Fractal dimension and time factors of sawdust pattern formation in sawmills

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Abstract: This paper presents the application of fractal theory, especially fractal dimension to the formation of sawdust particles during operation with four detailed cases, which helps us to understand the distribution of the sawdust particles inhaled by sawmill workers and remove the effect of toxicity on their body quickly. Pattern of formation of sawdust in human lungs and other parts of the body (in fast branching rate) is described with a practical case study in a developing country. As these sawdust particles settle down in the human body, cells are destroyed on a very fast rate by the toxic nature of sawdust particles. Thus, removing the effect of toxicity on the body will require special skill and is cost intensive. The pattern formation of sawdust particles follows random walking in 2-D Euclidean space using fractal dimension and time steps. Percentage total of average time steps required for aggregation of specified *n*-sawdust particles varies according to power law of percentage successive aggregation. Case 3 and its rules are the most reasonable if used in a real project since its percentage absolute error compared with the standard literature value of 1.71 is zero. The paper may be of great importance to occupational health scientist and those who control and monitor occupation health problems in small scale industries particularly where occupational hazards are well pronounced.

Keywords: fractal dimension; chaotic dynamics; woodworking problem; random walk; aggregation.

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1 Introduction

Understanding the chaotic dynamic behaviour of sawdust particles emitted through wood processing activities in sawmill using modelling and experimentation plays an important role in its control and in the reduction of employee health hazards at sawmills. Sawdust is of economic and experimental benefits in serving as absorbents and sorption materials (Hamdaoui, 2006; Šćiban et al., 2006; Hamadi et al., 2001; Jadhav and Vanjara, 2004; Taty-costodes et al., 2003) and has also been used as new base material for boilers (Akira et al., 2002). Other uses include as catalyst in the removal of mercuric ion from aqueous solutions (Ansari and Raofie, 2006). Studies on sawdust is increasingly becoming of great interest to researchers (Arif et al., 2003; Demers et al., 1997; Udoeyo and Dashibil, 2002; Ajayi and Owolarafe, 2007). For example, Hamid and Saffle (1965) identified the volatile fatty (i.e., acetic, propionic, butyric, isovaleric, n-valeric, isocaproie, and n-caproic) acids present in hickory sawdust smoke as a preliminary step to solving environmental pollution problem. Yet, relatively little progress has been made in the modelling and experimentation of the chaotic dynamic behaviour of sawdust particles as they are released into the work environment through wood processing activities at the sawmill (see Moon, 1987). Understanding this dynamics would help in planning and redesigning the work environment and in reestablishing the maximum inhale-able sawdust particles in the sawmill environment.

In the instance that a scientific approach is known with which government could control excessive exposure of sawmill workers to sawdust, a guideline for controlling activities could be issued by governments for easy control of occupational hazards at sawmills. Thus, while the complying sawmills may be commended, those not complying may have to be reprimanded. Therefore, it is only through a scientific tool that such a proper guidance could be obtained. This also suggests that governments may enforce the use of protective devices for nose protection and hearing loss avoidance. This indirectly reduces governments' health costs as those with health problems still seek assistance in government hospitals. Also, frequent reassignment of workers to other job points for those exposed to much sawdust could be arranged. In addition, regulation by governments may result in the redesign of a workplace that lacks proper ventilation.

Investigations on sawmill activities have however provided some useful insights to the contention that sawmill workers are at risk. These primarily concern the effects of emission of volatile compounds from stored woods on sawmill worker (Svedberg et al., 2004), the risk of childhood cancer by children of sawmill workers through their paternal exposure to harmful substances (Heacock et al., 2000), and prevalence of asthma in sawmill workers (Siracusa et al., 2007). The particular details of the above review are now given. Svedberg et al. (2004) investigated the emission of volatile compounds, particularly hexanal and carbon monoxide, from large- and small-scale storage of wood pallets. Such storage systems are predominantly found in sawmills. Heacock et al. (2000) established a relationship between the risk of childhood cancer and paternal occupational exposure to chlorophenate fungicides in British Columbian sawmills. Siracusa et al. (2007) evaluated the prevalence of asthma and its predictors in studies of several male working in cedar sawmills and observed the prevalence of asthma after employment in the industry.

