Development and Application of an Inflation-Based Productivity Model

S.A. Oke, M.Sc.[1*], I.O. Oyedokun[2], O.G. Akanbi, Ph.D.[3], and F.A. Oyawale, Ph.D.[3]

[1,2] Department of Mechanical Engiuneering, University of Lagos, Nigeria.
[3] Department of Industrial and Production engineering, University of Ibadan, Nigeria.

*E-mail: sa_oke@yahoo.com

ABSTRACT

The paper presents a productivity model with an inflation component based on an established productivity measurement theory. The motivation for writing this paper is the need for conflict resolution at the implementation of productivity incentive programs. A common complaint from trade unions and organizations is that productivity is inaccurately assessed when the traditional input-output approach is utilized. The support for this argument is that a general rise in prices of materials utilized for production activities, without corresponding added values to the materials takes away the work group productivity efforts. This does not reveal the true measure of productivity. From the results obtained, there is a significant difference between the values obtained when the traditional productivity formula is used to compute the performance of a work group compared with the formula proposed in this work. This may be a strong point and a justification for the trade union argument. The limitation of the study is the difficulty that exists in monitoring the inflation values of the multiple products utilized as inputs into the production activities. For computational activities, a factor is chosen. The novelty of the model could be traced to the fact that it is the first time that such an approach and a systematic analysis would be made through the incorporation of the inflation factor into the productivity models.

(Keywords: inflation, productivity, incentive, modeling, production, gain sharing)

INTRODUCTION

In recent years, there has been a dramatic change in science, technology, environment, economic structures, and industrialization all over the world. This has stimulated an increased business competition towards profitability of business outcomes. This mindset is often the direct result of various government policies such as International Standards Organization (ISO)-certification that monitors the quality of products and the environment in which the manufacturing takes place (Capman et al., 2000; Hale, 1997; Beechner and Koch, 1997; ISO, 1996; Withers and Ebrahimpour, 2000; Yahya and Goh, 2001). Consequently, owners of industrial facilities today are requiring more from their production assets. They need a faster rate of return on investment.

In turn, these transformations the world over have affected the structures and performance of industries at large. It has compelled business managers to embark on various performance – transforming and organizational restructuring initiatives. An important initiative in modern organizations is the concept of profit sharing through productivity schemes (Tangen, 2005).

Although significant success has been recorded on the implementation of productivity schemes, recent labor agitation in industries has led to a suspected common weakness in productivity incentive schemes. Labor/trade union leaders give support to the argument that the inflation component of performance analyses that involve costs usually affects the real results obtained if omitted. It is also argued that a scientific way is needed that allows the incorporation of inflation factors in productivity measurement schemes. Unfortunately, to date, no scientific lead has been offered in the literature on productivity (see Davis, 2004; Narain et al., 2004).

This important challenge is addressed in the current work. In particular, a model that incorporates inflation is developed based on sound mathematical principles. It is hoped that such a model would assist in quantifying the effect of inflation on productivity. With this information, an adequate and fair assessment of a work group using productivity schemes may be made. This then suggests that a prudent management style would be made by avoiding disputes between the

labor and the management with the use of scientific expressions that could convince both the management and the unions in productivity gain sharing.

In the next few paragraphs, this paper presents a scientific review of the literature on productivity. The review clearly shows that the current work has not been attempted in the productivity literature. The literature on productivity is wide ranging, covering areas primarily in manufacturing and service systems (Krishnasamy et. al., 2004; Oke, 2004). In the manufacturing systems, applications could be found in energy intensive industries (Mongia .et. al., 2001), natural gas industry (Hao and Wang, 2000), and precision engineering (Yan et. al., 1998).

Some of the extensive applications of productivity in service systems include the software industry (Morasca and Russo, 2001), insurance industry (Francalance and Galal, 1998), and construction industry (Sonmez and Rowings, 1998). Although productivity as a science entails measurement and analysis, control, and improvement, there is a vast amount of literature that relates to measurement and analysis with less attention paid to productivity improvement and control (Noor, 1998; Tangen, 2005).

In general, productivity has been measured, analyzed, controlled, and improved at the international, national, industrial, company, and individual levels. A few examples are given below. Capman et al. (2003) investigated into the primary productivity and its regulation in the pacific sector of the southern ocean. This is an example of international comparison of productivity.

One of the studies related to national productivity measurement is due to Ramstetter (2004) who investigated into labor productivity, wages, nationality and foreign ownership shares in Thai manufacturing between 1996 and 2000.

