A BICRITERIA MODEL FOR PRODUCTION PLANNING IN A TOOTHPASTE FACTORY

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ABSTRACT

This paper presents a bicriteria formulation of the material allocation to production facilities problem. The system under consideration is a toothpaste factory. Two objectives of the factory were identified, namely; (i) Minimization of the total sum of processing costs (ii) Maximization of the capacity utilization of production facilities. Linear Combination of Objective Functions (LCOF) method was used to solve the problem for the situation where the objectives were of equal importance. The solutions were compared to that of goal programming (GP) and they were found to be identical. The least utilized production facility was processing plant 1 with utilization of 20.32% followed by filling machine 1 with utilization of 43.85%. All the other production facilities were operated at 100% capacity. For LCOF, one problem was solved to obtain the solution while in the case of GP three problems were solved. LCOF is superior to GP in terms of simplicity and time savings.

Key Words: Bicriteria; Linear combination of objective functions, Production planning; goal programming

1. INTRODUCTION

Optimization situations with more than one objective are very common. Almost every important real world problem involves more than one objective (Roux et al, 2008, Rommelfanger 2007, Jozefowska and Zimniak 2008 and Carlos Gomes da Silva et al, 2006). For instance, in reservoir management the decision may be to increase electricity generation and irrigated surface area for agricultural benefits (Michalland et al, 1997), in plant design the objectives may be both cost minimization and minimization of the environmental impact (Dietz, 2006) and in oil refinery scheduling, the objectives might be minimize flaring of gasses, minimize high sulphur crude, minimize cost, etc (Steuer, 1986). These objectives are often in conflict with each other. As a result it is impossible to find a point in the decision space at which they assume their optimum values simultaneously. The common practice is for the analyst to seek a compromise solution according to the preference indices of management or decision maker.

The particular decision situation that this paper deals with is that of a toothpaste factory. The system under consideration is a multi-stage multi-facility production system. The problem therefore, is that of determining the allocation of process materials to production facilities in such a way that management objectives will be realized. Two objectives are considered, namely; (i) minimization of the total sum of processing costs and (ii) maximization of the capacity utilization of the individual production facilities. The second objective ensures that each facility is loaded to the maximum possible given the existing conditions. The interest of management is to find a solution that gives the best compromise among the objectives according to its preferences. In an earlier study, Adeyeye and Charles-Owaba (2008) used goal programming (GP).They solved three problems; first, the objectives were solved individually to determine their ideal values. Next, the respective ideal values of the objectives were used as the aspiration levels. Finally, a GP problem was formulated and solved. The exploration of the potentials provided by the objectives was necessary because a priori determination of goals could be difficult or too arbitrary. Arbitrary statement of goals can lead to suboptimal and even dominated solution. Although Adeyeye and Charles-Owaba's, (2008) approach ensured that suboptimal solution was not computed, it was however, tedious and lengthy.

In this study the Linear Combination of Objective Functions (LCOF) approach will be used and the result compared with that of GP to determine their relative merits.

2. BRIEF DESCRIPTION OF THE LINEAR COMBINATION OF OBJECTIVE FUNCTIONS (LCOF) APPROACH

The LCOF is made operational by combining the objective functions together using a weighting factor, w_i for each objective. The weighting factor denotes the relative importance of the objectives. The procedure of the LCOF is presented below (Adeyeye and Oyawale, 2010).

Step 1: Convert the minimizing objective to the maximizing form by multiplying it by -1.

Step 2: Normalize the objective functions. Normalization is very important because of the following;

i. The objectives are more often than not of different units. They must be dimensionally consistent before they can be combined together.

ii. The coefficients of the terms in the objective functions are often of different order of magnitude. Consider the objective functions $f_1(x)$ and $f_2(x)$. If β_{1l} and β_{2l} are the coefficients of the l^{th} term in $f_1(x)$ and $f_2(x)$ respectively, and $\beta_{1l} >>> \beta_{2l}$ for some or all the terms then, $f_1(x)$ dominates $f_2(x)$ if combined without normalization.

The objective functions may be normalized as follows (Adulbhan and Tabucanon, 1977 and Adeyeye and Oyawale, 2010).

