CHARACTERIZATION OF FLUID FLOW USING RANDOM WALK DYNAMICS

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ABSTRACT

There has been growing need to characterize the fluid flow through a simplified model such as random walk dynamics. The research work covered three distinct regions of fluid flow namely the laminar region, the transition region and the turbulent region. Appropriate flow charts and FORTRAN-90 source codes were developed to solve relevant fluid flow governing equations. Reynolds number was used as the control parameter to tune from laminar to turbulent flow while relevant solution was graphed using Microsoft Excel. The graph shows the distinct regions. The first region characterizes laminar region with three straight-line segments. The second region is the transition region, which is in form of wavy line segments. The third region is the turbulent region in which higher wavy line segments are shown. The degree of waviness and number of wavy line segments increases from transition region to turbulent region. The result shows that fluid flow can be characterized through the use of random walk dynamics.

Keywords: Fluid flow, random walk dynamics, Reynolds number, distinct regions, wavy line segments.

INTRODUCTION

A random walk is formalization in Mathematics, Computer Science and Physics of the intuitive ideas of taking successive steps, each in a random direction. The simplest random walk considers a walker that takes steps of length L to the left or right along a line while more complex random walks include fancies consideration such as given each step velocity and allowing the random walker to pause for random amount of time in between the steps.

Ojoawo (2007) investigated random walker in three dimensional Euclidean space. The random method to model the diffusion of vorticity was first proposed by Chorin(1978). In order to simulate the diffusion of vorticity in vortex flow, the positions of the vortices are given random displacements (Chorin and Marsden, 1990). The basic idea of the random walk method as applied to fluid flow is that the random displacements spread out the vorticity. Several studies investigated the theoretical and numerical aspect of the random walk method. Marchiora and Pulvienti (1982), Goodman (1987) and Long (1988) have shown that for flow in free space, the random walk solution converges to that of the Navier-Stokes equations as the number of vortices is increased, Cheer (1989) has applied the random walk method to flows over a cylinder. Lewis (1990) has used the random walk method for flow over airflow cascade while Chui (1993) used the random walk method to study thermal boundary layers. The random walk method has several advantages. It is simple to use and it can easily handle flows around complicated boundaries. The method also conserves the total circulation. This is in application to inviscid flows.

The soil erosion considered is the detachment of materials from the bed or sides of a channel. The water flowing through stream performs three types of geologic work. Moving water erodes materials from the bed or side of the channel and transports the eroded material to a new location and deposits it. After the material has been detached from the channel, it can be transported. As the particle size increases, the velocity needed to transport it also increases.

The Reynolds number governs laminar-turbulent transition. It also characterizes whether the flow conditions lead to laminar or turbulent flow. Transition to turbulent can occur over a range of Reynolds numbers depending on many factors such as surface roughness, heat transfer, vibration, noise and other disturbances.

The objectives of this work are to:

1. Characterize parameters for fluid flow using a set of random walkers.
2. Develop a simplified model to characterize the fluid flow through the use of random walk dynamics.

The study intends to explore the distinguishing features of the distinct regions in fluid flow through the use of random walk dynamics.

The research project is significant to the advancement of Science and Engineering. It is justified for the following reasons:

1. The random walk model can be used to analyze flows in floods.
2. The random walk dynamics is a simplified model that can help in analysis of farmland erosion and hence aid enhanced food production.

This paper reports a "random walk model" for characterization of fluid flow through the use of 'boundary layers by discrete vortex modeling'. The research work is expected to cover the three distinct regions namely laminar, transition and turbulent.

MODEL FORMULATION

The first practical scheme for simulation of a boundary layer by discrete vortices was proposed by Chorin (1978) based on his earlier conception of the random walk model for high Reynolds number bluff body wake flows. The boundary layer flow can be approximated by placing at appropriate location some vortices in a parallel flow. This forms the basis of the vortex element method.

The motion of a diffusing vortex of initial vorticity strength (Γ) entered on the origin of the (r,θ,ф) plane is described by the diffusion equation from which we may obtain the well known solution for subsequent vorticity w(r,t) in space and time.

$$\omega(r,t) = \frac{\Gamma}{4\pi\nu t} e^{\left(-r^2/4\nu t\right)} \qquad (1)$$

Vorticity strength is a function of radius r and time t.

For a vortex of unit strength split into N elements. Let us assume that n vortex elements are scattered into the small area rΔθΔфΔr after time t, the total amount of vorticity P_v in this element of area then follows from :

$$P_v = \frac{n}{N} = \left[\frac{1}{4\pi\nu t} e^{\left(-r^2/4\nu t\right)}\right] r\,\Delta\theta\Delta\phi\Delta r \qquad (2)$$

Where π is the ratio of the circumference of a circle to its diameter and υ is the kinematic viscosity.

