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THE WEST AFRICAN EXAMINATIONS COUNCIL, LAGOS.
A Seminar held in Lagos, on Friday 26th February, 2010

Published by

RESEARCH DIVISION & HEADQUARTERS OFFICE
THE WEST AFRICAN EXAMINATIONS COUNCIL
3, Watch-Tower Avenue, Onipanu, Lagos, Nigeria.
P. M. B. 1076, Yaba, Lagos.

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A PAPER PRESENTED AT THE WAEC MONTHLY SEMINAR HELD IN LAGOS ON FRIDAY 26TH FEBRUARY, 2010

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ABSTRACT

Facing Sequencing of Instructional content is one of the major problems mathematics teachers are encountering in schools. Attempt was made to review literature on the identified problem. Sufficient research findings revealed that there are substantial evidences to support the general theory of the hierarchical structure of knowledge. A learning hierarchy for teaching Bearing and distances in Senior Secondary School was constructed. Based on the hypothesized ordering of the nine subunits (eight cognitive entry characteristics and one criterion task), a diagnostic test was constructed to assess mastery of each of the eight units in the hierarchy and Summative test (Mathematics Achievement Test-MAT) was constructed to assess the mastery of the criterion task. The reliability coefficient (r) and difficulty level (p) of MAT are 0.80 and 0.4 respectively. The nine instruments were administered to Four hundred and ninety two SS 3 students selected through multi-stage random sampling technique from public co-educational secondary schools in three of the five local government areas in Ibadan Metropolis. The hierarchical orderings of the nine subtasks (eight CEC and one Criterion task) were generated in flow-chart form. Task analysis was basically used to establish links among units and criterion task. A correlation analysis technique was used to determine the indices of agreement of each pair of units as well as pair of each unit with criterion task. All the indices (r) were significant at 0.05 alpha level, hence all the subunits in the hierarchy were retained. It is sufficient to recommend for the entire practicing mathematics teacher that adequate sequencing of instructional objectives to reveal the prerequisite subtasks is an adequate procedure for enhancing better achievement in any mathematics topic.

1. INTRODUCTION

Sequencing instructional content is a fundamental issue in teaching. It is also important in the teaching of geometry in Mathematics. A team of Dutch educators, Pierre van Hiele and Dina van Hiele-Geldof, (Mason 2005) took note of the difficulties that their students had in learning geometry. These observations led the educators to develop a theory involving levels of instruction in geometry that students pass through as they progress from merely recognizing a figure to being able to write a formal geometric proof. The theory explains why many students encounter difficulties in geometry as a major subject in college mathematics. Mason (2005) was of the opinion that writing proofs requires thinking at a comparatively high level, and that many students need to have more learning experiences at lower levels before learning formal geometric concepts. According to Hieles (1982), there are five levels, which are sequential and hierarchical. These are: Visualization, Analysis, Abstraction, Deduction and Rigor (see Appendix I).

Cognitive entry Characteristics refer to pre-requisite learning needed by a learner for a particular learning task. Bloom(1976) explains that learning hierarchy does not only include specific knowledge and skills needed, but also more general cognitive skills and abilities that enable the learner achieve meaningfully in a particular learning task.

The studies of Abadom (1993) and Adeleke (2007) support Bloom's (1976) theory that majority of the variation in school learning is directly determined by Sequence of instruction. Abadom (1993) further explains that if students come into learning situations with adequate levels of Cognitive Entry Characteristics (CEC), virtually all students can attain a high degree of learning with little variation in their learning outcomes and that under this situation aptitude ceases to predict achievement. Abadom (1993) based her study on Secondary School Algebra, but this study sought to validate Learning Hierarchy that will involve sufficient CEC on Bearing as a topic in Mathematics. Why bearing and not any other topic?

