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Modelling effective rheologies for viscoelastic porous media with application to silt, and medium and coarse sand

J A Olowofela and J A Adegoke

Department of Physics, University of Ibadan, Ibadan, Nigeria

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Abstract
A modification of Biot's poroelastic differential equations is made to include matrix–fluid interaction mechanisms which assume a solid–fluid relaxation function coupling coefficient. Values of physical properties of sediments are incorporated into equations which define phase velocity and attenuation for porous media which are dependent on the composite densities of various media (silt, and medium and coarse sand). The results enable us to compare the attenuation and velocities of waves in these media. We observed that the density of coarse sand is greater than that of medium sand and this in turn is greater than that of silt—the same holds for the velocities of P-waves in these media but the situation is converse for shear waves in the same given media. As the densities of the media increase, their attenuation decreases as it was found that the attenuation of silt is the highest and that of coarse sand lowest for the media considered.

Keywords: Biot theory, phase velocity, attenuation, porous media

1. Introduction

The concepts of porous media have attracted a great deal of attention in recent years. The application covers a variety of fields, from physics to geophysics, engineering, soil mechanics and underwater acoustics. In particular, in the exploration of oil and gas reservoirs, it is important to predict the preferential directions of the fluid flow. These are closely related to the permeability of the medium and consequently to the geometric characteristics of the skeleton.

The wave propagation properties of synthetic porous media such as sintered glass beads were successfully described by Biot's theory of dynamic poroelasticity (Biot 1962). Discrepancies between Biot's theory and measurement are due to complex pore shapes, which are not present in simple synthetic media or in natural porous media such as sandstone (Gist 1994). This complexity gives rise to a variety of matrix–fluid interactions which contribute to the attenuation of different wave modes. Different matrix–fluid attenuation mechanisms are introduced into Biot's theory by substituting the fluid–solid coupling modulus with a time-dependent relaxation function based on the standard linear solid mode. The introduction of memory variables for avoiding the time convolutions yields a set of first-order differential equations for dynamic poroviscoelasticity (Carcione 1998).

According to Gurevich (1996) the value of poroelastic wave modelling is unclear without comparing its results to the corresponding simulation based on single-phase modelling. This is particularly important in the seismic range where poroelastic effects are relatively small. However, Gurevich and Lapotnikov (1995) have shown that attenuation levels and velocity-dispersion measurements can be explained by the combined effects and energy transfer between wave models.

This work compares the attenuation and phase velocities in different media (silt, medium and coarse sand) which largely depend on their composite densities.

The objective of this work is to verify the effect of porosity of media vis-à-vis their composite densities on the phase velocities and the attenuation of waves, laying emphasis on silt, and medium and coarse sand. The result of the work done by Carcione (1998) was used as a reference in the application to different media.

Data from the phase velocity and attenuation against log of frequency (for water and gas) have been extrapolated (tables 4 and 5) and plotted (figures 1 and 2), and an attempt has been made to calculate the peak values as obtained by Carcione (1998). The same set of equations have been used to determine composite densities, phase velocities and attenuation for silt, and medium and coarse sand, and these are later compared.

2. Theory

The constitutive equations for an inhomogeneous isotropic medium under plane strain conditions are given by (Biot and Willis 1957, Biot 1962)

$$\tau_{xx,t} = EV_{x,x} + (E - 2\mu)V_{z,z} + \alpha M_\epsilon + S_x \tag{1}$$

$$\tau_{zz,t} = (E - 2\mu)V_{x,x} + EV_{z,z} + \alpha M_\epsilon + S_x \tag{2}$$

$$\tau_{xz,t} = \mu(V_{x,z} - V_{z,z}) + S_{xz} \tag{3}$$

$$P_{,t} = -M_\epsilon + S_f \tag{4}$$

$$\epsilon = \alpha(V_{x,x} + V_{z,z}) + q_{x,x} + q_{z,z}; \tag{5}$$

here T_{xx}, T_{zz} and T_{xz} are the total stress components, P is the fluid pressure, V and q are the solid and fluid (relative to the solid) particle velocities and S_x, S_z, S_{xz} and S_f are the external sources of stress, for the solid and the fluid, respectively. The subscript x denotes $\frac{\partial}{\partial x}$. The elastic coefficients are given by

$$E = K_m + \tfrac{4}{3}\mu \tag{6}$$

$$M = \frac{k_s^2}{D - K_m} \tag{7}$$

$$D = K_s\left(1 + \phi\left(K_s K_f^{-1} - 1\right)\right) \tag{8}$$

$$\alpha = 1 - \frac{K_m}{K_s} \tag{9}$$

where K_m, K_s and K_f are the bulk moduli of the drained matrix, solid and fluid, respectively, ϕ is the porosity and μ is the shear modulus of the drained (and saturated) matrix. The stiffness E is the P-wave modulus of the dry skeleton, M is the coupling modulus between the solid and the fluid, and α is the poroelastic coefficient of effective stress.

