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Application and Comparison of Three Multiobjective Linear Programming Methods

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Abstract

A biobjective production planning problem was modelled using the Compromise Constraint Biobjective LP (CCBLP) method, the traditional Weighted-sum Scalarization (WSS) and Non-preemptive Goal Programming (NGP) approaches. Various preference indices were used to explore the tradeoff options and the L_1 distance metric was used to determine the best compromise solution and the appropriate preference indices. The solution of CCBLP was the closest to the ideal solution with L_1 metric of 0.326 and corresponding preference indices of $w_1 = 0.25, w_2 = 0.75$. Comparison of the results showed that the CCBLP is more sensitive to changes in preference indices than the WSS and NGP methods and hence it is more useful in helping the decision maker to make intelligent tradeoff decisions.

Keywords: Compromise constraint biobjective LP; Weighted-sum scalarization; Compromise solution; Tradeoff decisions; Preference indices; Goal programming

1. Introduction

Decision situations with more than one objective have been encountered with increasing frequency (Koski & Silvennoinen, 1990; Raju et al., 2003; Bilo et al., 2004; Angel et al., 2005; Chakraborty & Chandra, 2005; Da Silva et al, 2006; Francisco & Ali, 2006; Adeyeye & Charles-Owaba, 2008; and Hajkowicz & Higgins, 2008). For instance, in production planning, the Decision Maker (DM) may want to minimize overtime, finished goods inventory, backorders and maximize the total net revenue simultaneously. Some of these objectives often conflict with each other. The conflict arises because an improvement in one objective can only be made to the detriment of one or more of the other objectives. When the objective functions remain in conflict over the decision space, then it is impossible to find a point at which they would assume their optimum values simultaneously and consequently, the classical concept of a common optimal solution does not apply. In such decision situations, the DM has to seek a compromise solution.

Many methods exist by which the compromise solution can be determined. The traditional approach is to use the distance-function methods such as goal programming (GP), compromise programming (CP) and reference point method (RPM). The distance-function approaches use a certain target point in the decision space as a key element to model DM preferences. This point within GP is a vector of aspiration levels which represents the most desired values for the several objectives. Typical GP approaches aim to minimize the distance between the target point and the actual outcome values, thus implementing the strict satisficing model where no solution can be considered to be better than that generating the target values. Within RPM the target point is a vector of reference levels to be used in an interactive way by the DM while within CP it is an ideal point (normally infeasible) which corresponds to the optimum value of each objective. Another traditional approach is to employ the weighted-sum scalarization method (WSS) that performs the optimization of a combination of the objective functions (Romero et al., 1998; Ogryczak 2001; Adeyeye & Charles-Owaba, 2008; Adeyeye & Oyawale, 2010).

It has been argued that the WSS is a very simplistic approach to multicriteria optimization problems and that the solution of the WSS may at times fail to give the real compromise (Adulbhan &Tabucanon, 1977; Chen et al., 1998). Adulbhan and Tabucanon (1977) developed the Compromise Constraint Biobjective LP (CCBLP) method to overcome this drawback for biobjective situations. However, the literature is sparse on wide scale application of CCBLP. One possible reason for this rather poor acceptance may be lack of evidence on the efficacy of this approach. DMs need to know the relative merits of these approaches so that they can be guided aright in deciding which of these methods will best meet their decision needs. In this paper, we apply the CCBLP method to solve a biobjective production planning problem and the solutions are compared with that of the well known traditional WSS and non-preemptive goal programming (NGP) to determine their relative advantages. GP was chosen out of the distance-function models because most of the CP and RPM problems can also be modeled as a GP problem (Romero, 1985; Romero et al., 1998; Ogryczak, 2001). In biobjective decision situations, the DM often indicates his preference by using different weights of the objective functions. But in some complex decision situations the DM may want to explore more options. To arrive at an acceptable solution, sensitivity analysis is often performed by simulating several alternatives from which he may pick his preference. Therefore, the comparison of the models will be based on their sensitivity to changes in preference indices.

The paper is organized as follows; the description of the three approaches is presented in section 2 followed by the mathematical model of the problem. Next are the experimental study, discussion of results and the conclusion.