This gap between theory and practice of sawdust dynamics in sawmills is carefully addressed in the current paper. The objective of this study is to show how much computational time will be required to form a fractal pattern for sawdust particles in a sawmill environment using random-walking sawdust particles in 2-D Euclidean space. In addition, the study aims at understanding if the time taken to attach an additional sawdust particle will reduce or increase as fractal pattern formation progress in time and space. The article is structured as follows: the introduction presents the problem and the literature review. Section 2 discusses the methodology utilised for solving the problem. In Section 3, a practical case study is given to strengthen the quality of the paper and the results and discussions with explanations for the pattern of results obtained discussed. Section 4 concludes and states the future trends.

2 Methodology

The idea of the current study was motivated by the classic studies of Feder (1988) and Zmeškal et al. (2001) that attempted to simulate the electrolysis process whereby variation of voltage or current intensity drives metal atoms in a random manner towards disposition as attachment to growing pattern. However, these authors have omitted the time factor and the minimum distance (0.3-unit) for successive attachment of next random walk atoms (particles). It is this serious omission and gap that the current study attempts to bridge, with application to a practical problem of wood working problem. Thus, numerical experiment was carried out for the simulation of the sawdust particle problem. It should be noted that the comparison of estimated fractal disk dimension is reference to the model results of Feder (1988) and Zmeškal et al. (2001), which yielded 1.71. Thus, the models developed by these researchers are used and the data generated for experimentation was tested with the model.

2.1 *Model development rules*

The methodology utilised in the development of the sawdust particle dynamic chaotic model is hinged on a number of rules. Four important rules, labelled Cases 1, 2, 3 and 4, were developed based on random-walking principle applied to sawdust particles in circulation paths. The rules used for each of the four studied cases are as follow (Zmeskal et al., 2001; Feder, 1988):

Case 1

A random-walking 'sawdust particle' is released from the point of cutting the timber where the sawdust particles are produced (at the interaction of the powered machine blade and the timber logs already pushed to the band saw). The slicing of the timber into planks produces sawdust particles from the source position (zero-unit away from source). The release of the sawdust now positions it at a random location on a circle of radius ten-unit away from the 'seed sawdust particle' which was initially located at the circle center. This newly produced sawdust staggers with variable step size back and forth, up and down, until it is 0.3-unit closer to the 'seed sawdust particle'. For instance, it may stagger to 12 units away from the seed position in a forward movement. It may decrease by five units (backward movement), move by four units down and finally stayed ay 0.3-unit closer to the seed particle. Note that these movements are only four. Similarly the second, third, fourth, fifth, hundredth, thousandth walker is added to the growing pattern. That is a second particle is released from the seed's initial position, follow the same pattern and the addition continues until the thousandth sawdust particle is generated.

Case 2

This follows the description in Case 1 except that '*least 20-random steps must have been taken before next 'sawdust particle aggregation' may be allowed*'. That is, instead of the four steps taken by the sawdust particle as in Case 1, the number of movements extends to 20 in random combinations of back, forth, up and down movements. If less than 20-random steps are allowed for aggregation, the final fractal pattern formed and the corresponding dimension results will not compare satisfactorily with the literature results. The fact is that more particles will aggregate nearer to the circle circumference and as such prevent gradual aggregation with the seed particle. Similarly the second, third, fourth, fifth, hundredth, thousandth walker is added to the growing pattern.

Case 3

This is similar to Case 2 in description but the measurement of the sawdust particle movements are in radians. It is described as follows: a random-walking 'sawdust particle' is released at random location on a circle of radius ten units away from the 'seed sawdust particle' located on the circle centre and staggers with variable step size in any angular direction picked between zero-radian and 2π-radian until it is 0.3-unit closer to the 'seed sawdust particle'. If less than 20-random steps are allowed for aggregation there is an impact on the final result since it will not compare satisfactorily with the literature value. Similarly the second, third, fourth, fifth, hundredth, thousandth walker is added to the growing pattern.

Case 4

This is similar to Case 3 except that '*at least 20-random steps must have been taken before next 'sawdust particle aggregation' may be allowed*'. Similarly the second, third, fourth, fifth, hundredth, thousandth walker is added to the growing pattern.