Bearing is an aspect of Geometry where majority of students performed poorly in the West African Senior School Certificate Examination (WASSCE) (WAEC's Chief Examiners' reports). Geometry is an essential part of Mathematics. Unfortunately, according to evaluations of Mathematics learning, such as the National Assessment of Educational Progress (NAEP), college grade two students in Alexandria fail to understand basic geometric concepts and develop adequate geometric problem-solving skills (Carpenter, eta al, 1980; Fey eta al, 1984; Kouba eta al1988). This poor performance may be due, partly, to the lack of cognitive entry characteristics which focus on recognizing and naming geometric shapes and learning to write the proper symbols for simple geometric concepts (Carpenter et al. [1980]; Flanders [1987]). In contrast, it is believed that elementary geometry should be the study of objects, motions, and relationships in a spatial environment (Clements and Battista 1986). First, students' experiences with geometry should emphasize informal study of physical shapes and their properties and have as their primary goal the development of students' intuition and knowledge about their spatial environment. Subsequent experiences should involve analyzing and abstracting geometric concepts and relationships in increasingly formal settings. This is necessary to equip them adequately with CEC needed to achieve meaningfully in geometry topics. Based on this background, this study sought to validate Learning Hierarchy adequate to enhance students' achievement in Bearing.

2. STATEMENT OF PROBLEM

Mass failure in Mathematics is being recorded yearly in the WASSCE. Several attempts have been made by researchers in the past to look at the root cause of the problem. Some investigated instructional strategies, Learning materials and Learners' psycho socio variables that account for variation in students' achievement in Mathematics. It appears sufficient attention has not been given to the way teachers arrange their instructional content to support learning. Arranging instructional units sequentially to enhance achievement in school subjects especially Mathematics goes beyond paper and pen activities. It requires analytic approach that will establish the links and the magnitude of support they give to one another. Thus, this study was carried out to validate hierarchy of instructional content adequate for effective teaching of bearing in Mathematics.

3. RESEARCH QUESTIONS

The following research questions were raised for this study:

1. What is the hypothesized Learning Hierarchy for instructional activities on Bearing?

2. What Units of instruction are valid for enhancing student achievement in Bearing?

4. METHODOLOGY

(1) Design

This study used the cross-sectional survey design. An ex-post facto approach was used for the identification of CEC for learning bearing since none of the independent variables was manipulated. The procedures for hypothesizing the links adopts the format of American Instructional Design Project (IDP) (2005) on identification of Cognitive Entry Characteristics (CEC). IDP (2005) stipulates that the designer must identify CEC that must be passed along to the learners with the target task using procedural analysis or flowcharts or combination of the two if appropriate for the content. This study used the hierarchy of CEC as the flowchart. The flowchart is presented in Fig.1. Units 1 to unit 8 in Fig. 1 are the hypothesized cognitive entry characteristics (CEC) while unit 9 consist the target task that is distances, sizes of angles and bearing.

(2) Sample

Three Local Government Areas (LGA) were randomly selected from the five existing ones in Ibadan metropolis. Stratified sampling was also employed in which the selected LGAs formed the strata. Ten schools were selected from the three clusters using the method of sampling proportion to size that is the number of eligible co-educational senior secondary schools in each Stratum (selected LGA). An intact science class was randomly selected and used from each of the selected schools. Table I presents the summary of the distribution of the selected schools as well as the students for the study.

TABLE I: DISTRIBUTION OF THE SELECTED SCHOOLS AND STUDENTS FOR THE SURVEY

Ibadan LGA	No. of co-educational senior secondary Schools	No. of co-educational secondary school Selected for the survey	No. of students selected
North	23	4	219
North East	11	3	152
South West	19	3	121
Total	**53**	**10**	492

Source: Planning Research and Statistics Department- Statistics Unit Ministry of Education Oyo State.

(3) Instrumentation

Two different types of tests were used to gather data for this study. They are:

(a) Diagnostic Tests

Eight Diagnosis Tests were used for the study. Each Diagnotic Test is a 10-item formative test of 4 options scale that was used to measure students' achievement in each of the units that constitute the hypothesized learning hierarchy. These units were: Fraction, Decimal and Algebraic Fraction, Algebraic process, Angles and Triangle, Trigonometry, Specifying bearing, Presentation of bearing with diagram, Cosine Rule and Sine Rule.

(b) Mathematics Achievement Test (MAT)

This instrument was constructed and validated by the researcher. It consists of 20 items of multiple-choice type. The reliability coefficient was established using Kuder Richardson formula 20. The reliability coefficient and difficulty index (p) of the instrument were 0.8 and 0.4 respectively. A sample of 119 SSIII students similar to the target sample from co-educational secondary schools in Ibadan metropolis who have completed bearing in their Mathematics syllabus were used for the test Reliability and item analyses of MAT.