At the industrial level, studies have been conducted in various countries at the different sectorial levels. Some examples are in the electronics industry (Helo, 2004), software industry (Faulk et al., 2004; Snir and Bader, 2004; Kennedy et al., 2004; Kuck, 2004; Kepner, 2004), and construction industry (Hancher and Abd-Dlkhalek, 1998). Another level of measurement is carried out at the worker productivity level (Nembhard and Ramirez, 2004).

MODEL FRAMEWORK

The traditional productivity model refers to the ratio of the output obtained from the system (in terms of the products) to that of the input resources utilized in obtaining the output. If 'Q' is the output quantity from the system, 'U', the unit sales price of the products, 'I' the quantity of the various input resources utilized in the system and 'C' is the unit cost price of inputs, then the following mathematical expression about productivity (P) is true:

$$P = \frac{QU}{IC} \tag{1}$$

In simple terms, when inflation is considered, 'n_o' may be introduced as inflation factor, and 't; referred to as the time period for measurement. Then equation (1) may be redefined to incorporate the inflation factor '$(1-n_o)^t$' as given below:

$$P = \frac{QU}{IC(1-n_o)^t} \tag{2}$$

In performing mathematical analysis on the problem, the expressions '$\frac{Q}{C}$', '$\frac{Q}{I}$', and '$\frac{1}{(1-n_o)^t}$' are separated in the main equation (2) so that we can express each of these as a function of Q, I and t respectively. Therefore, the mathematical expression that relates equation (2) as different functions is given as:

$$P = Y(Q) \; X \,(I) \; T(t) \tag{3}$$

Now introducing a second order partial differential equation into the modeling such that all the functions of Q, I, and t are involved, we have the following expressions:

$$\frac{\partial^2 P}{\partial Q^2} + \frac{\partial^2 P}{\partial I^2} = \frac{\partial^2 P}{e^2 T t^2} \tag{4}$$

Note that in this expression 'e' is a constant. In an attempt to convert the partial differentials to differentials, we divide the first component of equation (4) by $\frac{1}{Y}$, the second component by $\frac{1}{X}$,

and the third component which is on the right hand side by $\frac{1}{T}$. Thus, the form of the expression in equation (4) changes a differential equation as shown in equation (5):

$$\frac{1}{Y}\frac{d^2Y}{dQ^2}+\frac{1}{X}\frac{d^2X}{dT^2}=\frac{1}{e^2T}\frac{d^2T}{dt^2} \tag{5}$$

In an attempt to carry out some mathematical manipulations, we observe that:

$$\frac{1}{Y}\frac{d^2Y}{dQ^2}=-m^2 \tag{6}$$

is an expression that could help us in order to progress with the modeling if we keep 'm' constant. We would make an effort to change the form of this equation to:

$$\frac{d^2Y}{dQ^2}+m^2Y=0 \tag{7}$$

Note that the solution for expression (7) is:

$$Y = A\sin(mQ) + B\cos(mQ) \tag{8}$$

However, we should note that $Y = \frac{Q}{C}$. Therefore, equation (8) could be rewritten as:

$$Y = \frac{Q}{C} = A\sin(mQ) + B\cos(mQ) \tag{9}$$

Now, considering the boundary conditions. If the expression $Y = \frac{Q}{C}$ is considered, we note that when the output (Q) from the system is zero, Y is obviously zero.

It is then interesting to find out what the value of B would be in the expression $Y = A\sin(mQ) + B\cos(mQ)$. We note that $Y = 0 = A\sin(0) + B\cos(0) = B$. Thus, when $Q = 0$ and $Y = 0$, then $B = 0$. Now let us consider a situation where $B = 0$ is substituted in the expression (9), i.e. $Y = \frac{Q}{C} = A\sin(mQ) + 0\cos(mQ)$. From here, we obtain the expression:

$$Y = A\sin(mQ) = \frac{Q}{C} \tag{10}$$

Now, let us consider a new boundary condition when one unit of output is produced in the manufacturing system. It means equation (9) would be changed to a new form where Q is replaced with 1 as follows:

$$Y = A\sin(m) = \frac{1}{C} \tag{11}$$

Also, if we consider the expression: $Y = A\sin(mQ)$, the differential of Y with respect to Q is as stated below:

$$\frac{dY}{dQ} = mA\cos(mQ) \tag{12}$$

Note that when $Q = 1$,

$$\left(\frac{dY}{dQ}\right)_{Q=1} = mA\cos(m) = \frac{1}{C} \tag{13}$$

Now from equations (11) and (13), we may state that:

$$\frac{A\sin(m)}{mA\cos(m)} = \frac{\frac{1}{C}}{\frac{1}{C}} \tag{14}$$

This reduces to $\frac{\sin(m)}{m\cos(m)} = 1$, form that suggests the application of Maclaurin's series in mathematics.