2.2 *Model for the estimation of fractal dimension*

The development of a model for characterising the chaotic behaviour of sawdust particles in the sawmill is based on the establishment of a relationship among the number of sawdust particles that could be counted in a specified radius (N), the radius of the circle being described (R), and the fractional dimension of the studied aggregated pattern formed using any of cases as the guideline (D). A number of questions may arise on the relationship of the specified circle radius, R, and the results. It may be interesting to know if the value of specified circle radius, R, has impact on the final results. In other words, why are the assumptions for rules very specific (i.e., generic approach) instead of using a generalised form of N units. For example why ten units away and not five or 15? Clearly, the choice of R is arbitrary, but the choice of higher value demand for higher computational time and vice versa. Note that successive attachment/aggregation is when two particles by random motion come close as 0.3-unit, which again is arbitrary, but at least ensures fine pattern formation. In essence, five units and 0.3-unit combination will be faster, but the branching may not show clearly. However, 15 units and 0.3-unit will take longer time to form but the branching will show clearly. It is a matter of compromise. That is, how do you want it without loosing focus (i.e., time factor in non-dimensional form).

Consequently, larger values of R implies that longer total time steps will be required to generate the ultimate fractal pattern, the dimension of which is the same statistically for any R. The relationship between N and R is direct proportionality. If we assume that the sawdust particle size is incompressible then N would increase proportionately with R. However, there is need to introduce the term D in the relationship. From practical observation, a power relationship seems applicable. It should be noted that the model developed is based on the estimation of chaotic behaviour through fractional dimension. The particular emphasis is the radius elimination method. The model is stated as (Feder, 1988; Zmeškal et al., 2001):

$$N = KR^D \tag{1}$$

where

N = number of sawdust particles counted in a specified circle radius

K = constant of proportionality

R = specified circle radius

D = fractal dimension of the studied aggregated pattern formed by any of the four cases described above.

The problem investigated may be viewed to encompass biological aspects and factors related to human health such as age of a person, respiration rate, etc. These may be considered in further studies in order to improve our understanding of the woodworking problem. In addition, consideration may be given to environmental factors such as wind velocity, temperature, type of tool used, type of wood used, etc. The argument here is that these parameters (wind velocity, temperature, etc.) are lumped together to generate the random walk of the sawdust particles. For example, if the wind velocity is unchanged, then the sawdust particles will experience zero motion. In addition, higher temperature means faster particle movement and haphazard too.

By taking the natural logarithm of both sides of equation (1) equation (2) can be obtained.

$$\text{Log}(N) = \text{Log}(K) + D\text{Log}(R) \tag{2}$$

Equation (2) is linear with Log (*R*) being the independent variable and Log (*N*) being the dependent variable. The slope of line of best fit to log-log plots as suggested in equation (2) gives the best estimate of the aggregated pattern fractal dimension. The various fractional patterns were formulated with the use of random walking particles principle in 2-D Euclidean space. In particular, fractional dimension and time steps analysis are used. The procedure is initiated by arbitrarily picking ten different random number generating seed values to drive random-walking particle in 2-D Euclidean space according to random rules (cases). The algorithm was coded in FORTRAN language while the emerged fractional pattern was analysed for its dimension using radius dimension method.

The empirical equation of the model used in the current work is based on the articles of Feder (1988) and Zmeškal et al. (2001). Although the equations are not new stars in research, however, in their applied forms, they present new information that has not yet been documented. The idea of the application sprang up from the observation of the structure and procedure of application of the Feder and Zmeškal et al.'s expressions. The chaotic experimentations of these references suggest a good resemblance of the behaviour of sawdust particles during its deposits in the lungs through the nostrils of the operator and mill workers in general. Certainly, different behavioural patterns of motion, collision of particles, speed of motion, sizes of sawdust particles, concentration of sawdust particles, density of particles and settlement (settling down) patterns of sawdust particles are observed in real life cases, which has motivated the current study in the application of established theory of fractal to the sawmill environment.

3 Case study, results and discussion

Since this paper deals with the formation of sawdust fractal pattern in human lungs, a case study dealing with the practical consideration of size fractal theory of emitted sawdust particles, which gives consideration to the mechanism of aggregation in the structure and relevant description of why and how the suggested cases would appear in human lungs is necessary. However, only a brief account which is sufficient for understanding the theoretical framework is presented. Numerical experimentation carried out resulted for the generation of results, which are presented as tables, each contains vital information that led to the conclusion made in this work. Table 1 emerged from using the seed value 9,876 to estimate dimensions and natural logarithm of intercepts. Ten different experimentations were carried out in which sawdust particles of a specified number were constrained according to rules 1 to 4, corresponding to Cases 1 to 4 stated at the beginning of Section 2. Thus, the number of particles, dimension and intercept for all the sample sizes ranging from 500 to 5,000 sawdust particles, and increasing in steps of 500 particles, were considered. For example, when 500 sawdust particles were experimented on, dimensions of 0.87, 0.87, 0.83 and 0.83 were obtained for Cases 1 to 4, respectively, while the intercept obtained were 4.54, 4.54, 4.60, and 4.60, respectively for Cases 1 to 4 (Table 1).