(4) Data Analysis

Task analysis was used to establish links among the subunits on the hierarchy. Correlation coefficients among the topics in the hierarchy were established using Pearson product Moment Correlation Analysis.

4. RESULTS

Research Question One: What is the hypothesized Learning Hierarchy for instructional activities on Bearing?

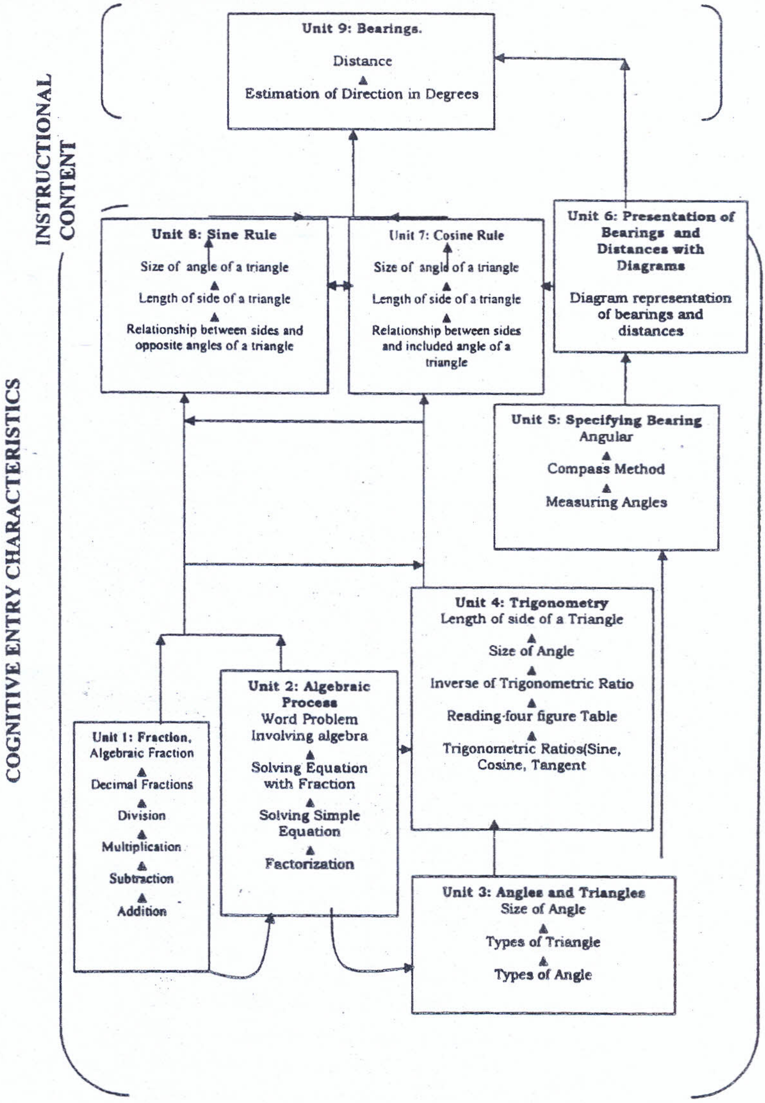

Fig. 1: HYPOTHESIZED LEARNING HIERARCHY IN BEARINGS

Fig 1 reveals the hypothesized units that may be included in instructional activities that constitute learning hierarchy that can enhance student achievement in Bearing. The first eight units serve as hypothesized Cognitive Entry Characteristics (CEC) that may be needed by students to learn Bearing to mastery. These Hypothesized CEC are: Fraction (Algebraic Fraction, Decimal \ Fractions, Division, Multiplication, Subtraction and Addition), Algebraic Process (Word Problem Involving Algebra, Solving Equation with Fraction, Solving Simple Equation and Factorization), Angles and Triangles (Size of Angle, Types of Triangle and Types of Angle), Trigonometry (Length of side of a Triangle, Size of Angle, Inverse of Trigonometric Ratio, Reading four figure Table, Trigonometric Ratios-Sine, Cosine and Tangent), Specifying Bearing (Angular, Compass Method and Measuring Angles), Presentation of Bearings and Distances with Diagrams (Diagram representation of bearings and distances), Cosine Rule (Size of angle of a triangle, Length of side of a triangle, Relationship between sides and included angle of a triangle), and Sine Rule(Size of angle of a triangle, Length of sides of a triangle and Relationship between sides and opposite angles of a triangle). The major learning task is Bearing (Distance and Estimation of Direction in Degrees).

