

A Bi-Criteria Algorithm for the Simultaneous Minimization of Makespan and Number of Tardy Jobs on a Single Machine with Sequence Dependent Set-up Time

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Abstract: Scheduling models based on efficient resources utilization without sufficient consideration for customer satisfaction or vice-versa is inadequate in many real-life multi-criteria scheduling problems. This work was aimed at the development of a bi-criteria algorithm for the simultaneous optimization of Makespan (C_{max}) and Number of tardy jobs (N_T) in a single machine problem with Sequence dependent set-up time. An existing single criterion algorithm, the Set Sequencing Algorithm, was adapted to solve the bi-criteria problem. Graphical User Interface (GUI) based software of the new algorithm was developed and its performance was evaluated with a real life problem and 150 randomly generated problems, with problem sizes ranging from between 20 and 150. The values of C_{max} and N_T of the solution sequences were analysed. The output sequences gave an average reduction of 32.10% in both measures of performance compared with the input sequences. The software was also shown to be user friendly. It is concluded that Set sequencing Algorithm is suitable for bi criteria scheduling optimization.

Key words: Algorithm, machine set-up, scheduling, set sequencing, traveling salesman

INTRODUCTION

Sequencing and scheduling have become crucial decisions of any effective planning function in both the manufacturing and service industries. Scheduling decision function is thus becoming more complicated with multiple, and often conflicting goals. Heightened business competition, due to globalization, tighter regulatory requirements, and increasing level of customers' tastes has increased the need for effective and robust planning function. For instance companies are under pressure to offer timely services for customers' satisfaction while at the same time strive for efficient utilization of available resources for profitability. These business objectives will often require conflicting scheduling criteria. Invariably these scenarios result into some multi objectives formulation.

One of the major criticisms of classical scheduling theory has been its use single objective scheduling models which are considered too idealistic for most problem environment (Pinedo, 1995; Hoogveen, 2005; Oyetunji and Oluleye, 2008). The literature on multi-criteria is really sparse. Most multi-objective scheduling problems are usually NP hard in complexity term, (Chen and Bulfin, 1993), with the modeling and development of solution methodologies for these scenarios posing additional challenge for operations researchers.

This work focuses on the development of a solution algorithm for the bi-criteria problem of the simultaneous

optimization of Makespan (C_{max}) and Number of tardy jobs (N_T) on single machine with Sequence dependent set-up time which, using scheduling notation, is represented as $1|S_{ij}|C_{max}, N_T$.

While Makespan (C_{max}) is the completion time of the last job, a job j is said to be tardy if it is completed after its due date d_j . The Makespan (C_{max}) is synonymous with efficient resources utilization while due date related criteria like Number of tardy jobs (N_T) are generally geared towards customer's satisfaction. Efficient resources utilization without sufficient consideration for due date related criteria may be inadequate for surviving in any competitive industry with growing customer demand for prompt service delivery. Hence varieties of algorithms and formulations are being developed for these various classes of problems.

Normally in a single machine scheduling environment with all jobs $j = 1, 2, 3, \dots, N$ having the same ready time (e.g., $r_j = 0$) and no sequence-dependent setup times, the maximum completion time or C_{max} is independent of the sequence and equal to the sum of the processing times of the N jobs. However, in many realistic problems there are sequence dependent setup times and in such situations C_{max} becomes a function of the sequence. In this work a set of N jobs with different due dates are available for sequencing under static scheduling environment. For any sequence S_i there is unique C_{max} (makespan) and a N_T (number of tardy jobs). Problems in bi-criteria scheduling literature usually

involve a combination of scheduling criteria minimized either simultaneously or sequentially. According to Chand and Schneeberger (1996), minimization of weighted completion time subject to minimal value of T_{max} was first considered by Smith who proposed an $O(N \log N)$ algorithm. They analysed Smith's algorithm and modified this heuristic for solving the problem of minimizing the weighted flow time constrained by T_{max} .