Table 1 Estimated dimensions and natural logarithm of intercepts with seed value of 9876

Z*	*Case 1*		*Case 2*		*Case 3*		*Case 4*	
	Dim	*Int*	*Dim*	*Int*	*Dim*	*Int*	*Dim*	*Int*
500	0.87	4.54	0.87	4.54	0.83	4.60	0.83	4.60
1000	1.26	4.39	1.26	4.39	1.18	4.46	1.18	4.47
1500	1.47	4.25	1.47	4.26	1.37	4.34	1.38	4.34
2000	1.60	4.16	1.60	4.16	1.47	4.27	1.50	4.26
2500	1.67	4.10	1.69	4.10	1.52	4.22	1.59	4.19
3000	*1.72*	4.06	1.75	4.04	1.56	4.19	1.65	4.14
3500	1.75	4.03	1.81	4.00	1.59	4.16	1.71	4.10
4000	1.77	4.01	1.85	3.97	1.61	4.14	1.75	4.06
4500	1.79	3.99	1.90	3.93	1.63	4.13	1.79	4.03
5000	1.81	3.98	1.93	3.90	1.65	4.11	1.83	4.00

Note: Z* means *N*-sawdust particles, Dim* is dimension, while Int* means intercept.

Consequently, there are ranges of readings for the various sample sizes. It is interesting to note that the sample size of 3000 sawdust particles yielded the dimension of 1.72, which is closer to 1.71 by 0.6% absolute errors. This literature value could also be obtained from Feder (1988). An attempt is made in Figure 1 to draw the pattern of movement of these sawdust particles when considering it in *X* and *Y* coordinates. The distribution pattern of these particles is interesting to note in Figure 1.

Figure 1 Case 1: 3000-agregated sawdust particles (see online version for colours)

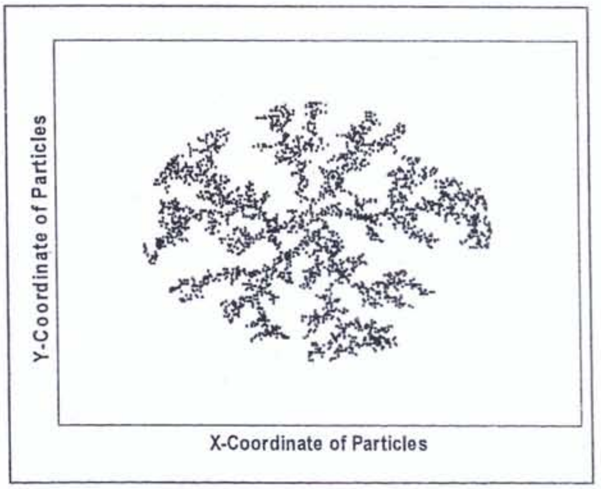


Figure 1 shared structural visual resemblance with what is supported by literature. Figures 2, 3 and 4 also relate to the structural visual resemblance of the aggregated sawdust particles.

Figure 2 Case 2: 3,000-agregated sawdust particles (see online version for colours)

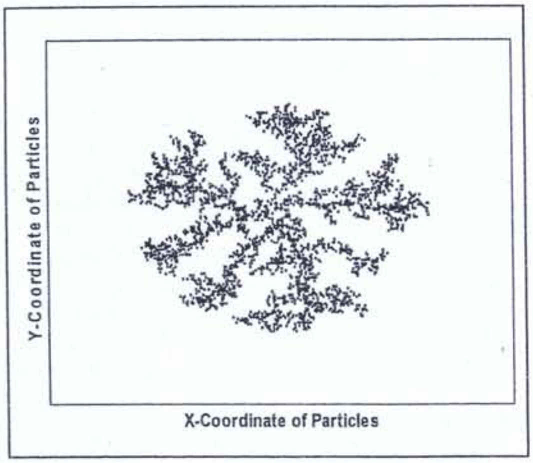


Figure 3 Case 3: 3000-agregated sawdust particles (see online version for colours)