Since the fluid is viscous, the motion is not instantaneous and energy dissipation occurs. Skeleton–fluid mechanisms are modelled by generalizing the coupling modulus M to a time-dependent relaxation function. We assume that E and μ are in general independent.

The term M_ϵ in (1), (2) and (4) is replaced by $\psi^*_{c,t}$, where

$$\psi_{(t)} = M\left(1 + \frac{1}{L}\sum_{l=1}^{L}\psi\right)^{-1}\left(1 + \frac{1}{L}\sum_{l=1}^{L}\exp(-\tau/T_{\delta l})\right)H(t), \tag{10}$$

where $H(t)$ denotes the Heaviside function

$$\psi = \frac{T_{\epsilon L}}{T_{\delta L}} \tag{11}$$

and $T_{\epsilon L}$ and $T_{\delta L}$ denote sets of relaxation times. Equation (10) corresponds to a parallel connection of standard linear solid elements. For higher frequencies ($t = 0^+$), $\psi = M$ as in single-phase viscoelastics in order to avoid the time convolutions.

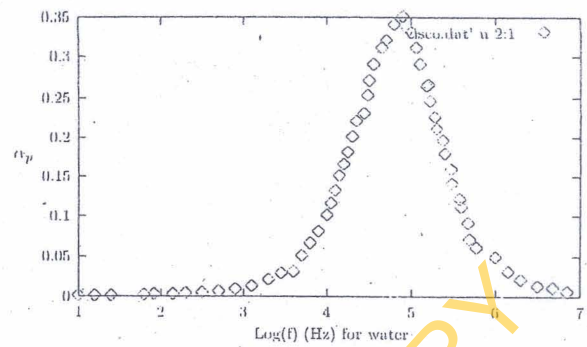

Figure 1. Plot of α_P versus Log(f) for water (Carcione 1998).

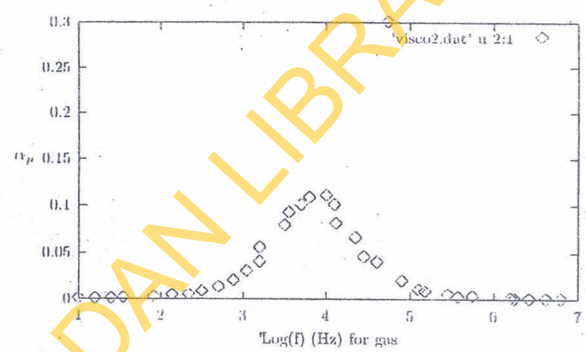

Figure 2. Plot of α_P versus Log(f) for gas (Carcione 1998).

Table 1. Values of density, attenuation, velocities of P- and S-waves for coarse sand, medium sand and silt.

Symbol (unit)	Coarse sand	Medium sand	Silt
ρ (kg m^{-3})	2155	1990	1907.5
α_s (dB)	2.18	3.06	3.66
C_p (m s^{-1})	2208.1	2206.9	1827.4
C_s (m s^{-1})	975.70	1030.60	1046.0

The poroviscoelastic equations of motion are (Biot–Newton dynamic equations; see Biot (1962))

$$\tau_{xx,x} + \tau_{xz,z} = \rho V_{x,t} + \rho_f q_{x,t} \tag{12}$$

$$\tau_{xx,x} + \tau_{zz,z} = \rho V_{z,t} + \rho_f q_{z,t} \tag{13}$$

where

$$\rho = (1 - \phi)\rho_s + \phi\rho_f \tag{14}$$

is the composite density and ρ_s and ρ_f are the solid and fluid densities, respectively.

When $M \to M_c$

$$M_c = M\left(L + \sum_{L=1}^{L}\psi_L\right)^{-1}\sum_{L=1}^{L}\frac{1 + i\omega T_{\epsilon L}}{1 + i\omega T_{\delta L}} \tag{15}$$

Table 2. Physical properties of silt, medium sand and coarse sand.