Wassenhove and Gelders (1990) suggested a pseudo-polynomial algorithm for the problem involving flow time (F) and maximum tardiness (T_{max}) criteria, note while F relates to inventory holding cost T_{max} relates to meeting due dates. They used the modified due date concept and modified Smith's algorithm to solve the problem. Nelson *et al.* (2001) extended their work to optimizing these three criteria simultaneously. Daniels (2004) used preference information on an interactive basis to solve the problem of optimizing the multiple criteria. He suggested that, along with providing a preferred schedule, preference information can be exploited to improve the computational effort to solve the problem. Dominance rules have been used based on the preference of the performance measures to develop the solution methodology to solve the problem.

Azizoglu and Kondacki (1991) used a branch and bound method to solve the total earliness and total tardiness problem for the single machine problem. The lower bounds were developed using shortest processing time rule, longest processing time rule and earliest due date rules. Wang (1997) minimized makespan and average completion Time on single machine with release dates jobs. While makespan and Due dates related objectives have been combined for single machine problem the literature is very sparse for the extension with sequence dependent set-up problem. This extension is the focus of this study.

METHODOLOGY

Study location: This study, carried out between January and September 2010, involved data gathering in from steel company located in Ikeja, southwest Nigeria. The company operates on made to stock basis with production planning based on market survey, only a maximum of five of the company's products are sufficiently demanded to warrant their production as at the time of the study and planning is on weekly basis to minimize frequency of setup.

Solution algorithm: The Set-Sequencing Algorithm (SSA) originally developed to solve the Single machine sequence dependent machine set up problem ($1|S_{ij}|C_{max}$) or the traveling salesman problem was adapted to solve the bi criteria problem $1|S_{ij}|C_{max}, N_T$. The SSA is a tour improvement algorithm in which a complete sequence S of N parts is viewed as comprising a set of N matrix

elements (links). Hence, set sequencing is defined as the transformation of a known sequence (S_i) to a new sequence (S_{i+1}) by feasibly replacing a subset of its links (Lr) with equal number (M) of candidate links (Lc) using the recursive function:

$$Va(S_i) = Va(S_{i-1}) + D(Lr, Lc, M)$$

where, $Va(S_i)$ and $Va(S_{i-1})$ are the respective sequence values and $D(Lr, Lc, M)$, the exact amount $Va(S_i)$ is changed by the replacement operation (Oladokun, 2006; Charles-Owaba and Oladokun, 2007; Oladokun and Charles-Owaba, 2008).

Modify set sequence algorithm steps: Generally two approaches can be used to develop a bi-criteria algorithm, these are:

- Simultaneous optimization of both criteria
- Sequential optimization, where by a primary criterion is first optimized followed by optimization of the secondary criterion

The proposed method is based on the first approach by adopting weighting factors α and β for C_{max} and N_T , respectively. The algorithm proposed provides solution to the problem and the steps shown in Fig. 1.

Software design methodology: The solution software is designed as a window based program with a mouse and keyboard driven input/output Graphical User Interface (GUI) developed using Visual Basic, Net programming language. The input module, Fig. 2, is capable of loading problem both interactively and through rich text file. The user specifies from the main menu the type of input system to use by clicking on the appropriate icons or by using some keys combination on the keyboard.

EVALUATION AND RESULTS

To examine the resulting algorithm's efficiency, 150 randomly generated Problem and a real life problem were solved. The values of C_{max} and N_T of the solution sequences were then compared with their equivalent values in the respective Input sequences. The input data for case study problem is as shown in Table 1. Relative setup time between different products as established from

Table 1: Set up times (mins), processing time and due date for case study problem

Jobs	1	2	3	4	5
1	-	6	1	3	1
2	7	-	7	7	3
3	8	3	-	1	5
4	1	3	2	-	5
5	8	3	7	6	-
Processing time	9	5	7	3	8
Due date	35	49	21	30	40