The normal form of $f_1(x)$ is given by;

$$f_1^N(x) = \left({w_1}\Big/{\sqrt{\sum_{l=1}^{L} \beta_{1l}^2}} \right) f_1(x) \qquad (1)$$

Similarly, the normal form of $f_2(x)$ is given by;

$$f_2^N(x) = \left({w_2}\Big/{\sqrt{\sum_{l=1}^{L} \beta_{2l}^2}} \right) f_2(x) \qquad (2)$$

The objective functions have now become dimensionless and the coefficients are now of comparable order of magnitude.

Step 3: Combine the normal forms of the objective functions into a single aggregate function and add the structural constraints.

$$\begin{aligned} &\text{Maximize } f_{12}(x) = f_1^{\,N}(x) + f_2^{\,N}(x) \\ &\text{Subject to;} \\ &\qquad x \in X \end{aligned} \tag{3}$$

3. PROCESS DESCRIPTION

The production process of toothpaste involves two major stages, namely; (i) Premix and (ii) Processing. Premixing is done in sealed mixing vessels (Premix Vessels) to prevent aeration of the paste. Distilled water, glycerin and carboxymethylcellulose (CMC) are the raw materials for the premix stage. The product of the premixed stage is immediately pumped into the processing plant. The processing plant comprises a highly effective vacuum mixer with mixing and dispersing system which can be used for individual toothpaste formulation. The raw materials for the processing stage are flavours, abrasives, preservatives and moisturizing agents (MA). Loses during paste production are negligible since mixing and processing are done in sealed vessels. The paste is pumped into a feed hopper of the Filling Machine at the end of processing. Figure 1 presents the process flow diagram of the factory under study while table 1 presents the major production facilities with their corresponding capacities and production cost coefficients. Table 2 presents the proportions of paste ingredients.

Table 1: Major Production Facilities with Corresponding Capacities and Cost Coefficients

Stage of Production	Facility Name	Capacity/Month (Kg)	Normalized cost Coefficient/Kg of Material Processed
Premix	Premix Vessel 1 (PM1)	9600	2.00
	Premix Vessel 2 (PM2)	14400	1.20
	Premix Vessel 3 (PM3)	24000	1.00
Processing	Processing Plant 1 (PP1)	25000	2.00
	Processing Plant 2 (PP2)	25000	1.80
	Processing Plant 3 (PP3)	40000	1.40
	Processing Plant 4 (PP4)	30000	1.60
Storage	Filling Machine 1 (FM1)	80000	0.30
	Filling Machine 2 (FM 2)	45000	0.45
	Filling Machine 3 (FM 3)	20000	0.20

Table 2: Raw Materials Required with their Respective Proportions

Stage	Raw Material	Proportion/ Ratio
Premix	Carboxymethylcellulose (CMC)	10% of glycerin
	Distilled Water	130% of glycerin
	Glycerin	
Processing	Intermediate product from premix stage	
	Moisturizing Agent (MA)	6.25% of intermediate product from Premix
	Preservatives	1.042% of intermediate product from Premix
	Abrasives	96% of intermediate product from Premix
	Flavour	5.21% of intermediate product from Premix