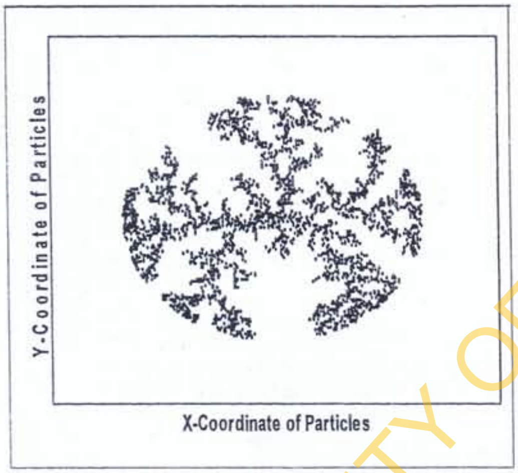
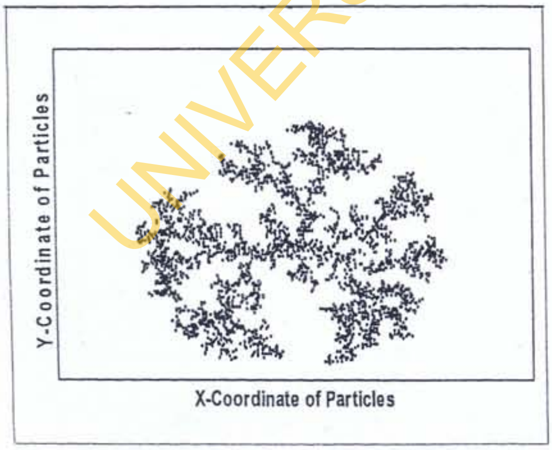


Figure 4 Case 4: 3000-agregated sawdust particles (see online version for colours)



The result presented in Table 2 is an outcome of the simulation based on different random number generator seed values. Recall that in Table 1, the sample size that produced the near-literature value of 1.71 was chosen, which corresponds to a sample size in which the seed value utilised is 9,876. In Table 2, it is observed that sawdust particles counted inside circle of specified radius increases as the specified radius increases.

Table 2 Number of sawdust particles counted for Case 1, seed value 9876 using radius dimension method

Radius	*Sawdust particles counted inside circle of specified radius*	*Natural logarithm of radius*	*Natural logarithm of sawdust particles counted*
1	59	0.0000	4.0775
2	185	0.6931	5.2204
3	379	1.0986	5.9375
4	660	1.3863	6.4922
5	883	1.6094	6.7833
6	1246	1.7918	7.1277
7	1648	1.9459	7.4073
8	2077	2.0794	7.6387
9	2559	2.1972	7.8474
10	3000	2.3026	8.0064

Table 3 Estimated fractal dimension for four (4) cases with ten-different random number generator seed values

S/N	*Seed values*	*Estimated fractal dimensions*			
		Case 1	*Case 2*	*Case 3*	*Case 4*
1	9876	1.72	1.75	1.56	1.65
2	6789	1.63	1.69	1.72	1.77
3	4567	1.67	1.72	1.75	1.80
4	5678	1.75	1.79	1.70	1.78
5	6784	1.60	1.66	1.67	1.72
6	3456	1.72	1.77	1.75	1.81
7	7865	1.68	1.77	1.81	1.86
8	3789	1.75	1.82	1.73	1.77
9	3467	1.61	1.69	1.77	1.83
10	7896	1.65	1.72	1.64	1.68
MEFD*		1.68	1.74	1.71	1.77
Standard deviation		0.05	0.05	0.07	0.06
AE*		1.75	1.75	0.00	3.51

Notes: AE* means % absolute error comparing mean estimated fractal dimension with the standard (1.71-literature); MEFD* could be written as mean estimated fractal dimension

What was done in the experiment that generated Table 3 was to use the seed values of 6,789, 4,567, 5,678, 6,784, 3,456, 7,865, 3,789, 3,467 and 7,896, respectively and closely monitoring the dimensions that are close to the literature value of 1.71. The results showed the mean estimated fractal dimension of 1.68, 1.74, 1.71, and 1.77 as well as the standard deviation of 0.05, 0.05, 0.07, and 0.06, for Cases 1, 2, 3 and 4, respectively. The percentage absolute errors when the mean estimated fractal dimension was compared with the standard (1.71-literature) were 1.75, 1.75, 0.00 and 3.51, respectively. In sum, the mean estimated fractal dimension obtained for the four studied cases range between 1.68 ± 0.05 and 1.77 ± 0.06. The percentage absolute error recorded comparing the mean estimated fractal dimension with the standard fractal dimension of 1.71 supported by literature range between 0.00% and 3.51% for all studied cases.