An appropriate strategy is to displace each element in the in the radial and angular directions by amounts r_i, θ_i and ϕ_i over time interval 0 to t. Thus we may define θ and ϕ values independently of r_i values by the equation :

$$\theta_i = 2\pi Q_i \qquad (3)$$

$$\phi_i = \pi Q_i \qquad (4)$$

Where Q_i is a random number within the range $0 < Q_I < 1.0$.

The probability P that an element will be within a circle of radius r is given by the equation

$$P = 1 - e^{\left(-r^2/4\nu t\right)} \qquad (5)$$

Thus for the n[th] vortex element equation (5) becomes

$$P_i = 1 - e^{\left(-r_i^2/4\nu t\right)} \qquad (6)$$

From which we obtain its radial random shift

$$r_i = \left(4\nu t \ln\left(\frac{1}{1-p_i}\right)\right)^{1/2} \qquad (7)$$

Considering diffusion over a succession of small time increments Δt, the displacements of element i during time Δt will then be

$$\Delta\theta_i = 2\pi Q_i \qquad (8)$$

$$\Delta\phi_i = \pi Q_i \qquad (9)$$

$$\Delta r_i = \left(4\nu t \ln\left(\frac{1}{1-P_i}\right)\right)^{1/2} \qquad (10)$$

Thus after the increment Δt, the new coordinate location (x_i', y_i', z_i') of the n[th] element will become

$$x_i' = x_i + \Delta r_i \sin\theta_i \cos\phi_i \qquad (11)$$

$$y_i' = y_i + \Delta r_i \sin\theta_i \cos\phi_i \qquad (12)$$

$$z_i' = z_i + \Delta r_i \cos\phi_i \qquad (13)$$

Where x_i = old x – coordinate of n[th] element
y_i = old y – coordinate of n[th] element
z_i = old z – coordinate of n[th] element

The displacement from the origin is given by the equation:

$$D_i = \sqrt{\left((x_i' - x_0)^2 + (y_i' - y_0)^2 + (z_i' - z_0)^2\right)} \qquad (14)$$

Where x_0, y_0 and z_0 are the origin

Boundary layers by Discrete vortex modeling

Convective motion were completely ignored for the diffusion point flow which have just been considered, an assumption which is permissible in view of symmetry in these special cases and justified for very low Reynolds numbers.

Boundary layer flows on the other hand are more complex involving:

(i). Externally imposed convection due to the main stream U, the significance of which is determined by

the body scale Reynolds number $\left(\frac{UL}{\nu}\right)$

L is the characteristic length of the particular flow.

(ii). Continuous creation of vorticity at the contact surface between fluid and wall, replacing the vorticity removed by diffusion and convection.

Random Number Generation

Algorithms were developed to produce long sequences of apparently random results, which are in fact completely determined by a shorted initial value known as a seed.

Application of Random Walk Method

The application of the random walk will result in the loss of half of the newly created vorticity due to diffusion across the walls and therefore out of the active flow domain if vorticity is not conserved during the diffusion and convection processes for each time step.

The single strength sheet is used through bouncing back vortices which attempt to cross the wall by assigning the value y_i = abs (y_i)

Selection of Element Size and Time Step

A reasonable approach to the selection of an appropriate time Δt is to focus attention on the average displacements of the discrete vortices due to convection and diffusion. The average convective displacement may be approximated by:

$$\delta_C = \frac{1}{2} U\Delta t \qquad (15)$$

The average diffusive displacement may be approximated by:

$$\delta_D = \sqrt{(4\nu\Delta t \ln 2)} \qquad (16)$$

To maintain equal discretisation of the fluid motion due to convection and diffusion we may equate δ_C and δ_D resulting in the expression

$$\Delta t = \frac{16L\ln 2}{U\,\mathrm{Re}} \qquad (17)$$

Where Re = $\frac{UL}{\nu}$ is the plate Reynolds number

It would also be reasonable to select surface element size Δs at twice δ_c leading to

$$\Delta s = U\Delta t \qquad (18)$$

$$\Delta s = \frac{16L\ln 2}{\mathrm{Re}} \qquad (19)$$

The required number of surface elements for satisfactory discretisation of the plate is then given by

$$M = \frac{L}{\Delta s} \qquad (20)$$

$$M = \frac{\mathrm{Re}}{16\ln 2} \qquad (21)$$

It is clear from this study that enforcing equal discretisation scales δ_C and δ_D for convention and diffusion will lead to computational difficulties at high Reynolds numbers. For example, the boundary layer considered for Re = 500, yields M = 45. On the other hand for a typical engineering system value of Re = 10^5, yields roughly M = 9017, thereby imposing severe pressure upon computational requirements. The related time increment Δt = 0.00011 would also require 10^4 time steps to achieve one flow pass. It is thus clear that practical computational limitations will rule out vortex modeling for typical engineering system Reynolds numbers if we attempt to impose the constraint $\delta_C = \delta_D$ to the foregoing calculation.