Research Question Two: What Units of instruction are valid for enhancing student achievement in Bearing

TABLE II: ZERO ORDER CORRELATION COEFFICIENTS

Correlation	Correlation Coefficient (r)
r_{12}	.556**
r_{13}	.444**
r_{14}	.50**
r_{15}	.43**
r_{16}	.277**
r_{17}	.194**
r_{18}	.273**
r_{19}	.204**
r_{23}	.499**
r_{24}	.512**
r_{25}	.409**
r_{26}	.299**
r_{27}	.274**
r_{28}	.258**
r_{29}	.283**
r_{34}	.508**
r_{35}	.397**
r_{36}	.409**
r_{37}	.218**
r_{38}	.265**
r_{39}	.223**
r_{45}	.376**
r_{46}	.399**
r_{47}	.202**
r_{48}	.299**
r_{49}	.290**
r_{56}	.338**
r_{57}	.146**
r_{58}	.192**
r_{59}	.308**
r_{67}	.234**
r_{68}	.295**
r_{69}	.276**
r_{78}	.244**
r_{79}	.259**
r_{89}	.222**

** = Correlation is significant at the 0.01 level (2 tailed)
Significant Link is significant when r_{ij} is significant at **0.05**.

Table II shows that unit 1, fraction is significantly related to the other seven cognitive entry characteristics (Algebraic Process, r=.556**; Angles and Triangles, r=..444**; Trigonometry, r=.50** Specifying Bearing, r=.43**; Presentation of Bearings and Distances with Diagrams, r=.277**; Cosine Rule, r=.194** and Sine Rule, r=.273**) and the major learning task, Bearing, r=.204**.

Similarly, Algebraic Process is found to be significantly related to other six cognitive entry characteristics (Angles and Triangles, r=.499**; Trigonometry, r=.512** Specifying Bearing, r=.409**; Presentation of Bearings and Distances with Diagrams, r=.299**; Cosine Rule, r=.274** and Sine Rule, r=.258**) and the major learning task, Bearing, r=.283**.

Angles and Triangles is another topic in the hypothetical hierarchy found to be significantly related to other five Cognitive Entry Characteristics (Trigonometry, r=.508**; Specifying Bearing, r=.397**; Presentation of Bearings and Distances with Diagrams, r=.409**; Cosine Rule, r=.218**; and Sine Rule, r=.265**) and the major learning task, Bearing, r=.223**

Trigonometric also is found to be significantly related to other four Cognitive Entry Characteristics (Specifying Bearing, r=.376**; Presentation of Bearings and Distances with Diagrams, r=.399**; Cosine Rule, r=.202**; and Sine Rule, r=.299**) and the major learning task, Bearing, r=.290**

Specifying Bearing is found to be significantly related to other three Cognitive Entry Characteristics (Presentation of Bearings and Distances with Diagrams, r=.338**; Cosine Rule, r=.146**; and Sine Rule, r=.192**) and the major learning task, Bearing, r=.308**.

Presentation of Bearings and Distances with Diagrams as a topic in Mathematics is found to be significantly related to other two

Cognitive Entry Characteristics (Cosine Rule, r=.234**; and Sine Rule, r=.295**) and the major learning task, Bearing, r=.276**.

Cosine Rule and Sine Rule are significantly related (r=.244**) and both are significantly related to bearing (r = .259** and .222** respectively).

Thus, all the hypothesized Units and links on the hierarchy were found to be significant. Meaning that acquisition of sufficient knowledge of Fraction (Algebraic Fraction, Decimal Fractions, Division, Multiplication, Subtraction and Addition), Algebraic Process (Word Problem Involving algebra, Solving Equation with Fraction, Solving Simple Equation and Factorization), Angles and Triangles (Size of Angle, Types of Triangle and Types of Angle), Trigonometry (Length of side of a Triangle, Size of Angle, Inverse of Trigonometric Ratio, Reading four figure Table, Trigonometric Ratios-Sine, Cosine and Tangent), Specifying Bearing (Angular, Compass Method and Measuring Angles), Presentation of Bearings and Distances with Diagrams (Diagram representation of bearings and distances), Cosine Rule (Size of angle of a triangle, Length of side of a triangle, Relationship between sides and included angle of a triangle), and Sine Rule(Size of angle of a triangle, Length of sides of a triangle and Relationship between sides and opposite angles of a triangle) leads to better achievement in Bearing. Hence all the hypothesized Units and links are retained to have the validated learning hierarchy for teaching and learning Bearing, Mathematics topics identified to be difficult. Thus, the validated Hierarchy is presented in Fig 2.