Figure 5 is the log-log plot of specified radius and the sawdust particles counted inside the circle which radius has been specified. The slope of the line of best fit is taken as the estimated fractal dimension for the aggregated sawdust particles (fractal pattern) using the radius dimension method. The estimated dimension for the aggregated sawdust particles is 1.72 to two-decimal as indicated by the equation of line of best fit in Figure 5. All other dimensions reported in this study were similarly obtained.

Figure 5 Estimated fractal dimension diagram for Case 1 with seed value of 9876

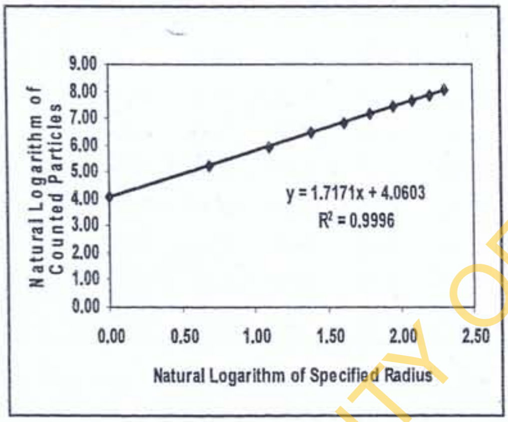
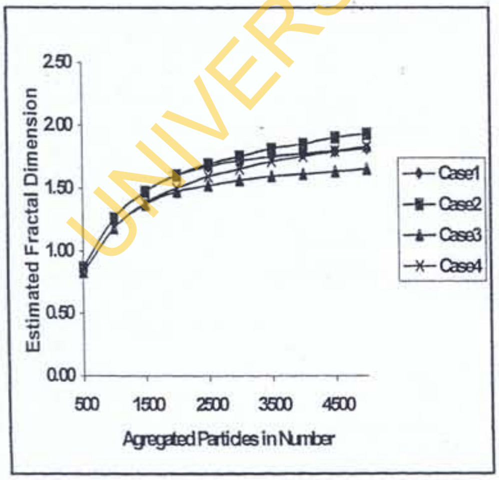


Figure 6 Estimated fractal dimension for the four cases



Referring to Figure 6, the estimated fractal dimension for Case 2 was consistently greater or equal to the estimated fractal dimension for Case 1. Similarly the estimated fractal dimension for Case 4 was consistently greater or equal to the estimated fractal dimension for Case 3. The estimated fractal dimension for the four cases tends toward a limiting value lesser than 2.0. The highest estimated fractal dimension is 1.93 in Case 2.

The sample results presented in Table 4 was drawn out of 3000 computer simulated results. The average was taken over ten different simulators represented by ten different random number generation seed values as in Table 3 column 2. It was observed for all studied cases that the average time steps taken before the successive attachment of *N*th sawdust particle statistically decreases as the aggregate size increases. Average time steps for Case 1 and Case 2 are same at the early stage of the sawdust particle aggregation, but differed at later stage. Similarly the average time steps for Case 3 and Case 4 are same at the early stage of the sawdust particle aggregation, but differed at later stage. Very shorter average time steps were recorded for all the cases at later stage which indicate saturation of the inside of specified circle with aggregated sawdust particles. This means that the probability of sawdust particle attachment increases drastically at later stage. Case 2 utilised highest number of total average time steps of 538080 before successful aggregation.

Table 4 Sample of average time steps taken before the successive attachment (aggregation) of N-th sawdust particle

N-th sawdust particle attached	*Average time steps taken before the success of sawdust particle attachment*			
	Case 1	*Case 2*	*Case 3*	*Case 4*
2	3343	3343	1433	1433
3	1699	1699	1855	1855
4	2195	2195	1876	1876
5	1486	1486	1241	1241
6	2320	2320	2302	2302
7	2205	2205	1708	1708
8	2279	2279	2352	2352
9	1371	1371	1451	1451
10	2638	2638	1385	1385
...	...	...	...	...
2996	4	22	5	21
2997	3	23	11	26
2998	7	26	4	25
2999	3	21	1	29
3000	9	23	2	21
TAT*	508926	538080	499170	525570

Notes: TAT* means total of average time steps required to attach all *N*-sawdust particles specified.