Physical properties	Symbols (unit)	Silt	Medium sand	Coarse sand
Kinematic viscosity of pore fluid	V ($m^2\,s^{-1}$)	1.0×10^{6}	1.0×10^{6}	1.0×10^{6}
Permeability	K_s (m^2)	1.0×10^{-12}	1.0×10^{-10}	1.0×10^{-9}
Porosity	ϕ	0.45	0.4	0.3
Added mass coefficient of skeleton frame	A	0.25	0.25	0.25
Dynamic shear modulus		1.0×10^{7}	5.0×10^{7}	1.0×10^{8}
Coulomb specific loss in the frame	δ,δ'	0.02	0.02	0.02
Poisson's ratio of skeletal frame	N	0.3333	0.3333	0.3333
Bulk modulus of fluid	K_f ($N\,m^{-2}$)	2.3×10^{9}	2.3×10^{9}	2.3×10^{9}
Bulk modulus of frame material	K_r ($N\,m^{-2}$)	3.6×10^{9}	3.6×10^{9}	3.6×10^{9}
Density of fluid	ρ_f ($kg\,m^{-3}$)	1.0×10^{3}	1.0×10^{3}	1.0×10^{3}
Density of grain material	ρ_s ($kg\,m^{-3}$)	2.65×10^{3}	2.65×10^{3}	2.65×10^{3}

Table 3. Material properties of the single constituents.

Solid	Bulk modulus, K_s	3.5×10^{9} N m^{-2}
	Density, ρ_s	26 650 kg m^{-3}
Matrix	Bulk modulus, K_m	1.7 GPa
	Shear modulus, N	1.8555 GPa
	Porosity ϕ	0.3
	Permeability, K	1D ≡ 1 darcy
	Tortuosity, T	1
Gas	Bulk modulus, K_g	0.022 GPa
	Density, ρ_g	100 kg m^{-3}
	Viscocity, η_g	0.015 cP
Water	Bulk modulus, K_w	2.4 GPa
	Density, ρ_w	1000 kg m^{-3}
	Viscocity, η_w	1 cP

where ω denotes the angular frequency. The relaxation times can be expressed in terms of a Q-factor, Q_{oL}, and a reference frequency, f_{oL}, as

$$T_{oL} = \frac{1}{2f_{oL}Q_{oL}}\left(\sqrt{Q_{oL}^2+1}+1\right) \quad (16)$$

and

$$T_{oL} = \frac{1}{2f_{oL}Q_{oL}}\left(\sqrt{Q_{oL}^2+1}-1\right). \quad (17)$$

The velocities of the fast (+ sign) and slow (− sign) compressional waves and shear waves are given by

$$V_{p\pm}^2 = \frac{A\pm\sqrt{A^2-4M_cE\rho_c\rho}}{2\rho_c\rho} \quad (18)$$

and

$$V_s^2 = \frac{N}{\rho_c} \quad (19)$$

where

$$A = M_c(\rho-2\alpha\rho_f)+\bar{\rho}(E+\alpha^2M_c) \quad (20)$$

$$\rho_c = \rho-\frac{\rho_f^2}{\bar{\rho}} \quad (21)$$

$$\bar{\rho} = \frac{T}{\phi}\rho_f-\frac{i}{2\pi f}\frac{\eta}{K} \quad (22)$$

where f denotes the frequency and $i=\sqrt{-1}$.

The phase velocity c is equal to the angular frequency $\omega=2\pi f$ divided by the real wave number. Then

$$C_{p\pm} = \left(\mathrm{Re}\left(\frac{1}{V_{p\pm}}\right)\right)^{-1} \quad (23)$$

$$C_s = \left(\mathrm{Re}\left(\frac{1}{V_s}\right)\right)^{-1} \quad (24)$$

where Re denotes the real part. Following Dutta and Ode (1983), we define the attenuation coefficients as

$$\alpha_{p\pm} = 17.372\pi\frac{\mathrm{Im}(V_{p\pm})}{\mathrm{Re}(V_{p\pm})} \quad (25)$$

$$\alpha_s = 17.372\pi\frac{\mathrm{Im}(V_s)}{\mathrm{Re}(V_s)} \quad (26)$$

where Im denotes the imaginary part.

3. Results and discussion

Wave velocities and attenuation were computed by first computing the values of porosity ϕ from table 1, in which the corresponding values of composite densities from which the velocities and the attenuation were calculated for different media using equations (23), (24) and (26) are given.

The results obtained for the various parameters of interest are shown in table 2.

The determination of the wave velocities and attenuation in porous media were considered, i.e. silt, medium sand and coarse sand, which are necessary to determine the level of energy loss for each of these media.

The results of this work for the considered media are in agreement with the previous work of Carcione (1993) in which sandstone was considered. In the work of Carcione (1993) for

Table 4. Extrapolated values of characteristic curve of attenuation α_p versus Log(f) (Hz) for water (Carcione 1998).