- Step 1 : Input the MSP problem matrix. Initialised counter $i=1$, set terminating counter k
- Step 2 : Generate a feasible initial Sequence S_i
- Step 3 : Obtain the aggregated criteria value of $f(S_i) = \alpha C_{max} + \beta N_T$
- Step 4 : Update $Sp = S_i$
- Step 3 : Using the SSA process identify new set of feasible candidates links L_c and form a new sequence S_{i+1} by replacing some incumbent L_u links in S_i with set L_c .
- Step 4 : Calculate the aggregated criteria $f(S_{i+1})$ for the new sequence S_{i+1}
- Step 6 : Obtain the weighted $f(S_{i+1}) = \alpha C_{max} + \beta N_T$
- Step 7 : if $f(S_{i+1}) < f(Sp)$ then update $Sp = S_{i+1}$, otherwise go step 8
- Step 8 : if $i = k$ go to step 12
- Step 9 : $i = i+1$. Go to Step 3
- Step 12 : Report sequence Sp as solution
- Step 13 : Stop

Fig.1:Description of revised SSA for Bi-criteria problem

	1	2	3	4	5	6	7	8	9	10	11	12
1	==											
2		==										
3			==									
4				==								
5					==							
6						==						
7							==					
8								==				
9									==			
10										==		
11											==	
12												==
Processing Time												
Due Date												

Fig. 2: Interactive problem loading men

Table 2: Evaluation of the input sequence $S_i = 1-2-3-4-5$

S.no. Jobs	Completion time	Due date	Tardy jobs
1	9	35	0
2	20	49	0
3	34	21	1
4	38	30	1
5	51	40	1

$C_{max} = 51$ $N_T = 3$

Table 3: Output Sequence $Sp = 3-4-1-5-2$

S.no. Jobs	Completion time	Due date	Tardy jobs	
1	3	7	21	0
2	4	11	30	0
3	1	21	35	0
4	5	30	40	0
5	2	38	49	0

$C_{max} = 38$ and $N_T = 0$

the records, historical data and engineer/supervisor’s opinion, Processing time and Due date are as shown in Table 1.

Table 2 is the evaluation of the current input sequence C_{max} and N_T , where the followings are obtained as follows:

$$C_j = C_{j-1} + P_j + S_{j-1,j} \text{ with } S_{0,1} = 1$$

So that $C_{max} = C_j$ for the last job to be processed.

$$N_T = \sum_{j=1}^N U_j \text{ where } U_j = \begin{cases} 1 & \text{if } C_j > d_j \\ 0 & \text{otherwise} \end{cases}$$

Output solution: Using the proposed Bi-criteria algorithm the following output Sequence Table 3 was obtained.

Computational analysis: One hundred and fifty (150) randomly generated problems were solved on a 2.0 GMZ

Table 4: Computation time and solution value using the Software

Problem size	Input composite criteria	Output composite criteria	Percentage reduction	Processing time (m.sec)
20	119.5	82	31.38	46
40	204	137.5	32.60	153
60	314	214.5	31.69	645
80	455.5	309.5	31.94	3149
100	559.5	377.5	32.53	7911
120	653	439	32.77	20953
140	770	513.5	33.25	43539

microprocessor Personal Computer. The objective values of the input sequences were compared with that of the output sequences shown in Table 4.

CONCLUSION

A bi-criterion scheduling algorithm based on the set-sequence algorithm has been developed and computerized for the simultaneous optimization of Makespan and Number of tardy jobs on single machine with sequence dependent set-up time. The algorithm software is suitable for implementation on a personal computer and achieving an average 32% reduction on both criteria. It is concluded that the adaptation of existing single criterion algorithms for multi-criteria problems is a feasible approach to extend their applications to many real life scheduling problems.

NOTATIONS

j	Index for Jobs $j = 1,2,3,\dots,n$
d_j	Due date of job 'j'
C_j	Completion time of job 'j'
U_j	Number of Tardy jobs
α	Weight for Makespan
β	Weight for Number of tardy jobs
$S_{j-1,j}$	Set-up time between job in position j and job preceding it in sequence S

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