2. Description of the three approaches

2.1 The weighted-sum scalarization approach (WSS)

In biobjective situations, if the objective functions remain in conflict over the decision space, then it is impossible to find a common optimum. The common practice for finding compromise solutions has been the WSS method that performs the optimization of the combined objectives. The corresponding WSS problem is;

$$\text{Minimize, } z = w_1 f_1^N(x) + w_2 f_2^N(x)$$
$$\text{Subject to:} \qquad (1)$$
$$h_t(x) \leq, =, \geq b_t, \forall t \in T$$

where, $f_1^N(x)$ and $f_2^N(x)$ are the normal forms of the objective functions $f_1(x)$ and $f_2(x)$ respectively. T is a set of structural constraints, while, $h_t(x)$ and b_t are constraint function and right hand value of constraint t respectively for each $t \in T$. In addition, $w_1, w_2 > 0$ and $w_1 + w_2 = 1$

Scalars w_1 and w_2 are the weights assigned to the objectives and determine the importance of each objective.

2.2 Brief Description of the compromise constraint biobjective linear programming (CCBLP) Method

The CCBLP is a modification of the WSS method. Apart from combining the objectives into a single objective, a new constraint called the compromise constraint is derived and added to the original constraint set. According to Adulbhan & Tabuçanon (1977), if $f_1(x)$ and $f_2(x)$ are two objective functions whose feasible maxima are $f_1^*(x)$ and $f_2^*(x)$ respectively and if the functions move toward the feasible region (that is both functions decrease in numerical value simultaneously) then the locus of the point of intersection of the functions is related to;

$$[w_1[f_1(x) - f_1^*(x)] - w_2[f_2(x) - f_2^*(x)]] = 0 \qquad (2)$$

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Equation (2) is referred to as the compromise constraint and is the locus of the point or region common to both objectives. The point where the locus first touches the feasible region is the maximum feasible compromise. When equation (2) is added to the original constraint set, it forces the two objectives to settle down to take where it first touches the feasible region. Actually, the locus passes through the feasible region, creating a new feasible region, it is only that the maximum compromise occurs at the point it first touches it. The procedure of the CCBLP is as follows;

Step 1: Formulate the objective functions;

$$f_1(x) = \sum_{j \in J} c_{1j} x_j \tag{3}$$

$$f_2(x) = \sum_{j \in J} c_{2j} x_j \tag{4}$$

where J is a set of activities

x_j : Level of activity j for each $j \in J$

c_{1j} and c_{2j} are the coefficients of activity j in objectives 1 and 2 respectively for each $j \in J$

Step 2: Formulate the structural constraints of the problem

Step 3: Convert the objective functions, $f_1(x)$ and $f_2(x)$ to their normal forms as follows;

$$f_1^N(x) = \left(w_1 \Big/ \sqrt{\sum_{j \in J} c_{1j}^2} \right) \sum_{j \in J} c_{1j} x_j \tag{5}$$

$$f_2^N(x) = \left(w_2 \Big/ \sqrt{\sum_{j \in J} c_{2j}^2} \right) \sum_{j \in J} c_{2j} x_j \tag{6}$$

with $w_1, w_2 \in IR_+^*$, normalization is done because the coefficients of the objectives are of differing magnitudes and the need to convert them to commensurable units before combining them into a single objective.

Step 4: Solve the resulting optimization problem with each of the objectives to determine their ideal solutions ($f_1^*(x)$ and $f_2^*(x)$) respectively.

Step 5: Combine the two objectives into a single objective as follows;

$$f_{12}^N(x) = \left(w_1 \Big/ \sqrt{\sum_{j \in J} c_{1j}^2} \right) \sum_{j \in J} c_{1j} x_j + \left(w_2 \Big/ \sqrt{\sum_{j \in J} c_{2j}^2} \right) \sum_{j \in J} c_{2j} x_j \tag{7}$$

The objectives are first converted to the same form before combining then into one. A minimizing objective may be converted to a maximizing objective by multiplying it by -1.