Now, we may wish to apply the principles of Maclaurin's series in the formulation of the model. Thus, we note that $\sin m = \left(m - \frac{m^3}{6}\right)$, and $\cos m = \left(1 - \frac{m^2}{2} + \frac{m^4}{24}\right)$, therefore $\frac{\sin m}{m\cos m}$ would give us

$$\frac{m - \frac{m^3}{6}}{m\left(1 - \frac{m^2}{2} + \frac{m^4}{24}\right)}$$

which could be equated to 1.

This is in line with the previous definition of the relationship between sin m and m cos m. After cross multiplying and doing the manipulations, m is obtained as $\sqrt{8}$. In furtherance of the modeling, we need to substitute the value of m = $\sqrt{8}$ in the expression: Y = Asin(mQ). This gives Y = Asin($\sqrt{8}$ Q). If we differentiate Y with respect to Q, then we have $\frac{dY}{dQ}$ = A$\sqrt{8}$ cos $\sqrt{8}$ Q. This expression equals $\frac{1}{C}$. Therefore, at Q = 0, A$\sqrt{8}$ = $\frac{1}{C}$. Now, having known the expression for A, we could then substitute new values in the expression Y = Asin(mQ) to obtain Y = $\frac{1}{C\sqrt{8}}$ sin $\sqrt{8}$ Q.

The process of working through the model with respect to Y would now be followed with respect to X so that we could obtain new result. The starting point is to define the differential equation involving X and a constant n as:

$$\frac{1}{X}\frac{d^2X}{dI^2} + n^2 = 0 \qquad (15)$$

By multiplying through each component of the expression with X, we obtain a rewritten form of the differential equation as:

$$\frac{d^2X}{dI^2} + n^2X = 0 \qquad (16)$$

The solution of this differential equation is:

$$X = Dsin(nI) + Ecos(nI) \qquad (17)$$

This could be equated as $\frac{U}{I}$. Note that D and E are constants. However if the value of D and E are the same, and are equated to λ.

Since we have equated D and E to λ, therefore the expression X = Dsin(nI) + Ecos(nI) could be rewritten as X = λ(sin(nI) + cos(nI)). Note that this expression equals $\frac{U}{I}$.

Now if we introduce boundary conditions of I = 1, and X = U, we could rewrite the expression for X

given previously as X = λ(sin(n) + cos(n)) = U. If we then differentiate X with respect to I, and set a boundary condition at I = 1, then we have:

$$\left(\frac{dX}{dI}\right)_{I=1} = \lambda n(\cos(n) - \sin(n)) = -U \qquad (18)$$

If we divide the expression X = λ(sin(h) + cos(n)) = U by that in equation (18), then we have:

$$\frac{\sin(n) + \cos(n)}{n(\cos(n) - \sin(n))} = -1.$$

From this expression, we note that:

sin(n) + cos(n) = nsin(n) – ncos(n). (19)

If we fall back to Maclaurin's series, we would recall that $\sin(n) \approx n - \frac{n^3}{6}$ and $cos(n) \approx 1 - \frac{n^2}{2}$.

If we substitute the value of n as observed in Maclaurin's series into equation (19), then we have:

$$\left(n - \frac{n^3}{6} + 1 - \frac{n^2}{2}\right) = -n\left(1 - \frac{n^2}{2} - n + \frac{n^3}{6}\right).$$

By solving equation (19) we obtain:

$$\frac{n^4}{6} - \frac{4n^3}{6} - \frac{3n^2}{2} - 2n + 1 = 0 \qquad (20)$$

If we consider $\left(\frac{n^4}{6} - \frac{4n^3}{6}\right)$ to be negligible, if we consider small values of n, then we have a quadratic equation of the form $\frac{-3n^2}{2} - 2n + 1 = 0$ which would give a value of n = 0.387. if we then substitute this value in the expression X = (sin(n) + cos(n)) = U, then we would obtain $\lambda = \frac{U}{\sin 0.387 + \cos 0.837}$, which gives λ = 0.993U. This value of λ could be substituted in the expression X = λ(sin(n) + cos(n)) to obtain a new equation X = 0.993U(sin0.387I + cos0.387I).