Table 5 shows sample of total aggregation and total average time steps expressed in percentage. The percentage of particles aggregation required ranges from 10.00% to 100.00% in steps of 1.00%. For example, for a 10.00% of sawdust particles, the percentage of total of average time steps required for aggregation of *N*-specified sawdust particles for Cases 1 to 4 were 47.72, 45.13, 46.48, 44.14, respectively with its average being 45.87. The percentage of total of average time steps required for 50% aggregation of *N*-specified sawdust particles range between 88.42% and 92.96% while the average over the studied cases is 90.50%. Referring to Table 5 the percentage total of average time steps required for 50% aggregation of *N*-specified sawdust particles range between 88.42% and 92.69% while the average over the studied cases is 90.50%.

Table 5 Sample (out of 2,999-data set) of total aggregation and total average time steps expressed in percentage

% of sawdust particles aggregation required	*% of total of average time steps required for aggregation of N-specified particles.*				
	Case 1	*Case 2*	*Case 3*	*Case 4*	*Cases average*
10.00	47.72	45.13	46.48	44.14	45.87
20.00	68.17	64.49	67.11	63.77	65.88
30.00	80.23	76.02	79.73	75.76	77.94
40.00	87.63	83.46	87.70	83.31	85.52
50.00	92.41	88.47	92.69	88.42	90.50
60.00	95.49	92.10	95.76	91.93	93.82
70.00	97.49	94.71	97.65	94.58	96.11
80.00	98.70	96.75	98.81	96.66	97.73
90.00	99.52	98.48	99.56	98.45	99.00
100.00	100.00	100.00	100.00	100.00	100.00

Figure 7 Percentage of successive aggregation and percentage total of average time steps required for aggregation of N-specified sawdust particles (see online version for colours)

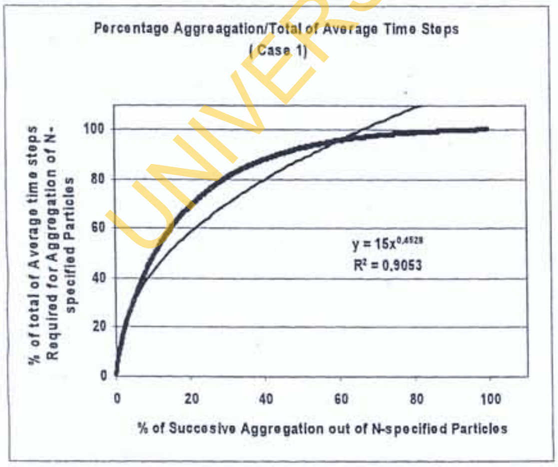


Figure 7 was obtained using the whole data set for which Table 5 is a sample. The curves of best fit to the data set are obtained respectively for the four cases as shown in Table 6. Observe that Table 6 contains the results obtained from the derivation of the power law equation and the coefficient of correlation. Four cases are considered: Cases 1 to 4. The correlation coefficient was least for Case 3 (i.e., R^2 = 0.8970), while it was the best for Case 2 (i.e., R^2 = 0.9213). However, the power exponent was least for Case 1 (i.e., 0.4528) and the highest was recorded for Case 4 (i.e., 0.4827). It was noticed that the mean value of the power law exponent is 0.4683 ± 0.0125.

Table 6 Power law equations for cases

Case	*Power law equation and coefficient of correlation*	*Power law exponent*
1	$y = 15 \times {}^{0.4528}$; $R^2 = 0.9053$	0.4528
2	$y = 13.787 \times {}^{0.4656}$; $R^2 = 0.9213$	0.4656
3	$y = 13.89 \times {}^{0.4722}$; $R^2 = 0.8970$	0.4722
4	$y = 12.868 \times {}^{0.4827}$; $R^2 = 0.9113$	0.4827

Reference to Table 6, it is shown that the power law exponent for the cases studied range between 0.4528 and 0.4827 while the mean value is 0.4683 ± 0.0125.