Some Considerations for high Reynolds Number Flows

One way to reduce these difficulties for high Reynolds number would be to select different time steps for diffusion (Δt_D) and convection (Δt_C). Since convection now dominates the flow, it will be preferable to select the scale of convection displacements through:

$$k = \frac{\delta_c}{\Delta s} \qquad (22)$$

Where k can be set to be equal to 0.5

The convective time step is:

$$\Delta t_c = \frac{2k\Delta s}{U} \qquad (23)$$

$$\Delta t_c = \frac{2k}{M}\left(\frac{L}{U}\right) \qquad (24)$$

Although it would be perfectly in order to perform both the convection and random walk processes over the same time step Δt_C, a saving in computational effort could be achieved by undertaking only one random walk for every N_t convection step with

$$\Delta t_D = N_t\,\Delta t_C \qquad (25)$$

The upper limit of N_t obtained from equating the scales δ_C and $\delta_D N^t$ is

$$N_t = \frac{k\mathrm{Re}}{8M\ln 2} \qquad (26)$$

SIMULATION

The governing equation is developed for the fluid flow. The Reynolds number served as the control parameter that governed the laminar-turbulent transition. This is followed by the formulation of algorithms for the model, which is illustrated by the flow chart. The flow chart is used in writing the FORTRAN-90 program. The program is then run to generate desire output. The result obtained were used to plot the graphs through Microsoft Excel.