As previously shown for other parameters, let us consider similar manipulations for parameters involving e, E, t, T, D, r and n_o. In order to do this, let us suppose that:

$$\frac{1}{e^2 T}\frac{D^2 t}{dt^2} + r^2 = 0 \qquad (21)$$

Here, r is considered a constant. By solving this problem in a similar way as we did for previous work, the solution is presented as:

$$T = D\sin(ert) + E\cos(ert) = (1 - n_o)^{-t} \qquad (22)$$

If we introduce the boundary conditions, then at t = 0, T = 1. Again, noting that E and D are constants, if we set E = 1, then we have an equation that is slightly different from equation (22). It could be labeled as equation (23) as shown below:

$$T = D\sin(ert) + \cos(ert) = (1 - n_o)^{-t} \qquad (23)$$

Now, if we different T with respect to t then we have an expression stated as:

$$\left(\frac{dT}{dt}\right)_{t=0} = D(er)\cos(ert) - (er)\sin(ert) = -(1 - n_o)^{-t}\ln(1 - n_o) \qquad (24)$$

By further simplification, we have:

$$D(er) = -(1 - n_o)^{-t}\ln(1 - n_o) \qquad (25)$$

However, we note from Pythagoras theorem in mathematics that $r = \sqrt{m^2 + n^2}$. The value of m could be obtained from the early part of the work in the text between equations (14) and (15).

This is given as m = $\sqrt{8}$. Also from the text following equation (20), we note that n = 0.387. By using these values in the expression for r, we obtain r = 2.86. Now we would referred to equation (25). If we assume that e = 1, then by substituting r = 2.86 and e = 1 into expression (25), then we have:

$$D = -(1 - n_o)^{-t}\frac{\ln(1 - n_o)}{2.86} \qquad (26)$$

We also need to refer to equation (23). Here, we would substitute the new values of D (as stated in equation (26)), e = e and r = 2.86 in equation (23) to obtain:

$$T = -\frac{(1 - n_o)^{-t}}{2.86}\left(\frac{\ln(1 - n_o)}{2.86}\right)\sin(2.86t) + \cos(2.86t) \qquad (27)$$

This could be further simplified as:

$$T = \frac{-(1 - n_o)^{-t}}{2.86}\left(\ln(1 - n_o)^{-0.3497}\right)\sin 2.86t + \cos 2.86t \qquad (28)$$

Thus, the equation that represents the mathematical model for calculating productivity is given as:

$$P = \frac{1}{C\sqrt{8}}\sin\sqrt{8}Q \bullet 0.993U\left(\sin 0.387I + \cos 0.387I\right) \times$$
$$\left(\frac{(1 - n_o)^{-t}}{2.86}\ln(1 - n_o)\sin 182.86t + \cos 2.86t\right) \qquad (29)$$

In order to make expression (29) simpler we multiply the actual values of $\frac{1}{\sqrt{8}}$ and 0.993 to obtain 0.351. This then replaces the two terms as seen in equation (30).

$$P = \frac{0.351U}{C}\sin\sqrt{8}Q\left(\sin 0.38I + \cos 0.387I\right)$$
$$\left[\frac{(1 - n_o)^{-t}}{2.86}\ln(1 - n_o)\sin 182.86t + \cos 2.86t\right] \qquad (30)$$

Therefore, equation (30) is the final form of the mathematical model that represents productivity for the system being measured.

Recall that we are interested in tracking the inflation component of the model. Since the simplest form of the productivity model is expressed in equation (1), by subtracting equation (1) from equation (30) would give the inflation component of the productivity model. This result is shown in equation (31), and represented as P*.

$$P* = \frac{0.351U}{C}\sin\sqrt{8}Q\left(\sin 0.387I + \cos 0.387I\right)$$
$$\left[\frac{(1 - n_o)^{-t}}{2.86}\ln(1 - n_o)\sin 182.86t + \cos 2.86t\right] - \frac{QU}{IC}. \qquad (31)$$

Now, we would try to expand equation (31) so that we can obtain a neater expression that would permit us to find the turning points. This is stated in equation (32)

$$P* = \frac{0.351U\sin\sqrt{8}Q\sin 0.387I}{C} + \frac{0.351U\sin\sqrt{8}Q\cos 0.387I}{C}$$
$$\left[\frac{(1-n_o)^{-1}1}{2.86}\ln(1-n_o)\sin 182.86t + \cos 2.86t\right] - \frac{QU}{IC}.$$

(32)