7 Conclusions and future work

This paper describes the formation of sawdust particles in the lungs of humans and in other parts of the human body where harmful residue sawdust may accumulate as a fast branching rate in fractal patterns and also considers the mechanism of aggregation in structure. These formed patterns (Cases 1 to 4) describe the settled state of the sawdust particles. The implication is that as these sawdust particles settle down in the human body these cells are destroyed on very fast rate by the toxic nature of sawdust particles. Thus, an attempt to remove the effect of toxicity on the body will require special skill and it will be cost intensive.

The estimated fractal dimension of emerged fractal pattern of 3,000-sawdust particles aggregated in a circle of ten-unit radius, sawdust particles closeness not greater than 0.3-unit, random number generating seed value of 9,876 and using Case 1 rule agreed by 0.6% absolute error comparing with literature result of 1.71. The mean of the estimated fractal dimension for all studied cases range between 1.68 ± 0.05 and 1.77 ± 0.06 while the percentage absolute error comparing with standard fractal dimension of 1.71 range between 0.00% and 3.51%. This study further shows that average time steps required to successively attach an additional sawdust particle decreases as the percentage successive attachment increases. That is the probability of an additional sawdust particle attachment increases drastically at later stage than at earlier stage of growing fractal pattern using any of the studied rules. Also successive aggregation of 3,000 particles took total average time steps of 538,080, the highest recorded for Case 2. This study also shows that the percentage of total of average time steps required for execution of 50% aggregation range between 88.42% and 92.69% while the average taken over the studied cases is 90.50%. Percentage of total of average time steps required for aggregation of specified n-sawdust particles varies according to power law to percentage successive aggregation. The mean value of the power exponents was 0.4683 ± 0.0125. Case 3 and its rules are most reasonable if used in a real project since its percentage absolute error compared with mean fractal dimension with the standard literature value is 1.71 was zero. This is evidenced in Table 3. Thus, growing fractal pattern in 2-D Euclidean space using any of the studied rules demand lot of computational time and nothing better can be projected in 3-D Euclidean space.

Concerning future investigations, we submit as follows. A first future investigation that must be pursued rigorously concerns detailed practical studies in a number of sawmills where actual real life data could be collected and analysed in order to verify the working of the model in reality. Recall that only numerical experiments were conducted here and the study was related to practice but not to the extent of collecting practical data from the field. Implementing a practical study would be exciting as some interesting results may be obtained. Viewing from another perspective, it is observed that extensive analysis based on R^2 has been done in this work. This reflects the coefficients of determination, which shows the relationship between two sets of values. However, for model accuracy the use of R^2 may not be sufficient for testing the statistical significance of models. Several other statistical parameter error indicators which were not considered in the current work may be evaluated with their results compared to R^2. This way, the error measurements may be monitored and the model adjusted each time to ensure that models with minimum errors using the wide array of parameters available are developed. Such parameters that could be compared with R^2 include model bias (MB), normalised mean square error (NMSE), fractional bias (FB), and index of agreement (IA). A further area of improvement relates to the presentation of results. It is noted that in the current work, four patterns (Cases 1 to 4) are presented. However, it would have been better to superimpose one case on the other in a particular diagram and colour these cases differently. Unfortunately, this situation is challenging and efforts are being made to achieve this. The successful efforts on this would aid proper explanation of the results of the work. However, for the current purpose of reporting the results obtained for the different fractal pattern deposits of emitted sawdust in the lungs, the diagrams are separated into four.

Logistics equation and Lyapunov exponent estimation function have been well established in the literature with noteworthy applications in stability and permanence studies of physical systems, microbial growth, single-species populations, disaster response activities, and in studying chaotic situations (see Lai and Chen, 1998; Stefanski, 2000). Efforts are being made to combine them and find their suitable applications for the sawdust deposit patterns in operators at sawmills as a means to further understand this problem. Simulation experiments would be performed and the work will be analytically treated for good results. In addition, since there is an increased level of operational activities at sawmills, which translates into increased sawdust generation. There is a further motivation to study the current problem. A method that studies the dynamics of sawdust movements in sawmills which could aid understanding its control strategy using defined parameters space of forced Duffing's dynamic system could be statistically investigated for the probability of total parameter points that would exhibit chaotic behaviour using maximum Lyapunov exponent value indicator. This approach would involve randomly picking parameter points that would be used to solve a second order non-linear differential Duffing's equation numerically using Runge-Kutta Algorithms, which may be coded in a computer language. Thus, cycles of solutions based on forcing period would be obtained.

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