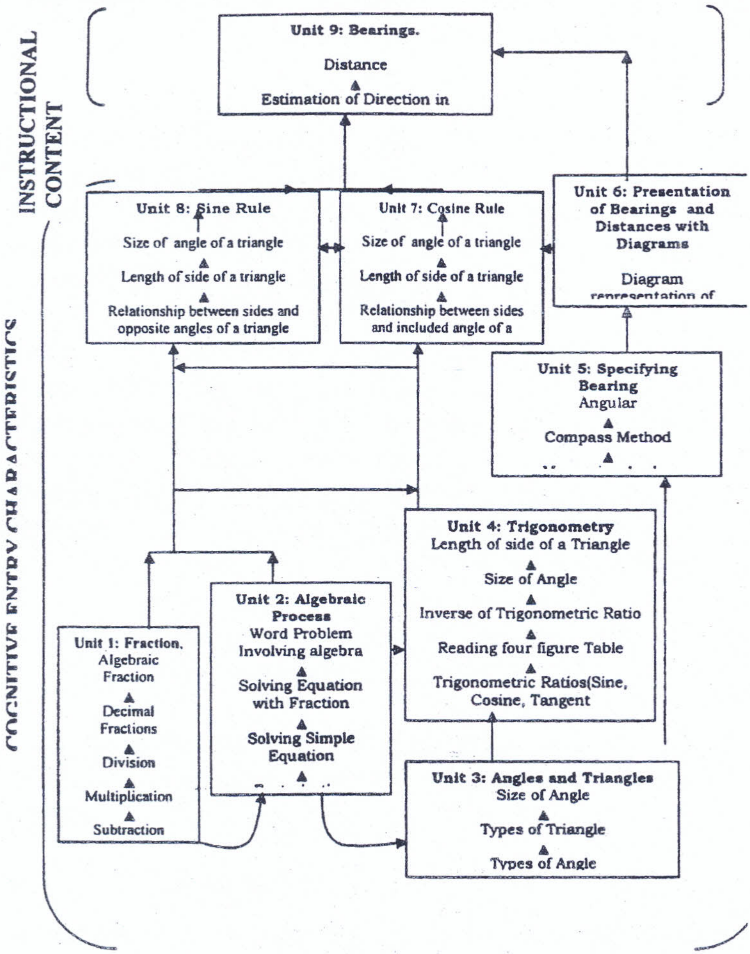

Fig. 2: VALIDATED LEARNING HIERARCHY IN BEARINGS

5. DISCUSSION

This study, shows that possession of adequate levels of Cognitive Entry Characteristics leads to high cognitive achievement in Mathematics. This is in agreement with Bloom's (1974, 1981) theory, Abadom's (1993) and Adeleke's (2007) findings. The students that performed better in the units identified to be entry characteristics equally performed better in the major learning task. Adeleke (2007) found out that the highly enhanced Cognitive Entry Characteristics group performed better than students in partially enhanced CEC and the control group. This reveals the efficacy of CEC.

Sequencing learning tasks hierarchically leads to some improvement in performance. This finding is in agreement with Mason (2005) that learning experiences at lower level are basic prerequisite for learning formal geometric concepts. However the sequencing of learning tasks hierarchically can be complemented with an appropriate remediation strategy to promote better achievement among the learners. Abadom (1993) aptly summarizes this finding in her assertion that presenting learning tasks hierarchically is a necessity but however not sufficient condition for high levels of cognitive achievement. Mason went further to assert that, it seems that if the prerequisite tasks have not been learned to a high degree, it will not make much impact on the summative achievement test. This finding provides some explanation on the learning of geometry. The sequence of presentation of learning materials assists learners build competence to cope with higher learning task.