Equation (32) could be further broken down as follows:

$$P* = \left[\frac{0.351U \times 0.0493Q \times 0.006751}{C} + \frac{0.351U \times 0.0493Q \times 0.9991}{C}\right]$$

x

$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin 182.86t + \cos 2.86t\right] - \frac{QU}{IC}$$

(33)

When similar terms in equation (33) are merged together, we have the expression below:

$$P* = \left[\frac{0.0001168IQU}{C} + \frac{0.01729IQU}{C}\right]$$
$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin 182.86t + \cos 2.86t\right] - \frac{QU}{IC}.$$

(34)

By simplifying the first component of the expression so that we could obtain a common denominator C, we have expression (35) below:

$$P* = \left[\frac{0.0001168IQU + 0.01729IQU}{C}\right]$$
$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin 182.86t + \cos 2.86t\right] - \frac{QU}{IC}$$

(35)

The result of the simplification is further displayed as equation (36) below:

$$P* = \left[\frac{0.01741IQU}{C}\right]$$
$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin 182.86t + \cos 2.86t\right] - \frac{QU}{IC}$$

(36)

In order to ease the simplification process, letter "I" is brought to the numerator as shown below:

$$P* = \left[\frac{0.01741IQU}{C}\right]$$
$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin 182.86t + \cos 2.86t\right] - \frac{QUI^{-1}}{C}$$

(37)

Further simplification of the problem leads us to the following equation:

$$P* = \left[\frac{0.01741IQU}{C}\right]$$
$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin 182.86t + \cos 2.86t\right] - QUI^{-1}C^{-1}$$

(38)

By differentiating P* with respect to I and keeping all other quantities constant, we have:

$$\frac{dP*}{dI} = \left[\frac{0.01741QU}{C}\right]$$
$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin 182.86t + \cos 2.86t\right] - \left[-\frac{QUI^{-2}}{C}\right]$$

(39)

This is further simplified as:

$$\frac{dP*}{dI} = \left[\frac{0.01741QU}{C}\right]$$
$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin 182.86t + \cos 2.86t\right] + \frac{QU}{CI^2}$$

(40)

The interpretation of equation (40) is the rate of change of inflation with the quantity of input resources utilized in the system. This measures the effect of inflation on the quantity of input resources utilized in the system. In order words, it shows how much the quantity of products is purchased due less to the shrinking purchasing power of money than as a result of inflation. In numerical values, assuming that the money used for purchases of input without inflation could buy 10 units of product, now with inflation, it could buy less, say 8 units of products.

Mathematically, we may be interested in expressing the effect of inflation on the quantity of output produced. This is obtained by finding the differential of P* with respect to Q. Thus, differentiating P* with respect to Q, and making all other factors constants would give.

$$\frac{dP^*}{dQ} = \left[\frac{0.01741IU}{C}\right]$$
$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin182.86t + \cos2.86t\right] - \frac{U}{IC}$$

(41)

The meaning of this expression (41) is that prior to inflation, the quantity of output produced would be more than what is produced when inflation is considered.

Now, we are interested in calculating the effect of inflation on the unit sales price of products. This is done by differentiating P* relative to U while other factors are kept constant. This expression explains how much of the inflated prices of products that the consumer may have to absorb. That is the difference in the price the consumer would buy the goods if the cost of inputs were normal, and when inflation sets in. This is expressed mathematically as:

$$\frac{dP^*}{dU} = \left[\frac{0.01741IQ}{C}\right]$$
$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin182.86t + \cos2.86t\right] - \frac{Q}{IC}$$

(42)

Another investigation that could be attempted is to find out the possible behavior of the changes in inflation relative to the unit cost price of inputs. This is the rate of change of the additional cost on the input prices due to inflation. This indicates how much extra cost is added to the unit price of inputs. Mathematically, we state that we differentiate P* with respect to C. This is shown as follows:

$$\frac{dP^*}{dC} = \left[-\frac{0.01741IQ}{C^2}\right]$$
$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin182.86t + \cos2.86t\right] \cdot \left[-\frac{Q}{IC^2}\right]$$

(43)

Equation (43) is further simplified to obtain a new expression (44) as follows:

$$\frac{dP^*}{dC} = \left[-\frac{0.01741Q}{C^2}\right]$$
$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin182.86t + \cos2.86t\right] + \frac{Q}{IC^2}$$

(44)