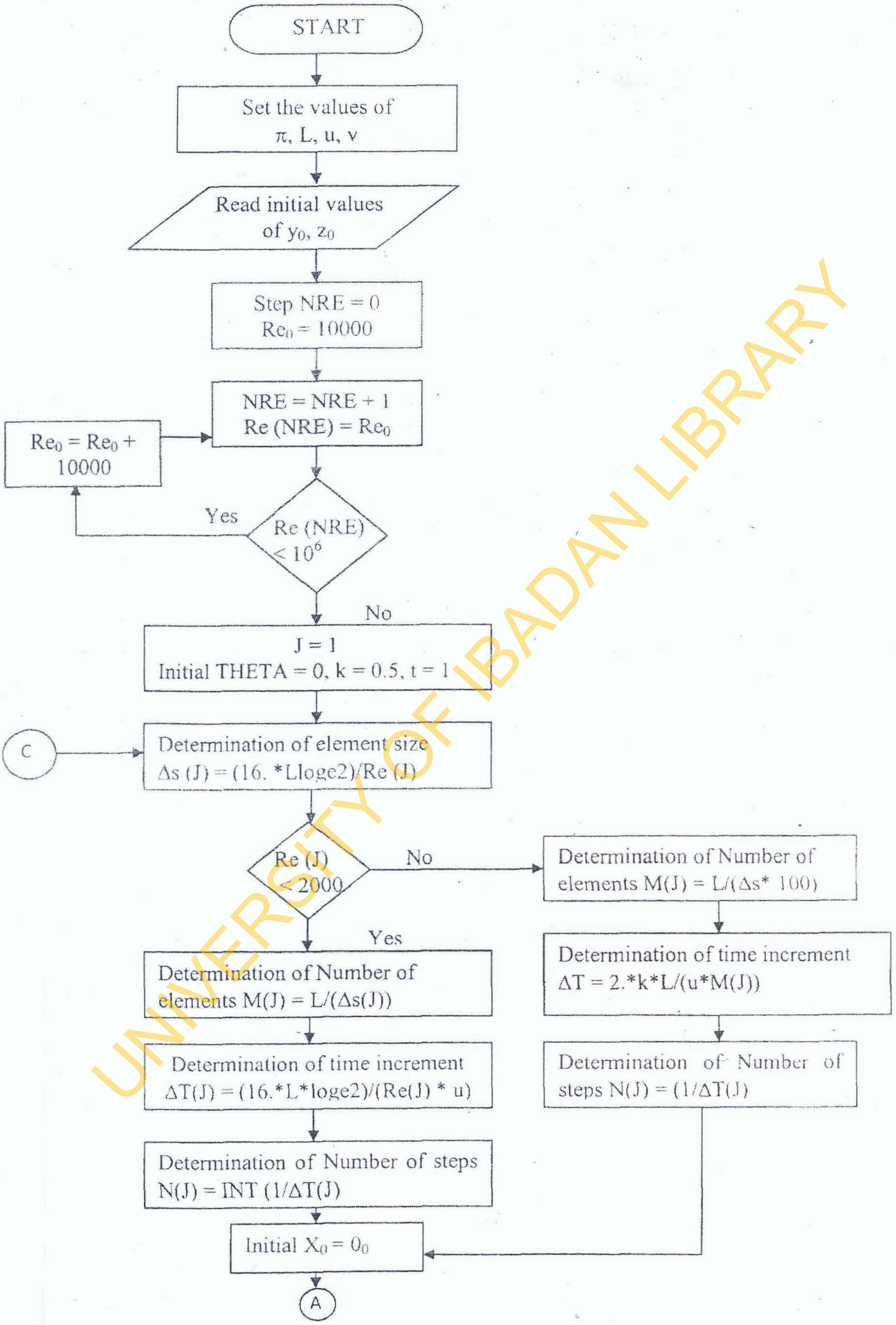
RESULTS AND DISCUSSION

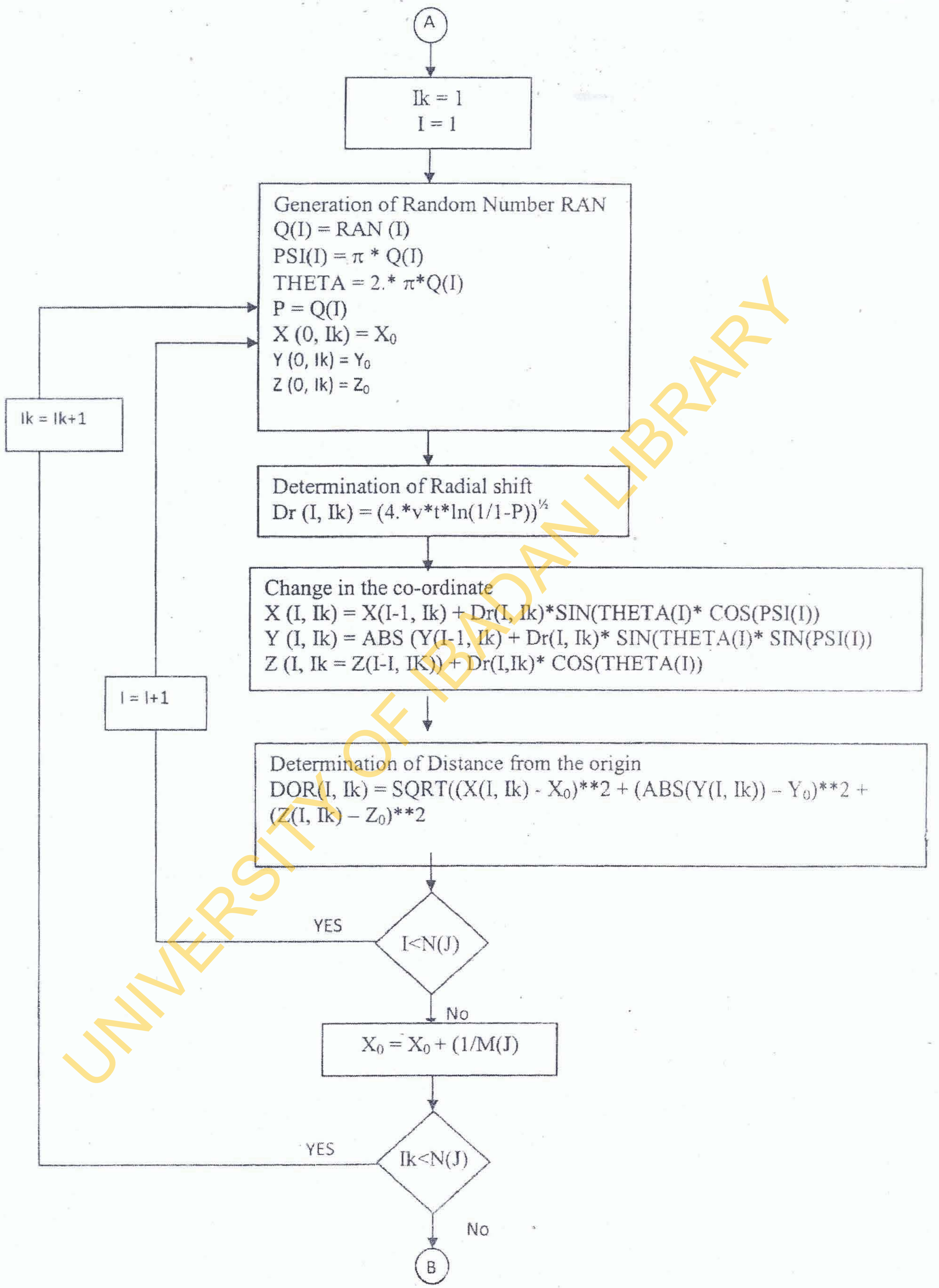
Table 1 shows the result of Reynolds number and time increment. It also shows the number of time steps, number of elements and log of average distance against log of time steps. The index is the slope obtained from the graph of log of average distance against log of time steps. The time increment decreases with increase in Reynolds number. The Reynolds number increases with increase in the number of time steps and number of elements or trials. Initially, the Reynolds number increases with the index, but from the Reynolds number of 70,000 there is onset of fluctuation in index.

The characterizing parameters are the index and Reynolds number. The graph of Index and Reynolds number displayed three distinct regions (fig.2) The concept of critical Reynolds number proves quite useful in demarcating the regimes of laminar and turbulent flows. The lower limit of critical Reynolds $(\mathrm{Re})_{cr}$ exists and its value is approximately 70,000. The upper limit of critical value of $(\mathrm{Re})_{cr}$ which characterizes full attainment of transition lie between 90,000 and 310,000. The lower critical Reynolds number is of greater engineering importance as it defines the limit below which all turbulence, no matter how severe, entering the flow from any source will eventually be damped out by viscous action.