Step 6: Derive the compromise constraint and add it to the original constraint set. The compromise constraint may be expressed as;

$$\left(w_1 \Big/ \sqrt{\sum_{j\in J} c_{1j}^2} \right)\left(\sum_{j\in J} c_{1j}x_j - f_1^*(x) \right) + \left(w_2 \Big/ \sqrt{\sum_{j\in J} c_{2j}^2} \right)\left(\sum_{j\in J} c_{2j}x_j - f_2^*(x) \right) = 0 \quad (8)$$

Or simplified as;

$$\left(w_1 \Big/ \sqrt{\sum_{j\in J} c_{1j}^2} \right)\sum_{j\in J} c_{1j}x_j + \left(w_2 \Big/ \sqrt{\sum_{j\in J} c_{2j}^2} \right)\sum_{j\in J} c_{2j}x_j = \frac{w_1 f_1^*(x)}{\sqrt{\sum_{j\in J} c_{1j}^2}} + \frac{w_2 f_2^*(x)}{\sqrt{\sum_{j\in J} c_{2j}^2}} \quad (9)$$

As with the case of the combined objectives, a minimizing objective is converted to a maximizing objective by multiplying it by -1 before the derivation of the compromise constraint. Any of the original objectives or the combined objectives may be used as the objective after the addition of the compromise constraint to the original constraint set. Hence, the problem may be stated as (Adulbhan & Tabucanon 1977);

Maximize (any one of the three objective functions);

$$f_1(x) = \sum_{j\in J} c_{1j}x_j$$

$$f_2(x) = \sum_{j\in J} c_{2j}x_j,$$

$$f_{12}^N(x) = \left(w_1 \Big/ \sqrt{\sum_{j\in J} c_{1j}^2} \right)\sum_{j\in J} c_{1j}x_j + \left(w_2 \Big/ \sqrt{\sum_{j\in J} c_{2j}^2} \right)\sum_{j\in J} c_{2j}x_j$$

Subject to; (10)

$$\left(w_1 \Big/ \sqrt{\sum_{j\in J} c_{1j}^2} \right)\sum_{j\in J} c_{1j}x_j + \left(w_2 \Big/ \sqrt{\sum_{j\in J} c_{2j}^2} \right)\sum_{j\in J} c_{2j}x_j = \frac{w_1 f_1^*(x)}{\sqrt{\sum_{j\in J} c_{1j}^2}} + \frac{w_2 f_2^*(x)}{\sqrt{\sum_{j\in J} c_{2j}^2}}$$

$$h_i(x) = \sum_{j\in J} a_{ij}x_j \le, =, \ge b_i\ ,$$

$x_j \ge 0$, for each $j \in J$

where a_{tj} is the coefficient of activity j in constraint t for each $j \in J$ and $t \in T$

2.3 Non-preemptive goal programming (NGP) method

Generally, any biobjective model can be transformed to a GP model by assigning a reasonably low aspiration level to the minimizing objective and a reasonably high aspiration level to the maximizing objective function. However, in some cases, the ideal solutions of the objectives $f_1^*(x)$ and $f_2^*(x)$ are taken as the aspiration levels of objectives 1 and 2 respectively because a priori determination of goals may be difficult without the previous exploration of the potentials provided by the objectives. If the aspiration levels are set arbitrarily, suboptimal or even dominated solution may be computed in such situations. The GP does not attempt to maximize or minimize the objectives directly. Rather, it seeks to minimize the deviations between the desired goals and the compromise solutions to be obtained according to the assigned weights. For the case under study, the DM wishes to minimize the total sum of production costs and also maximize the capacity utilization of production facilities. Hence, we need to minimize the positive deviation (d_1^+) and negative deviation (d_2^-) from the cost and capacity utilization goals respectively. Therefore the NGP model is given as:

Minimize, a = $w_1 d_1^+ + w_2 d_2^-$

Subject to:

$$f_1(x) + d_1^- - d_1^+ = f_1^*(x)$$

$$f_2(x) + d_2^- - d_2^+ = f_2^*(x)$$

$$h_t(x) \leq, =, \geq b_t, \forall t \in T \quad (11)$$

where $d_1^+, d_1^-, d_2^+, d_2^- \geq 0, d_1^+ \times d_1^- = 0$ and $d_2^+ \times d_2^- = 0$

3. Mathematical model of the problem

The case to which we apply the CCBLP is a biobjective decision situation in which a manufacturer wants to determine the quantities of each raw material to feed into each production facility at each stage of production such that his objectives are realized. The decision maker is interested in two objectives, namely;

(i) Minimization of the total sum of production costs

(ii) Maximization of the capacity utilization of the production facilities

3.1 Assumptions of the model

The following assumptions are set to construct the mathematical model of the problem.