Now, we are interested in finding out the turning points. These are to reflect the maximum and minimum points on the curves that may represent the change behaviors. First, we start with the calculation of turning points for changes in inflation relative to the quantity of inputs purchased for productive activities. We note that from equation (40),

$$\frac{dP^*}{dI} = \left[\frac{0.01741QU}{C}\right]$$
$$\left[\frac{(1-n_o)^{-1}}{2.86}\ln(1-n_o)\sin182.86t + \cos2.86t\right] + \frac{QU}{CI^2}$$

We therefore find the double differential of the expression (40) to obtain

$$\frac{d^2P^*}{dI^2} = -2QUC^{-1}I^{-3} = \frac{-2QU}{CI^3}$$

(45)

However, for maximum and minimum points $\frac{dP^*}{dI} = 0$. This means that the changes in P* relative to the changes in I at these turning points is zero. We then equate expression (40) to zero so that we could obtain the value of I. The value of I at this point, referred to as I* is the optimal quantity that the company could purchase economically in the period of inflation. Therefore, from expression (40), we obtain

$$I^2 = \left[0.0061(1-n_o)^{-1}\ln(1-n_o)\sin182.86t + \cos2.86t\right]^{-1}$$

(46)

Finally, we obtain

$$I^* = \left[0.0061(1-n_o)^{-1}\ln(1-n_o)\sin182.86t + \cos2.86t\right]^{\frac{1}{2}}$$

(47)

It should be noted that traditionally, at maximum point, $\frac{d^2P^*}{dI^2} = -ve$, $P^* = +ve$.

Also, at minimum point,

$$\frac{d^2 P*}{dI^2} = +ve, \qquad P* = -ve \quad .$$

Note that in expression (45), we obtained the double differential of P* relative to I. This is the maximum point. If we then attempt to find the *optimum values, we would consider I**. Then the expression $\frac{d^2 P*}{dI*^2} = \frac{-2QU}{CI*^3}$ would be solved.

Therefore at maximum point, we would substitute equation (47) in expression (45) to obtain

$$\frac{d^2 P*}{dI*^2} = \frac{-2QU}{C}$$

$$\left[\frac{1}{0.0061(1-n_o)^{-1} \ln(1-n_o)\sin 182.86t + \cos 2.86t} \right]^{-\frac{3}{2}} \tag{48}$$

It should be noted that the $\frac{d^2 P*}{dI^2}$ considered here is -ve. This is the convention used in mathematics. On calculating the minimum point, we consider the positive part of the differential $\frac{d^2 P*}{dI^2}$.

Mathematically, we calculate it as follows:

$$\frac{d^2 P*}{dI*^2} = \frac{2QU}{C}$$

$$\left[\frac{1}{0.0061(1-n_o)^{-1} \ln(1-n_o)\sin 182.86t + \cos 2.86t} \right]^{-\frac{3}{2}} \tag{49}$$

Having determined $\frac{d^2 P*}{dI^2}$, the next task is to determine $\frac{d^2 P*}{dQ^2}$.

In determining $\frac{d^2 P*}{dQ^2}$, we need to revisit equation (41).

Unfortunately, since expression (41) does not contain any parameter Q, $\frac{d^2 P*}{dQ^2} = 0$. The next task is to find out $\frac{d^2 P*}{dU^2}$. For this situation, we may have to refer to equation (42).

Again, $\frac{d^2 P*}{dU^2} = 0$ since equation (42) does not contain any expression of U. Lastly, we are interested in calculating $\frac{d^2 P*}{dC^2}$. In doing this, we refer to equation (43).

Therefore, the answer obtained is as follows:

$$\frac{d^2 P*}{dC^2} =$$

$$2(0.01741QC^{-3})\left[\frac{(1-n_o)^{-1}}{2.86} \ln(1-n_o)\sin 182.86t + \cos 2.86t \right]$$
$$- 2QI^{-1}C^{-3} \qquad . \tag{50}$$

This could be simplified as:

$$\frac{d^2 P*}{dC^2} =$$

$$(0.03482IQC^{-3})\left[\frac{(1-n_o)^{-1}}{2.86} \ln(1-n_o)\sin 182.86t + \cos 2.86t \right]$$
$$- 2QI^{-1}C^{-3} \tag{51}$$

CASE STUDY AND DISCUSSION OF RESULTS

This section presents a case study of a real life manufacturing company based in Lagos, Nigeria. For the purpose of maintaining confidentiality of the company's name, the organization shall be called Dynamics Building Products Limited (DBP). The company is a member of a large conglomerate with over thirty sister companies in Nigeria. An important characteristic of this group of companies is that the output of one company is the input of the other. Therefore, the company study receives input from two different organizations in the group. The main input to DBP's manufacturing process is aluminum and steel coils. A sister company that specializes in the production of rolled steel/aluminum coils supplies this. The other company helps DBP to coat the roofing sheet in

different colors of red, pale blue, orange, etc. The rolled coils may also be plain.