α (dB) (water)	Log(f) (Hz)	α (dB) (water)	Log(f) (Hz)
0.000	1.00	0.290	4.55
0.000	1.20	0.310	4.65
0.000	1.40	0.320	4.70
0.001	1.80	0.340	4.80
0.002	1.92	0.350	4.90
0.002	2.15	0.330	5.00
0.003	2.30	0.310	5.06
0.004	2.50	0.290	5.10
0.005	2.70	0.264	5.18
0.008	2.90	0.265	5.20
0.012	3.10	0.245	5.22
0.020	3.30	0.225	5.28
0.028	3.45	0.210	5.30
0.030	3.60	0.195	5.38
0.050	3.70	0.178	5.40
0.065	3.80	0.158	5.48
0.080	3.90	0.140	5.49
0.100	4.00	0.120	5.58
0.115	4.05	0,110	5.60
0.131	4.10	0.090	5.68
0.150	4.15	0.070	5.70
0.165	4.20	0.060	5.78
0.180	4.25	0.048	6.00
0.200	4.30	0.030	6.15
0.220	4.35	0.020	6.30
0.230	4.44	0.012	6.50
0.252	4.48	0.010	6.68
0.270	4.50	0.005	6.85

Table 5. Extrapolated values of characteristic curve of attenuation α_p versus Log(f) (Hz) for gas (Carcione 1998).

α (dB) (gas)	Log(f) (Hz)	α (dB) (gas)	Log(f) (Hz)
0.000	1.00	0.100	4.10
0.000	1.20	0.080	4.12
0.000	1.40	0.065	4.35
0.001	1.55	0.045	4.45
0.002	1.92	0.040	4.60
0.004	2.14	0.300	4.74
0.005	2.34	0.020	4.90
0.008	2.50	0.010	5.10
0.013	2.70	0.008	5.18
0.020	2.88	0.005	5.46
0.030	3.04	0.003	5.75
0.040	3.20	0.002	5.58
0.055	3.20	0.001	6.20
0.078	3.50	0.000	6.24
0.092	3.55	0.000	6.42
0.100	3.70	0.000	6.62
0.108	3.80	0.000	6.80
0.110	4.00		

sandstone filled with water the wave velocities for P- and S-waves are 2205 m s^{-1} and 928 m s^{-1}, respectively, while the attenuation of shear waves is given as 2.044 dB.

In this work the velocities of P-waves for silt, medium sand and coarse sand are 1827.4 m s^{-1}, 22060.9 m s^{-1}, and 2208.1 m s^{-1}, respectively, while that of S-waves was obtained to be 1046.0 m s^{-1}, 1030.6 m s^{-1} and 975.70 m s^{-1}, respectively.

The values of velocities for these media are greatly influenced by their composite densities for sandstone filled with water, 2155 kg m^{-3}, silt, 1907.5 kg m^{-3}, medium sand, 1990 kg m^{-3} and coarse sand, 2155 kg m^{-3}.

4. Conclusion

In this work, Biot's theory has been extended in order to include the relaxation mechanism arising from grain–fluid interaction in which silt, medium sand and coarse sand filled with water have been investigated.

The values of the porosity of coarse sand, 0.3, and medium sand, 0.4, are the major determinants of the composite densities of these media. Coarse sand, being more porous, has the highest density and silt, which is fine grained has the lowest density of the considered media.

Table 1 clearly shows that the more coarse the grain, the greater the density in kg m^{-3} and it decreases in the following order: coarse sand, medium sand and silt. The velocities of P-waves increase with composite density. Conversely, in the case of S-waves, the velocities of S-waves decrease with composite density. We should recall that shear waves cannot propagate through ordinary fluid and gas; perhaps this property might be one of the reasons for this behaviour.

Moreover, the attenuation of shear waves was also considered, and it was observed that it increases as the densities of the media decrease and is in agreement with the available literature.

References

Biot M A 1962 Mechanics of deformation and acoustic propagation in porous media *J. Appl. Phys.* 33 1482–98

Biot M A and Willis D G 1957 The elastic coefficients of the theory of consolidation *J. Appl. Mech.* 24 594–601

Carcione J M 1998 Viscoelastic effective rheologies for modelling wave propagation in porous media *Geophys. Prospect.* 46 249–70

Carcione J M 1993 A 3-D time-domain wave equation for viscoacoustic saturated porous media *Eur. J. Mech.* A 12 53–71

Dutta N C and Ode H 1983 Seismic reflections from a gas–water content *Geophysics* 48 148–62

Gist G A 1994 Fluid effects on velocity and attenuation in sandstones *J. Accoust. Soc. Am.* 96 1158–73

Gurevich B 1996 On 'Wave propagation in heterogeneous porous media: a velocity stress finite-difference method' by Dai, Vafidis and Kanasewich (Geophysics 60 327-40) *Geophysics* 61 1230–2

Gurevich B and Lopatnikov S L 1995 Velocity and attenuation of elastic waves in finely layered porous rocks *Geophys. J. Int.* 121 933–47