(i) The single product produced by the company requires many raw materials. We shall denote the raw material index by, $i \in \{1,2,...,I\}$

(ii) The production is in stages with each stage having several machines that perform similar functions. We shall denote the machine index by, $g \in \{1,2,...,G\}$ and the stage index by, $k \in \{1,2,...,K\}$.

(iii) Each production facility at stage k requires raw materials and/ or intermediate products from the preceding stage (stage $k-1$) and supplies output to the next stage (stage $k+1$).

(iv) In-process inventory is not allowed

(v) Losses of materials during processing are negligible

(vi) The unit production cost vary from machine to machine within a stage

(vii) No limitation on raw material availability

3.2 Notations

M : Set of raw materials

N : Set of production facilities

P : Set of production stages

x_{igk} The quantity of raw material i fed into facility g of stage k of production, for each $i \in M, g \in N$ and $k \in P$

y_{gk} The quantity of intermediate product fed into facility g of stage k of production, for each $g \in N$ and $k \in P$

c_{gk} : *The unit production cost of facility* g *of stage* k *of production, for each* $g \in N$ *and* $k \in P$

D_{gk} : *The available capacity of facility* g *of stage* k *of production, for each* $g \in N$ *and* $k \in P$

w_1, w_2 : *Weights assigned to objectives 1 and 2 respectively*

3.3 Objectives of the model

The two key objectives considered are:

(i) Minimization of the total sum of production costs

(ii) Maximization of the capacity utilization of the production facilities

The cost minimization objective Z_1:

The total production cost is the sum of the products of the unit variable costs and the quantity of material processed by each facility. The criterion is;

$$\text{Minimize, } Z_1 = \sum_{k \in P} \sum_{g \in N} \sum_{i \in M} C_{gk} x_{igk} \tag{12}$$

Maximization of capacity utilization Z_2:

The capacity utilization function is the summation of individual utilization factor (i.e. load divided by maximum capacity).

$$\text{Maximize } Z_2 = \sum_{k \in P} \sum_{g \in N} \left(\frac{\sum_{i \in M} x_{igk}}{D_{gk}} \right) \tag{13}$$

3.4 Constraints of the Problem

In addition to the objectives associated with this problem, the model structure for this decision process will consist of the following constraint types:

(i) Available production capacity of each facility at each stage of production
(ii) Material proportion constraints
(iii) Balance equations of materials throughout the process
(iv) Full capacity constraint

Capacity constraint:
The total amount of materials fed into a facility should not exceed the capacity of the facility.

$$\sum_{i \in M} x_{igk} + y_{gk} \le D_{gk} \quad ; \text{for each } g \in N \text{ and } k \in P \text{, and } y_{gk} = 0 \text{ for } k = 1 \qquad (14)$$

Material proportion constraint:

The quantity of material i fed into facility g of stage k of production is measured as a ratio of a base material r_b for that stage.

$$x_{igk} \Big/ x_{r_b gk} = \gamma_{igk} \; ; r_b \ne i \text{ and for each } g \in N \text{ and } k \in P \qquad (15)$$

The linear form of equation (15) is given by

$$x_{igk} - \gamma_{igk} x_{r_b gk} = 0 \qquad (16)$$

Balance Equations of Materials:

Since loses are negligible and in-process inventory is not allowed, the quantity of materials fed into the stage $k-1$ of production is equal to the output fed to stage k of production as intermediate product. The material balance constraint is given by;

$$\sum_{g \in N} \sum_{i \in M} x_{igk} + y_{gk} = \sum_{g \in N} y_{g,k+1} \; ; \text{for each } k \in P \qquad (17)$$

Full Capacity Constraint:
Management requires that the factory operates at its full capacity. The bottleneck stage determines the full capacity of the factory.

$$\sum_{g \in N} \sum_{i \in M} x_{igs} + y_{gs} = \sum_{g \in N} D_{gs} \; ; \text{ where } s \text{ is the bottleneck stage.} \qquad (18)$$

3.5 The case study

We consider tooth paste manufacturing situation in which the paste manufacturer wants to determine the material mix for each facility at each stage of production such that he will get maximum realization of his objectives. The production process of toothpaste can be generally considered as composed of three major stages, namely; premix, processing and storage. The raw materials with their required proportions are given in table I while the production cost coefficients and capacities of respective production facilities are presented in table II.