In order to motivate workers towards improved performance, the management of the organization introduced a productivity incentive scheme that encourages workers to perform optimally. Targets are set above which achievers are compensated. The incentive scheme has been in place for three years with success. During the first year of implementing this incentive scheme, a reasonable number of employees with outstanding performance were rewarded.

There was a sharp decline in the subsequent two years in performance rewards. When the labor union of the organization confronted the management about the decrease in the number of recipients of the award, the management promised to check their computations. After a thorough check, feedback was given to the labor union leaders that the computations were not in error. Management said that the computation is done automatically to calculate the productivity of work groups in the organization. The labor union was not satisfied with the explanation since available records show that the production output during these two periods of decline in performance were higher than in the previous periods.

As a result of intense discussion, the labor union demanded the method of calculation. Management said that production output is different from productivity. They emphasized that the measure of productivity is not the number of output produced by a group of workers but the judicious use of output resources in the achievement of the output. The management then defined the equation used for calculating productivity as $P = \dfrac{QU}{IC}$.

There was an intense argument between the labor union and the management. The point raised is that inflation has reduced their effort. They remarked that it is not sufficient to use the above formula which was stated in equation (1). They proposed a new formula that incorporates inflation. This is stated in equation (2) as $P = \dfrac{QU}{IC(1 - n_o)^t}$.

The union insisted that they would like to know the effect of inflation on the results computed. After many deliberations, they agreed to bring a consultant from a local university who developed the inflation component of the model as stated in equation (38). This is mathematically expressed as:

$$P^* = \left[\frac{0.01741 IQU}{C} \right]$$
$$\left[\frac{(1 - n_o)^{-1}}{2.86} \ln(1 - n_o) \sin(182.86t) + \cos(2.86t) \right] - QUI^{-1}C^{-1}$$

In order to demonstrate how the productivity measurement scheme is applied, the consultant gave an example using the company's products.

For a particular amount, the output quantity of roofing sheets produced is 20,000 units. The average sales price for a unit of product is N750,000 (note that $1 = N157, Nigerian currency). The quantity of various input averages N180,000. The average cost price of inputs is N55,000. Based on this, the simple productivity index of $P = \dfrac{QU}{IC}$

was applied to give $\dfrac{20,000 \times 750,000}{180,000 \times 55,000} = 1.52$.

In order to calculate the value of the inflation of the company's goods we may need to apply equation (38). The expression contains important values such as I, Q, U, C, n_o, and t. In addition to the values of I, Q, U, and C, which have been known and substituted in the prior equation, we set n_o as 0.18. This is on the knowledge of the inflation factor or rate of inflation, which is 18%.

This means that if materials for production cost a value NZ in the current month, it is expected to increase by 18% in the next month. Thus, the new price of those materials would be NZ + 18%. It would be noted that this is the average value but inflation may take place not on a monthly basis but on a much longer period.

If we calculate $QUI^{-1}C^{-1}$ as 0.52, then the actual amount of inflation is obtained from:

$$P^* = \left(\frac{0.01741 IQU}{C} \right)$$
$$\left[\frac{(1 - n_o)^{-1}}{2.86} \ln(1 - n_o) \sin(182.86t) + \cos(2.86t) \right] - QUI^{-1}C^{-1}$$

This is on the assumption that $n_o = 0.18$, and $t = \dfrac{1}{12} = 0.0833$.

Therefore $P^* = \left(\dfrac{0.01741 \times 180,000 \times 20,000 \times 750,000}{55,000} \right)$

x

$\left[\dfrac{(1-0.18)^{-1}}{2.86} \ln(1-0.18)\sin(182.86 \times 0.0833) + \cos(2.86 \times 0.0833) \right] - \dfrac{QU}{IC}$

Thus,
$P^* =$
$854672727.3 \left[0.4264 \ln 0.82 \sin 15.23 + \cos 0.2382 \right] - 1.52$

$= N835,666,763$

We are interested in testing other values of $n_o = 0.18$, and $t = 31$. This gives

$P^* =$
$854672727.3 \left[0.4264 \ln 0.82 \sin(182.86 \times 31) + \cos(2.86 \times 31) \right] - 1.52$

Furthermore, $P^* =$
$854672727.3 \left[0.4264 \ln 0.82 \sin 5668.66 + \cos 88.66 \right] - 1.52$
$= 92288943.36$