The first region characterized laminar region with straight-line segment (fig.3) In this region, the Reynolds number is less than 70,000.The second region is the transition region which is in form of way-line segments (fig.4) There is onset of waviness (moving to and fro or up and down of lines in series). This region is from Reynolds number of 70,000 to 310,000. The third region is the turbulent region in which higher wavy-line segments are shown (fig.5) The region starts from Reynolds number of 320,000. Hence in laminar region, there is no waviness while the degree of waviness and number of wavy line segments increase from transition region to turbulent region.

Fig.1. Random Walk Model Algorithms/Flowchart




A
Ik = 1
I = 1
Generation of Random Number RAN
Q(I) = RAN (I)
PSI(I) = π * Q(I)
THETA = 2.* π*Q(I)
P = Q(I)
X (0, Ik) = X0
Y (0, Ik) = Y0
Z (0, Ik) = Z0
Ik = Ik+1
Determination of Radial shift
Dr (I, Ik) = (4.*v*t*ln(1/1-P))½
Change in the co-ordinate
X (I, Ik) = X(I-1, Ik) + Dr(I, Ik)*SIN(THETA(I)* COS(PSI(I))
Y (I, Ik) = ABS (Y(I-1, Ik) + Dr(I, Ik)* SIN(THETA(I)* SIN(PSI(I))
Z (I, Ik = Z(I-I, IK)) + Dr(I,Ik)* COS(THETA(I))
I = I+1
Determination of Distance from the origin
DOR(I, Ik) = SQRT((X(I, Ik) - X0)**2 + (ABS(Y(I, Ik)) – Y0)**2 +
(Z(I, Ik) – Z0)**2
YES
I<N(J)
No
X0 = X0 + (1/M(J)
YES
Ik<N(J)
No
B