4. Experimental study

4.1 Experiment

Experiments were performed by simulating different alternatives with various preference indices. Three different treatment combinations of the preference indices were used to study the behaviour of the three methods (Table III). The deviations were weighted equally, hence the L_1 distance metric were computed for the preference indices and the solution with the minimum L_1 metric selected as the best compromise.

Mathematically, the L_1 distance metric in its discrete form is given as;

$$L_1 = w_1\left|\frac{f_1^*(x) - f_1^{'}(x)}{f_1^*(x) - f_1^{**}(x)}\right| + w_2\left|\frac{f_2^*(x) - f_2^{'}(x)}{f_2^*(x) - f_2^{**}(x)}\right| \quad (19)$$

$f_1^{'}(x)$ and $f_2^{'}(x)$ are the objective values for the compromise solution of objectives 1 and 2 respectively while $f_1^{**}(x)$ and $f_2^{**}(x)$ are the objective values of the anti-ideal solutions of objectives 1 and 2 respectively.

4.2 Results and Discussion

The three methods have been able to help the DM to determine the material mix for each facility at each stage of production. However, their sensitivities to preference indices or relaxation in the objectives vary, hence their usefulness in helping the DM to make intelligent tradeoff decisions about the different objectives also vary. A summary of the solutions is given in Table IV. The L_1-distances were not computed for cases where the so-called compromise solutions are identical to any of the ideal solutions. Due to the lack of sensitivity of the WSS and NGP to small relaxations in the objectives their alleged 'best' compromise solution could mislead the DM. The compromise solution for the CCBLP model was sensitive to changes in preference indices (Table IV). The solution of CCBLP was the closest to the ideal solution (L_1 metric=0.326) with the corresponding preference indices of $w_1 = 0.25, w_2 = 0.75$.

The differences in the sensitivities of the methods to relaxations in the objectives are due to their constraint sets. In the case of WSS, the solution merely takes one of the extreme points of the feasible region. This could be misleading because the point of the real best compromise may not coincide with any extreme point of the convex set. In the case of NGP, the addition of goal constraints to the structural constraints changes the feasible region and creates new vertices. Once the goal constraints have been added, the feasible region is defined and relaxations in the objectives does not change the feasible region hence the solution is limited to the already defined vertices. The vertices of the new feasible region consist of some of the vertices of the original feasible region and the new vertices introduced by the goal constraints. This appears to be the reason for the identical solution of WSS and NGP model when w_1 takes the two different values: 0.75 and 0.5 (Table IV). The difference in the solution of WSS and NGP models for $w_1 = 0.25$ might be that the NGP solution takes one of the new vertices introduced by the goal constraints. Since the best compromise solution may not coincide with any of the vertices, the WSS and NGP cannot locate the real compromise except where the real compromise coincides with one of the vertices.

In the case of CCBLP the compromise constraint passes through the original feasible region and forces both objectives to settle on a common point in the boundary of the original feasible region, even other than the extreme points of the convex sets. The CCBLP gives the real compromise solution. Since each relaxation of the objectives requires that the compromise constraint be derived a new different relaxation of the objectives cannot give identical solutions unlike the WSS and NGP.

The formulation of the WSS model is straight forward and easy. It is not necessary that the ideal solutions be known before hand and is not limited to biobjective decision situations. In NGP, once the goals have been added relaxations in the objectives are achieved by assigning appropriate weights to the deviational variables in the achievement function. Although the CCBLP is very sensitive to relaxations in the objectives, its formulation is more involving. The ideal values of each objective must be determined and used together with the assigned weights to construct the compromise constraint. Each relaxation requires that the compromise constraint be derived anew. Further studies are required to extend the CCBLP to handle more than two objectives and also to determine the efficiencies of the various methods in terms of solution time.