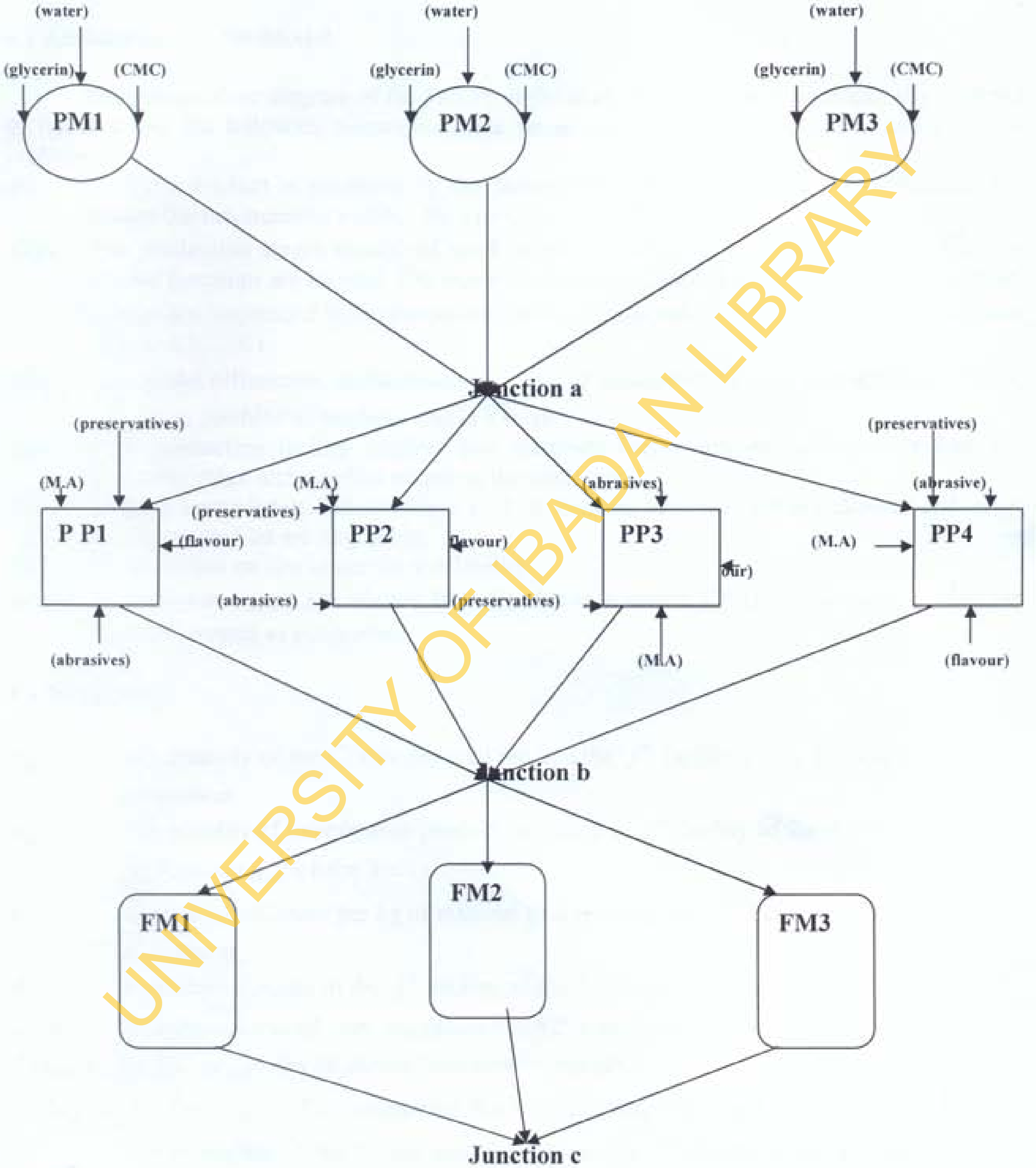

Figure 1: Process Flow Diagram for Toothpaste Production

4. MATHEMATICAL MODEL OF THE PROBLEM

4.1 Assumptions of the Model

The process flow diagram of the factory under study in this paper is schematically depicted in figure 1 and the following assumptions are set to construct the mathematical model of the problem.

(i) A single product is produced by the factory but many raw materials are required. We denote the raw material number by $i, (= 1,2,..., I)$.

(ii) The production stages consist of work centers in which several machines that perform similar functions are located. The machine number is denoted by $j, (= 1,2,..., J)$. The work centers are sequenced in the production technological order. The stage number is denoted by $k, (= 1,2,..., K)$.

(iii) Due to the differences in the model and age of machines, the unit production cost (c_{jk}), vary from machine to machine within a stage.

(iv) Each production facility requires raw materials and/or intermediate product from the preceding stage and supplies output to the next stage.

(v) Stage k immediately follows stage $k-1$. In-process inventory are not allowed and losses during production are negligible.

(vi) No limitation on raw materials availability.

(vii) The weights w_1, w_2 are elicited from the decision maker (DM) by the analyst. They are therefore treated as exogenous.