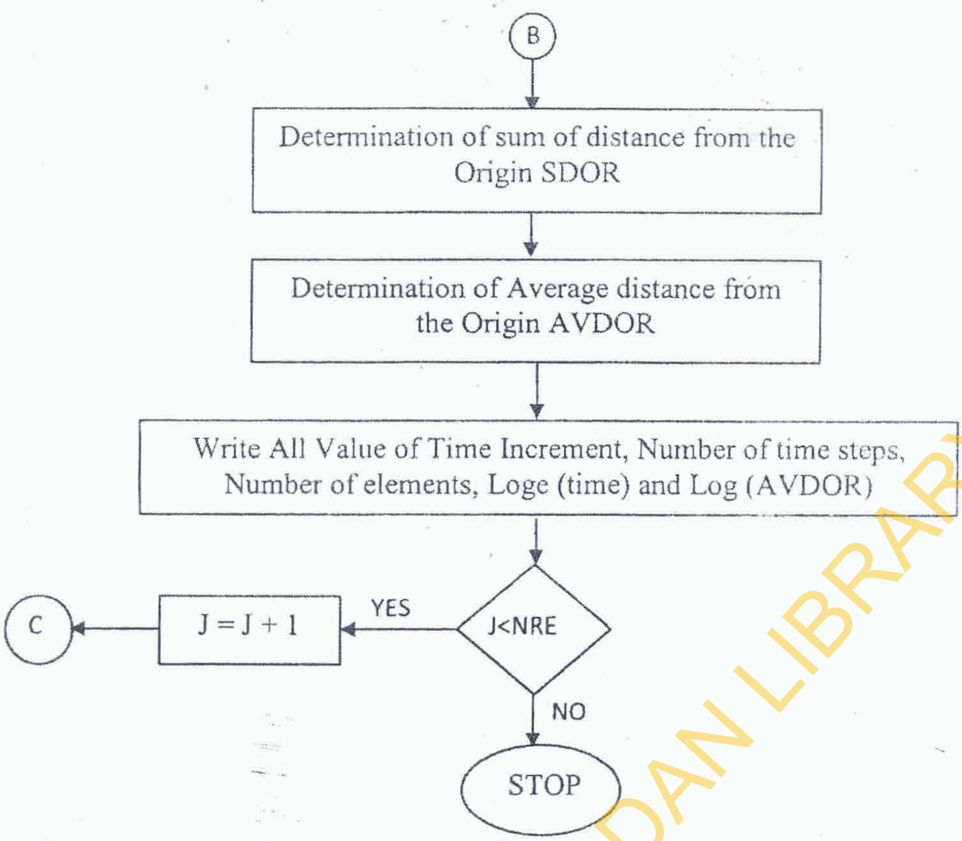


TABLE 1: CHARACTERIZATION OF FLUID FLOW

Reynolds number (Re)	Time Increment (Δt)	Number of time steps (N)	Number of elements or trials (M)	Log of average distance against log of time steps (y = mx + c)	Index (m)
10,000	0.11090	9	9	y = 0.5865 x + 0.2821	0.5865
20,000	0.05545	18	18	y = 0.7218 x + 0.2165	0.7218
30,000	0.03697	27	27	y = 0.8182 x + 0.1355	0.8182
40,000	0.02773	36	36	y =0.8243 x + 0.1274	0.8243
50,000	0.02218	45	45	y = 0.8266 x + 0.1647	0.8266
60,000	0.01848	54	54	y = 0.8281 x + 0.1517	0.8281
70,000	0.01584	63	63	y = 0.8558 x + 0.1131	0.8558
80,000	0.01386	72	72	y = 0.8817 x + 0.9490	0.8817
90,000	0.01232	81	81	y = 0.875 x + 0.1003	0.875
100,000	0.01109	90	90	y = 0.8908 x + 0.0874	0.8908
110,000	0.01008	99	99	y = 0.8859 x + 0.0230	0.8859
120,000	0.00924	108	108	y = 0.8806 x + 0.0929	0.8806
130,000	0.00853	117	117	y = 0.8906 x + 0.0863	0.8906
140,000	0.00792	126	126	y = 0.8953 x + 0.0713	0.8953
150,000	0.00739	135	135	y = 0.8948 x + 0.0780	0.8948
160,000	0.00693	144	144	y = 0.0919 x + 0.0722	0.0919
170,000	0.00652	153	153	y = 0.9125 x + 0.052	0.9125
180,000	0.00616	162	162	y = 0.9087 x + 0.0561	0.9087
190,000	0.00584	171	171	y = 0.921 x + 0.0361	0.921
200,000	0.00555	180	180	y = 0.9232 x + 0.0317	0.9232
210,000	0.00528	189	189	y = 0.9170 x + 0.0456	0.9170
220,000	0.00504	198	198	y = 0.9236 x + 0.0346	0.9236
230,000	0.00482	207	207	y = 0.9250 x + 0.0311	0.9250
240,000	0.00462	216	216	y = 0.9193 x + 0.0450	0.9193
250,000	0.00444	225	225	y = 0.9220 x + 0.0468	0.9220
260,000	0.00427	234	234	y = 0.9268 x + 0.0361	0.9268

Table 1: Cont'd

Reynolds number (Re)	Time Increment (Δt)	Number of time steps (N)	Number of elements or trials (M)	Log of average distance against log of time steps (y = mx + c)	Index (m)
270,000	0.00411	243	243	y = 0.9293 x + 0.0295	0.9293
280,000	0.00396	252	252	y = 0.9367 x + 0.0118	0.9367
290,000	0.00382	261	261	y = 0.933 x + 0.0238	0.933
300,000	0.00370	271	271	y = 0.9338 x + 0.0208	0.9338
310,000	0.00358	280	280	y = 0.9295 x + 0.0353	0.9295
320,000	0.00347	289	289	y = 0.9375 x + 0.0158	0.9375
330,000	0.00336	298	298	y = 0.9329 x + 0.0264	0.9329
340,000	0.00326	307	307	y = 0.9357 x + 0.0217	0.9357
350,000	0.00317	316	316	y = 0.9373 x + 0.0172	0.9373
360,000	0.00308	325	325	y = 0.9368x + 0.0192	0.9368
370,000	0.00300	334	334	y = 0.9413 x + 0.0107	0.9413
380,000	0.00292	343	343	y = 0.9454 x + 0.0035	0.9454
390,000	0.00284	352	352	y = 0.9406 x + 0.0136	0.9406
400,000	0.00277	361	361	y = 0.9445 x + 0.005	0.9445
410,000	0.00270	370	370	y = 0.9450 x + 0.002	0.9450
420,000	0.00264	379	379	y = 0.9450 x + 0.0032	0.9470
430,000	0.00258	388	388	y = 0.9470 x - 0.0003	0.9470
440,000	0.00252	397	397	y =0.9457 x + 0.0038	0.9457
450,000	0.00246	406	406	y = 0.9548 x – 0.0173	0.9548
460,000	0.00241	415	415	y = 0.9479 x – 0.0008	0.9479
470,000	0.00236	424	424	y = 0.9470 x + 0.0001	0.9470
480,000	0.00231	433	433	y = 0.9539 x – 0.0129	0.9539
490,000	0.00226	441	441	y = 0.9495 x – 0.0028	0.9495
500,000	0.00222	451	451	y = 0.9525 x – 0.0095	0.9525
510,000	0.00217	460	460	y = 0.954 x – 0.0134	0.954
520,000	0.00213	469	469	y = 0.9531 x – 0.0109	0.9531