Based on the findings of this study, it could be said that Bloom's Theory of school learning is applicable to the learning of geometry in the Senior Secondary School especially with respect to cognitive achievement. When students come into a learning situation with high levels of necessary cognitive entry characteristics, they attain high levels of cognitive achievement.

It would seems that any student with learning difficulties in topics that make up CEC (Fraction, decimal and Algebraic fraction, Algebraic Process, Angles and triangle, Trigonometry, Specifying Bearing, presentation of bearing with diagram, cosine rule and Sine rule) may not be able to solve problems on bearing and distances adequately. Abadom's (1993) explanation on the relationship between CEC and cognitive achievement paints a plausible picture of what goes on in the teaching learning situation. According to Abadom:

> *The partially enhanced CEC and control groups had a more complex learning situation. Many of them had to learn the process and figure out what was being done when each of those pre requisite skills was being brought into the solution of the problem. From the beginning, too many of the students in the two groups had more learning difficulties as they progress in the target learning tasks. The misunderstanding or difficulty encountered with any of the sub skills or process further complicated the learning.*

This explains the reason why positive relationships exist among the CECs and finally with the criterion task, bearing and distances.

6. RECOMMENDATIONS

The findings of this study informed the following recommendations:

1. Teachers' responsibilities go beyond presentation of instructional content but sequencing it to support learning. Hence, teachers (especially mathematics teachers) should prepare their instructional content to include necessary Cognitive Entry Characteristics which should be sequenced to support learners achieve significantly in any target learning task.

2. Experts in Pedagogy and Instructional Contents should be engaged from time to time by Teachers' employers (government, Missions and Individuals) in organizing Workshops and Seminars on structuring of instructional content. This will assist every teacher in the preparation and dissemination of instructional content that yields better achievement in school subjects especially Mathematics.

3. The current curriculum that is in use in the Senior Secondary Schools should be reviewed to include necessary CECs.

4. School Inspectors and Supervisors should not be left out in knowledge updates. They should be trained on the efficacy of sequencing instructional contents. This will equip them sufficiently to assist deficient teachers while on field.

5. Sequencing of learning tasks hierarchically should be supported with appropriate remediation strategy to enhance better achievement among the learners.

7. CONCLUSION

Students in Nigerian schools need to have the opportunity to engage in learning experiences that enhance mastery of educational goals and standards. Knowledge of mathematics and the ability to apply Mathematical skills will not make all the students mathematicians later in life but support them to solve problems especially those that involve geometrical manipulation, basic operations (addition, subtraction, and multiplication, division) and everyday calculations. These could actually be possible if solid foundation for success in Mathematics is laid by teachers in schools by sequencing instructional content to foster effective learning.

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APPENDIX I

Level 1 (Visualization): Students recognize figures by appearance alone, often by comparing them to a known prototype. The properties of a figure are not perceived. At this level, students make decisions based on perception, not reasoning.

Level 2 (Analysis): Students see figures as collections of properties. They can recognize and name properties of geometric figures, but they do not see relationships between these properties. When describing an object, a student operating at this level might list all the properties the student knows, but not discern which properties are necessary and which are sufficient to describe the object.

Level 3 (Abstraction): Students perceive relationships between properties and between figures. At this level, students can create meaningful definitions and give informal arguments to justify their reasoning. Logical implications and class inclusions, such as squares being a type of rectangle, are understood. The role and significance of formal deduction, however, is not understood.

Level 4 (Deduction): Students can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. At this level, students should be able to construct proofs such as those typically found in a high school geometry class.

Level 5 (Rigor): Students at this level understand the formal aspects of deduction, such as establishing and comparing mathematical systems. Students at this level can understand the use of indirect proof and proof by contrapositive, and can understand non-Euclidean systems.

In addition to this, Clements and Battista (1992) proposed the existence of Level 0, which they call *pre-recognition*. Students at this level notice only a subset of the visual characteristics of a shape, resulting in an inability to distinguish between figures. They gave an example that they may distinguish between triangles and quadrilaterals, but may not be able to distinguish between a rhombus and a parallelogram. If ability to differentiate among symbols serves as entry behaviour (Cognitive entry Characteristic) to a learning task then difficulty is bound to set in.