4.2 Notations

x_{ijk}: The quantity of the i^{th} raw material fed into the j^{th} facility of the k^{th} stage of production

y_{jk}: The quantity of intermediate product fed into the j^{th} facility of the k^{th} stage of production ($y_{jk} = 0$ for $k = 1$).

c_{jk}: The cost coefficient per kg of material processed by the j^{th} facility of the k^{th} stage of production

d_{jk} : Available capacity of the j^{th} facility of the k^{th} stage of the production process

w_1, w_2: Weights associated with objectives 1 and 2 respectively

$f_1(x_{ijk}, y_{jk}), f_2(x_{ijk}, y_{jk})$: The objective functions of objectives 1 and 2 respectively

$f_1^N(x_{ijk}, y_{jk}), f_2^N(x_{ijk}, y_{jk})$: The normalized functions of objectives 1 and 2 respectively

γ_{ijk} : The proportion of the i^{th} raw material fed into the j^{th} facility of the k^{th} stage of production

4.3 Objectives of the model

The two key objectives considered are:

(i) Minimization of the total sum of production costs.

(ii) Maximization of the capacity utilization of the production facilities.

The cost minimization objective:

The total production cost is the sum of the processing costs of the various production facilities. The criterion is;

$$\min imize, f_1(x_{ijk}, y_{jk}) = \sum_{k=1}^{K}\sum_{j=1}^{J}\sum_{i=1}^{I} c_{ijk}(x_{ijk} + y_{jk}) \ ; \ y_{jk} = 0 \text{ for } k = 1 \quad (4)$$

Maximization of capacity utilization:

The capacity utilization function is the summation of individual utilization factor (i.e. load divided by maximum capacity).

$$\min imize, f_1(x_{ijk}, y_{jk}) = \sum_{k=1}^{K}\sum_{j=1}^{J}\left(\frac{\sum_{i=1}^{I}(x_{ijk} + y_{jk})}{d_{jk}}\right) \quad (5)$$

4.4 Constraints of the Problem

Three essential sets of constraints are considered, namely:

(i) Available production capacity of each facility at each stage of production

(ii) Material proportion constraints

(iii) Balance equations of materials throughout the process

4.4.1 Capacity constraint

The total amount of materials fed into a facility should not exceed the capacity of the facility.

$$\sum_{i=1}^{I}(x_{ijk} + y_{jk}) \le d_{jk} \ ; \text{for each } j \in (1,2,...,J) \text{ and each } k \in (1,2,...,K) \quad (6)$$

It is the decision of management to operate the factory at full capacity. The bottleneck stage determines the full capacity of the factory.

$$\sum_{j=1}^{J}\sum_{i=1}^{I}(x_{ijs} + y_{js}) = \sum_{j=1}^{J} d_{js} \ \text{:where } s \text{ is the bottleneck stage} \quad (7)$$

4.4.2 Material proportion constraints

The quantity of i^{th} raw material fed into the j^{th} facility of the k^{th} stage of production is measured as a ratio of a base material r_b for that stage.

$$x_{ijk} \big/ x_{r_b jk} = \gamma_{ijk} \; ; r_b \neq i \text{ and for each } j \in (1,2,\ldots,J) \text{ and } k \in (1,2,\ldots,K) \quad (8)$$

The linear form of equation (8) is given by

$$x_{ijk} - \gamma_{ijk} x_{r_b jk} = 0 \quad (9)$$

4.4.3 Balance Equations of Materials

The intermediate product from stage k of production must be fed into the facilities at stage $k+1$. Since in-process inventory is not allowed and losses during processing are negligible, the material balance at junctions (depicted in figure 1 as junctions a and b respectively) are;

$$\sum_{j=1}^{J}\sum_{i=1}^{I}(x_{ijk} + y_{jk}) = \sum_{j=1}^{J} y_{j,k+1} \text{ ;for each } k \in (1,2,\ldots,K) \quad (10)$$

5. MODEL APPLICATION

The problem is to find the quantities of raw materials and intermediate products to be fed into production facilities at each stage of production such that management would have maximum realization of its objectives. The data in tables 1 and 2 together with fig.1 were used to model the bicriteria problem. Management decided to have equal relaxation on the objectives, that is, $w_1 = w_2 = 0.5$. The cost objective was converted to a maximizing objective by multiplying it by -1. The objectives were normalized in order to make them commensurable and dimensionally consistent (see section 2). They were combined into a single objective and solved subject to the structural constraints.