Also, For $n_o = 0.5$, and $t = 31$,

$P^* =$
$854672727.3 \left[\dfrac{(1-0.5)^{-1}}{2.86} \ln(1-0.5)\sin(182.86 \times 31) + \cos(2.86 \times 31) \right] - 1.52$

We further have $P^* =$
$854672727.3 \left[0.6993 \ln 0.5 \sin 5668.66 + \cos 88.66 \right] - 1.52$

That is $P^* = 4.34148570.5$

We also note that $n_o = 1$, and $t = 31$ gives

$P^* =$
$854672727.3 \left[0 \ln(1-1.00)\sin(187.86 \times 31) + \cos 88.66 \right] - 1.52$

This finally gives $P^* = 19986759.51$

This section has presented a case study of a roofing sheets manufacturing company in Nigeria. The case discusses the problem that arose in real practice due to productivity incentives scheme that was not properly evaluated. As a result, we demonstrated how productivity of a work group was calculated. Again, we showed how the inflationary factor is incorporated into the model. Based on this, we calculated the effect of inflation on the company's resources. This case study could be adapted to similar situations with minor modifications in the data collection instruments,

CONCLUSIONS

In the current century, there is an increasing convergence of issues and approaches on productivity across national frontiers. This development belies the recognition that the sustenance of global competitiveness at the international market-level by manufacturing companies requires an adequate tool for performance measurement. It is now widely recognized in many parts of the world that inflation is not only wiping away profits, but also the real measurement of the state of performance of a work group when implementing productivity improvement schemes. In order to correct this anomaly and the resulting problem of labor unrest in industries, this work proposes a mathematical model that defines the actual value of inflation in a productivity measurement scheme.

Three prominent questions may readily come to mind in understanding the basis of the presentation proposed here: (1) What are we going to learn from the article that we do not already know?; (2) How can the model improve my organizational performance?; and (3) What ideas of relevant future research are required? This article is new and presents a clearly distilled framework that helps managers in curbing industrial unrest in situations where the labor union feels cheated.

Since it presents a scientific basis for distilling the inflation component from the productivity measurement model, it gives a true position of productivity measure. This is an idea that is ripe for investigation. Unfortunately, no documentation seems to have been made in this regard. Since productivity models are expected to judge the level of performance against which production workers are punished or commended, the model serves as an encouragement to production workers since the target set for obtaining productivity incentives is fair and attainable. This is a strong motivation for improved performance at work. Again, since it is meant to reduce labor agitation with respect to unfair standards, possible periods of down time are avoided. This saving of time is reflected in company profits.

There are a large number of areas where future research may be applied in order to improve and extend the framework presented here. An immediate area of research is the possible application of a soft computing tool to the model proposed here. This is necessary in view of the uncertain nature of data that may be used as input into the model, particularly during productivity planning. Soft computing tools such as fuzzy logic,

artificial neural network, neuro-fuzzy may serve as useful applications.

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ABOUT THE AUTHORS

Sunday A. Oke, M.Sc. graduated in Industrial Engineering from the University of Ibadan, Nigeria with a Bachelor and Master's degrees in 1989 and 1992, respectively. He worked for the IDM Services Limited as a consultant. Mr. Oke lectures in the Department of Mechanical Engineering, University of Lagos. He has reviewed papers for several international journals.

I.O. Oyedokun, is an undergraduate student of the Department of Mechanical Engineering, University of Lagos.

O.G. Akanbi, Ph.D. received his BS from Ohio State University, USA, his M.Sc. and an MBA from Oklahoma State University, and his Ph.D. from the University of Ibadan. He is a Member of Nigerian Institute of Industrial Engineering, Member of the Nigerian Society of Engineers, and a Registered Engineer with the Council of Registered Engineers of Nigeria. He was the General Manager of Conpole Nigeria, Limited. He has presented several international and local papers and reviewed journals. Currently, he Lectures at the University of Ibadan, Nigeria.

F.A. Oyawale, Ph.D., MNSE, Reg COREN, MNIIE is a lecturer in Industrial and Manufacturing Engineering in the Faculty of Technology, University of Ibadan. He is a Member of the Nigerian Institute of Industrial Engineers. He is also a Registered Engineer and specializes in Production Engineering. He is currently the Post-Graduate Coordinator in the Department of Industrial and Production Engineering, University of Ibadan. He has published several books for undergraduate courses and has several years of teaching and research experience. His research interests also include local substitution, renewable energy, and welding.

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