Table 1: Cont'd

Reynolds number (Re)	Time Increment (Δt)	Number of time steps (N)	Number of elements or trials (M)	Log of average distance against log of time steps (y = mx + c)	Index (m)
530,000	0.00209	478	478	y = 0.9534 x – 0.0110	0.9534
540,000	0.00205	487	487	y = 0.9551 x – 0.0050	0.9551
550,000	0.00202	496	496	y = 0.9524 x – 0.0070	0.9524
560,000	0.00198	505	505	y = 0.9570 x – 0.0179	0.9570
570,000	0.00195	514	514	y = 0.9541 x – 0.0117	0.9541
580,000	0.00191	523	523	y = 0.9538 x – 0.0095	0.9538
590,000	0.00188	532	532	y = 0.9574 x – 0.0199	0.9574
600000	0.00185	541	541	y = 0.9569 x – 0.0168	0.9569
610,000	0.00182	550	550	y = 0.9578 x – 0.0194	0.9578
620,000	0.00179	559	559	y = 0.9587 x – 0.0212	0.9587
630,000	0.00176	568	568	y = 0.9593 x – 0.0230	0.9593
640,000	0.00173	577	577	y = 0.9620 x – 0.0029	0.9620
650,000	0.00171	586	586	y = 0.9582 x – 0.0180	0.9582
660,000	0.00168	595	595	y = 0.9572 x – 0.0161	0.9572
670,000	0.00166	604	604	y = 0.9595 x – 0.0235	0.9595
680,000	0.00163	613	613	y = 0.9589 x – 0.0203	0.9589
690,000	0.00161	622	622	y = 0.9618 x – 0.0277	0.9618
700,000	0.00158	631	631	y = 0.9599 x – 0.0242	0.9599
710,000	0.00156	640	640	y = 0.9586 x – 0.0194	0.9586
720,000	0.00154	649	649	y = 0.9628 x – 0.03	0.9628
730,000	0.00152	658	658	y = 0.9604 x – 0.0224	0.9604
740,000	0.00150	667	667	y = 0.9600 x – 0.0212	0.9600
750,000	0.00148	676	676	y = 0.9625 x – 0.0273	0.9625
760,000	0.00146	685	685	y = 0.9612 x – 0.0235	0.9612
770,000	0.00144	614	614	y = 0.9652 x – 0.0357	0.9652
780,000	0.00142	703	703	y = 0.9624 x – 0.0268	0.9624

Table 1: Cont'd

Reynolds number (Re)	Time Increment (Δt)	Number of time steps (N)	Number of elements or trials (M)	Log of average distance against log of time steps (y = mx + c)	Index (m)
790,000	0.00140	712	712	y = 0.9627 x – 0.0274	0.9627
800,000	0.00139	721	721	y = 0.9625 x – 0.0275	0.9625
810,000	0.00137	730	730	y = 0.9630 x – 0.028	0.9630
820,000	0.00135	739	739	y = 0.9650 x – 0.0333	0.9650
830,000	0.00134	748	748	y = 0.9611 x – 0.0226	0.9611
840,000	0.00132	757	757	y = 0.9625 x – 0.0251	0.9625
850,000	0.00130	766	766	y = 0.9637 x – 0.0298	0.9637
860,000	0.00129	775	775	y = 0.9649 x – 0.0321	0.9649
870,000	0.00127	784	784	y = 0.9634 x – 0.0274	0.9634
880,000	0.00126	793	793	y = 0.9658 x – 0.0348	0.9658
890,000	0.00125	802	802	y = 0.9642 x – 0.0302	0.9642
900,000	0.00123	812	812	y = 0.9653 x – 0.0323	0.9653
910,000	0.00122	821	821	y = 0.9644 x – 0.03	0.9644
920,000	0.00121	830	830	y = 0.9632 x – 0.0268	0.9632
930,000	0.00119	839	839	y = 0.9681 x – 0.0402	0.9681
940,000	0.00118	848	848	y = 0.9684 x – 0.0398	0.9684
950,000	0.00117	857	857	y = 0.9672 x – 0.0371	0.9672
960,000	0.00116	866	866	y = 0.9688 x – 0.0415	0.9688
970,000	0.00114	875	875	y = 0.9666 x – 0.035	0.9666
980,000	0.00113	884	884	y = 0.9694 x – 0.0417	0.9694
990,000	0.00112	893	893	y = 0.9697 x – 0.0432	0.9697
1000000	0.00111	902	902	y = 0.9696 x – 0.0428	0.9696