6. DISCUSSION OF RESULTS

The LCOF method has been able to help management to determine the quantities of materials to be fed to each production facilities at each stage of production (Table 3). A corollary summary is presented in table 4 which shows the percentage utilization of each production facility and the associated costs. The solution of the LCOF was the same with that of GP with equal relaxations on the objectives. The least utilized production facility was processing plant 1(PP1) with utilization of 20.32% followed by filling machine 1 (FM 1) with utilization of 43.85%. The associated production cost was ₦ 254,416.48. The LCOF method was able to arrive at the compromise solution without the evaluation of the ideal solutions of the individual objectives. In this regard, the LCOF method is easier, straight forward and saves time because only one problem was solved.

In the case of GP, Adeyeye and Charles-Owaba (2008) had to evaluate the ideal values of the objectives individually before the statement of goals. This was done to avoid arbitrariness in statement of goals and the attendant problems of suboptimal or dominated solutions. Consequently, they had to solve three problems resulting in a tedious and lengthy process. In situations like this where statement of goals is difficult, LCOF approach may be superior to the GP method. Although LCOF and GP approach arrived at identical solution to the problem under study, it may be misleading to conclude that LCOF and GP will always give identical solutions. Further study is required to know whether LCOF and GP will always give identical solutions in every situation.

Sometimes management is interested in exploring trade-off options. In such situations the analyst simulates several alternatives using different preference structures elicited from management. The results are then presented to management so that it can pick the one that best meet its needs. The method the analyst will use for such simulation must be very sensitive to changes in the preference structures. The performance of GP and LCOF in terms of their sensitivities to changes in preference structure is beyond the scope of this study. It is therefore recommended for further study.

Table 3: Monthly Material Allocation to Production Facilities When Management Decided *To* Have Equal Relaxation on the Objectives (W_1=W_2= 0.5)

Stage	Raw Material	Monthly Allocation to Facility (kg)			
Premix		Premix Vessel 1	Premix Vessel 2	Premix Vessel 3	
	CMC	400	600	1000	
	Water	5200	7800	13000	
	Glycerin	4000	6000	10000	
Processing		Processing Plant 1	Processing Plant 2	Processing Plant 3	Processing Plant 4
	Intermediate product from stage 1	2437	11990	19184	14388
	Moisturizing Agent	152.3	749.4	1199	899.3
	Preservatives	25.4	124.9	199.9	149.9
	Abrasives	2339.5	11510.4	18416.6	13812.5
	Flavour	127	624.7	999.5	749.6
Filling		Filling Machine 1	Filling Machine 1	Filling Machine 1	
	Paste	35081	45000	20000	

Table 4: Summary of the Percentage Utilization of Production Facilities with Associated Costs

Facility Name	Ideal solution of Cost objective (W_1= 1;W_2= 0) Adeyeye and Charles-Owaba (2008)	Equal Relaxation on Objective (W_1=W_2= 0.5)	Ideal solution of Capacity Utilization Objective (W_1=0;W_2=1) Adeyeye and Charles-Owaba (2008)
Premix Vessel 1	100	100	100
Premix Vessel 2	100	100	100
Premix Vessel 3	100	100	100
Processing Plant 1	20.32	20.32	100
Processing Plant 2	100	100	100
Processing Plant 3	100	100	50.20
Processing Plant 4	100	100	100
Filling Machine 1	100	43.85	43.85
Filling Machine 2	0.18	100	100
Filling Machine 3	100	100	100
	Cost = 247678.4	Cost = 254416.48 (2.72% increase)	Cost = 266367.68 (7.55% increase)

7. CONCLUSION

Linear combination of objective functions method was able to achieve the same solution as that of goal programming for the production planning problem by solving only one problem instead of three in the case of goal programming. In situations where statement of goals is difficult, the LCOF method is superior to GP in terms of simplicity and time savings. The relative performance of GP and LCOF in terms of sensitivity to changes in preference indices is recommended for further study.

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