5. Conclusion

A biobjective production planning problem was modelled and solved using the CCBLP, WSS and NGP methods. The exploration of the various tradeoff options was made by using different preference indices for the objectives to generate the possible solutions. The CCBLP was found to be more sensitive to changes in preference indices than the WSS and the NGP models and hence more useful in helping the DM to make intelligent tradeoff decisions. The CCBLP is limited to biobjective decision situations whereas the WSS and NGP can handle two or more objectives. The CCBLP needs to be extended to handle cases involving more than two objectives. The comparison of the models with an application of solver like CPLEX or AMPL and the efficiencies of the various methods is recommended for further study.

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Table I: Raw Materials Required with their Respective Proportions

Stage	Raw Material	Proportion/ Ratio
Premix	Carboxymethylcellulose (CMC)	10% of glycerin
	Distilled Water	130% of glycerin
	Glycerin	
Processing	Intermediate product from premix stage	
	Moisturizing Agent (MA)	6.25% of intermediate product from Premix
	Preservatives	1.042% of intermediate product from Premix
	Abrasives	96% of intermediate product from Premix
	Flavour	5.21% of intermediate product from Premix

Table II: Major Production Facilities with Corresponding Capacities and Cost Coefficients

Stage of Production	Facility Name	Capacity/Month (Kg)	*Normalized cost Coefficient/Kg of Material Processed*
Premix	Premix Vessel 1 (PM1)	9600	2.00
	Premix Vessel 2 (PM2)	14400	1.20
	Premix Vessel 3 (PM3)	24000	1.00
Processing	Processing Plant 1 (PP1)	25000	2.00
	Processing Plant 2 (PP2)	25000	1.80
	Processing Plant 3 (PP3)	40000	1.40
	Processing Plant 4 (PP4)	30000	1.60
Storage	Filling Machine 1 (FM1)	80000	0.30
	Filling Machine 2 (FM 2)	45000	0.45
	Filling Machine 3 (FM 3)	20000	0.20

Table III: Preference Indices for the Experiments

S/N	Preference Indices	Remark
1	$w_2 = 1$	Only objective 2 is important to the DM.
2	$w_1 = 0.25, w_2 = 0.75$	Objective 2 is three times more important than objective 1
3	$w_1 = w_2 = 0.5$	The two objectives are considered to be of equal importance
4	$w_1 = 0.75, w_2 = 0.25$	Objective 1 is three times more important than objective 2
5	$w_1 = 1$	Only objective 1 is important

Table IV: Summary of the Results Showing Costs, Utilization (%) of Production Facilities and L_1 metric

Facility Name	$w_1=1$	$w_1=0.75$, $w_2=0.25$			$w_1=0.5$, $w_2=0.5$			$w_1=0.25$, $w_2=0.75$			$W_2=1$
	Ideal Solution Of $f_1(x)$	LCOF	NGP	CCBLP	LCOF	NGP	CCBLP	LCOF	NGP	CCBLP	Ideal Solution of $f_2(x)$
PM 1	100	100	100	100	100	100	100	100	100	100	100
PM 2	100	100	100	100	100	100	100	100	100	100	100
PM 3	100	100	100	100	100	100	100	100	100	100	100
PP 1	20.3	20.3	20.3	20.3	20.3	20.3	25.2	100	100	58.4	100
PP 2	100	100	100	100	100	100	100	100	100	100	100
PP 3	100	100	100	100	100	100	97	50.2	100	76.2	50.2
PP 4	100	100	100	100	100	100	100	100	33.6	100	100
ST 1	100	100	100	66.8	43.9	43.9	43.9	43.9	43.9	43.9	43.9
ST 2	0.18	0.18	0.18	59.1	100	100	100	100	100	100	100
ST 3	100	100	100	100	100	100	100	100	100	100	100
$f_1(x)$= Cost (₦)	247678	247678	247678	252046	254416	254416	255141	266367	262381	260130	266367
$f_2(x)$= Capacity	328201	328201	328201	338577	345670	345670	346401	357622	350983	351365	357622
Distance Metric (L_1)	-	-	-	0.337	0.388	0.388	0.391	-	0.366	0.326	-
Increase In Cost (%)		0.0	0.0	1.8	2.7	2.7	3.0	7.5	5.9	5.0	7.5