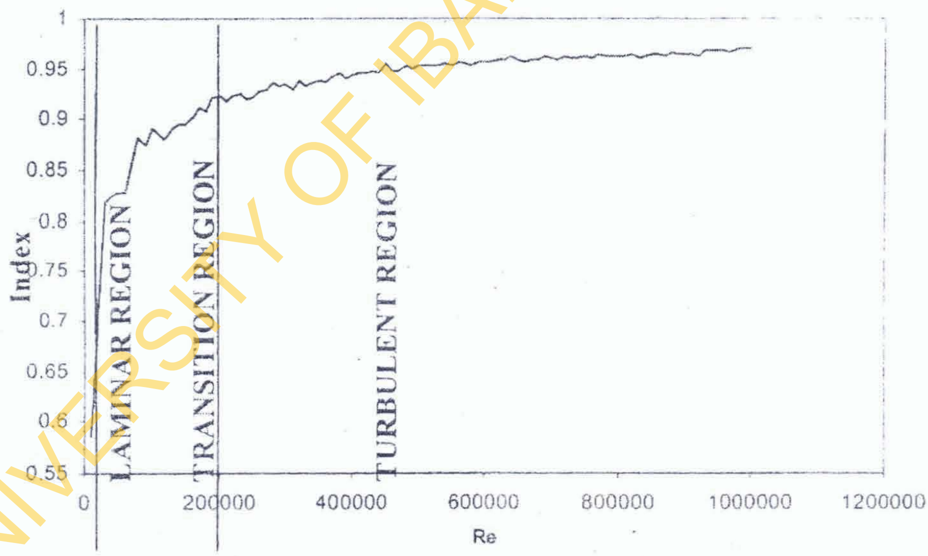


Fig 2: Laminar, transition and turbulent region

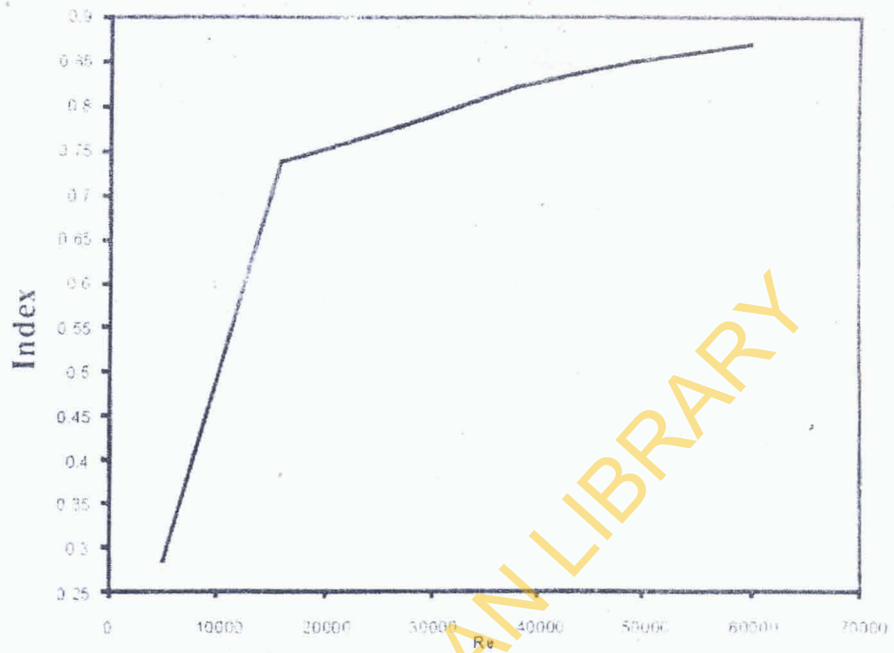


Fig. 3: Laminar region

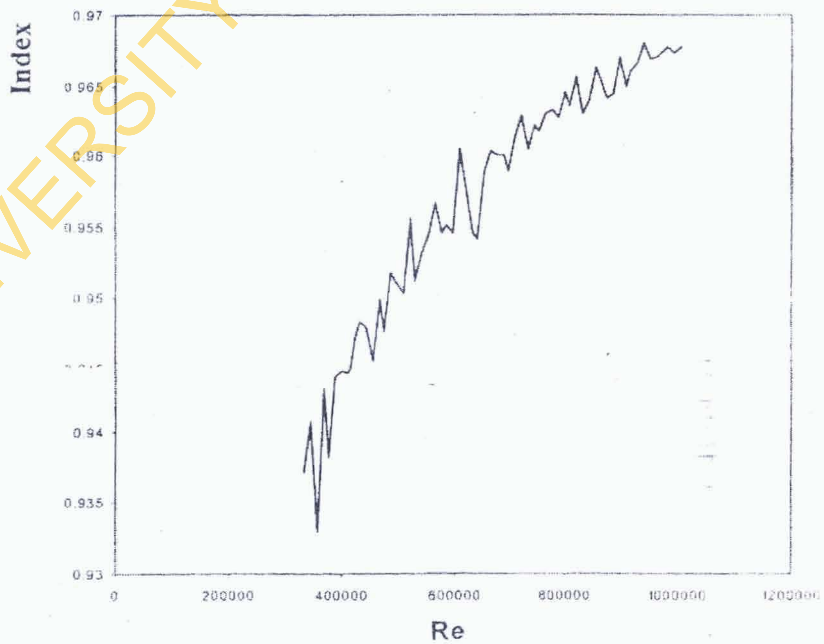


Fig. 5 : Turbulent region

CONCLUSION

The study has explored the use of random walk model to characterize the fluid flow (soil erosion). The index number increases with increase in Reynolds number. Rate of increase of index number is highest in the laminar region and smallest in turbulent region.

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