$ as the number of distinct fuzzy subgroups of $G$ . By simply applying the technique of computing $h(G)$, using the application of the Inclusion-Exclusion Principle, we have that:

$$h(G) = 2\left(\sum_{r=1}^{t} h(M_r) - \sum_{1\le r_1<r_2\le t} h\left(M_{r_1} \cap M_{r_2}\right) + \cdots + (-1)^{t-1} h\left(\bigcap_{r=1}^{t} M_r\right)\right) \quad (1.1)$$

In [4], (1.1) was used to obtain the explicit formulas for some positive integers $n$.

**Theorem 1.1. [5]** The number of distinct fuzzy subgroups of a finite $p$-group of order $p^n$ which have a cyclic maximal subgroup is:

1. $h(\mathbb{Z}_{p^n}) = 2^n$
2. $h(\mathbb{Z}_p \times \mathbb{Z}_{p^{n-1}}) = h(M_{p^n}) = 2^{n-1}[2 + (n-1)p]$

**3. The number of fuzzy subgroups for $\mathbb{Z}_8 \times \mathbb{Z}_8$**

**Lemma 2.1.** Let $G$ be abelian such that $G = \mathbb{Z}_4 \times \mathbb{Z}_4$ Then, $h(G) = 2h(\mathbb{Z}_2 \times \mathbb{Z}_{2^2}) = 48$

**Proof:** By the use of GAP (Group Algorithms and Programming), $G$ has three maximal subgroups in which each of them is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^2}$. Hence, we have that: $\frac{1}{2}h(G) = 3h(\mathbb{Z}_2 \times \mathbb{Z}_{2^2}) - 3h(\mathbb{Z}_2 \times \mathbb{Z}_{2^2}) + h(\mathbb{Z}_2 \times \mathbb{Z}_{2^2}) = h(\mathbb{Z}_2 \times \mathbb{Z}_4)$. And by theorem (*), $h(\mathbb{Z}_2 \times \mathbb{Z}_{2^2}) = 24. \Rightarrow h(\mathbb{Z}_4 \times \mathbb{Z}_4) = 48$.

**Corrolary 2.1.** Following the last lemma, $h(\mathbb{Z}_4 \times \mathbb{Z}_{2^5}), h(\mathbb{Z}_4 \times \mathbb{Z}_{2^6}), h(\mathbb{Z}_4 \times \mathbb{Z}_{2^7})$ and $h(\mathbb{Z}_4 \times \mathbb{Z}_{2^8})$ = 1536, 4096, 10496 and 26112 respectively.

**Proposition 2.1 [3]** Suppose that $G = \mathbb{Z}_8 \times \mathbb{Z}_{2^n}, n \geq 2$. Then, $h(G) = 2^n[n^2 + 5n - 2]$

**Proof:** $G$ has three maximal subgroups of which two are isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^n}$ and the third is isomorphic to $\mathbb{Z}_4 \times \mathbb{Z}_{2^{n-1}}$.

Hence,

$h(\mathbb{Z}_4 \times \mathbb{Z}_{2^n}) = 2h(\mathbb{Z}_2 \times \mathbb{Z}_{2^n}) + 2^1h(\mathbb{Z}_2 \times \mathbb{Z}_{2^{n-1}}) + 2^2h(\mathbb{Z}_2 \times \mathbb{Z}_{2^{n-2}}) + 2^3h(\mathbb{Z}_2 \times \mathbb{Z}_{2^{n-3}}) + 2^4h(\mathbb{Z}_2 \times \mathbb{Z}_{2^{n-4}}) + \cdots + 2^{n-2}h(\mathbb{Z}_2 \times \mathbb{Z}_{2^2})$

$$= 2^{n+1}[2(n+1) + \sum_{j=1}^{n-2} [(n+1) - j]$$

$$= 2^{n+1}[2(n+1) + \tfrac{1}{2}(n-2)(n+3)] = 2^n[n^2 + 5n - 2], n \geq 2$$

We have that: $h(\mathbb{Z}_4 \times \mathbb{Z}_{2^{n-1}}) = 2^{n-1}[(n-1)^2 + 5(n-1) - 2] = 2^{n-1}[n^2 + 3n - 6], n > 2$.

**Theorem 2.1. [2]** Let $G = D_{2^n} \times \mathbb{C}_2$, the nilpotent group formed by the cartesian product of the dihedral group of order $2^n$ and a cyclic group of order 2. Then, the number of distinct fuzzy subgroups of $G$ is given by : $h(G) = 2^{2n}(2n+1) - 2^{n+1}, n > .3$

**4. The number of fuzzy subgroups for $D_{2^n} \times \mathbb{C}_8$**

**Proposition 4.1.** Suppose that $G = D_{2^n} \times \mathbb{C}_8$. Then, the number of distinct fuzzy subgroups of $G$ is given by :

$$2^{2(n-2)}(64n + 173) + 3\sum_{j=1}^{n-3} 2^{(n-1+j)}(2n + 1 - 2j$$

**Proof :**

$$\begin{aligned}
\frac{1}{2}h(D_{2^n} \times C_8) &= h(D_{2^n} \times C_4) + 2h(D_{2^{n-1}} \times C_8) + h(D_{2^{n-1}} \times C_8) + 2h(D_{2^{n-1}} \times C_4) \\
&\quad + \mathbb{Z}_{2^{n-1}} - 4h(D_{2^{n-1}} \times C_2) + h(\mathbb{Z}_4 \times \mathbb{Z}_{2^{n-1}}) - 2h(\mathbb{Z}_2 \times \mathbb{Z}_{2^{n-1}}) \\
&\quad - 2h(\mathbb{Z}_4 \times \mathbb{Z}_{2^{n-2}}) + 8h(\mathbb{Z}_2 \times \mathbb{Z}_{2^{n-2}}) + h(\mathbb{Z}_{2^{n-1}}) - 4h(\mathbb{Z}_{2^{n-2}}) \\
h(D_{2^n} \times C_4) &= (n-3).2^{2n+2} + 2^{2(n-3)}(1460) + 3[2^n(2n-1) + 2^{n+1}(2n-3) \\
&\quad + 2^{n+2}(2n-5) + \cdots + 7(2^{2(n-2)})] \\
&= (n-3).2^{2n+2} + 2^{2(n-3)}(1460) + 3\sum_{j=1}^{n-3} 2^{n-1+j}(2n+1-2j) \\
&= 2^{2(n-2)}(64n + 173) + 3\sum_{j=1}^{n-3} 2^{n-1+j}(2n+1-2j)
\end{aligned}$$

## 5. Conclusion

In this work, the explicit formulae for the number of distinct fuzzy subgroups of the cartesian product of the dihedral group of order $2^n$ with a cyclic group of order 8 is established.

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