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PREDICTION OF CENTRAL LOCATION OF THE STATE WITH IRREGULAR BOUNDARY

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ABSTRACT

Mathematics has diverse applications as decision making tool. This study investigated the use of a proposed pure mathematical formulation (i.e. excluding human factor) for citing appropriately the location of political capital city of an inhabited designated area. Digitized data of Scale map of Nigeria as a case study was used as input to a FORTRAN 90 programme codes of an equivalent least squares method mathematical formulation. The resulting simultaneous equations involving the political capital city coordinates as unknown variables were solved using Gauss Elimination Algorithm. The political capital city Cartesian coordinate prescribed as (26, 24) in grids unit for the studied case of scale map of Nigeria compared visually satisfactorily with Abuja location on the Map. The Nigeria land area cover was under-estimated by 2.3% referencing 923,768 sq km obtained from Microsoft Encarta Premium (2009). Similarly the Nigeria coastline was estimated as 25.3% of the country's estimated perimeter (3369 km). This mathematical tool can be used to cite the centrally located capital city or capital city relocations economically, timely, accurately and reliably. This programme can also be used for citing of capital cities for other countries and center of any irregular shapes on two dimensional plane.

Keywords: Irregular Shape, Digitization, Mathematical tools, FORTRAN,

INTRODUCTION

Mathematics plays several important roles in diverse disciplines including political decision making. It helps us not only to count votes, but also sheds light on the question of whether some voting and apportionment systems are fairer than others, Bennett and Briggs (2001).

For political reasons among others the acceptable mathematical statement that 12 2/3 approximated 13 during the petition proceeding of Nigeria 1979 Presidential election was a subject of post election hot debate, Reserved Judgements of the Supreme Court of Nigeria (1979). The inherent danger of not accepting this mathematical settlement is better imagined than witnessed. Nigeria may have been unable to move forward provided that the stalemate can be managed. Political stalemate of this nature may degenerate the nation to a war torn country.

Political office holders ruling group of people living together on designated land area herein referred to as "state" usually have their operating offices located at political capital city. The political capital city, not necessarily the economic state capital, is any location within the designated land area as agreeable to all state inhabitants' representatives.

Generally, the state boundary used to be irregular. The irregularity coupled with political factors can make political capital city location tasking. Associated problems of political capital city location may discourage the creation of newer states and makes state slower development inevitable. Nigeria enjoyed newer states creation in 1939, 1963, 1967, 1976 and 1987 respectively, Jaiyesimi (2009). Although newer state creation is good for developmental purposes, the continued agitation for more new state creation in Nigeria in recent times remained unimplemented. The reasons may not be more than political, among others.

The present study proposes a mathematical formulation that is capable of prescribing appropriate political capital city location using rigorous procedure and the Cartesian coordinates of the state irregular boundary.

A substitute circular boundary close to all points on the irregular boundary was sought. The centre of the circular boundary is the same as the political capital city location. This formulation procedure is analogous to finding best line of fit to experimental data using least squares method. Least squares linear regression method uses a rigorous mathematical procedure to find a line that is close to all the data points (Holtzapple and Dan Reece, 2003).

Purposely, all the state inhabitants living along the edge of the irregular state boundary are fairly considered for "equal" access distance to political city location in economic rate per kilometre travelled. Unbalanced consideration may lead to various agitations and tensed state.

Figure 1 refers:

Let the Cartesian coordinate of political capital city be represented as PCC (x_c, y_c).
Let P_i be an arbitrary point on the irregular boundary with Cartesian coordinate (x_i, y_i).
Let R be the radius of a substitute circular boundary with centre at PCC (x_c, y_c).
Let G_i be the radii line that connects P_i and PCC.

Then

$$G_i^2 = (x_i - x_c)^2 + (y_i - y_c)^2 \qquad (1)$$

Expand the right hand side of equation (1) yields:

$$G_i^2 = x_i^2 + y_i^2 + x_c^2 + y_c^2 - 2x_i x_c - 2y_i y_c \qquad (2)$$

Let error be:

$$\varepsilon_i = R^2 - G_i^2 \qquad (3)$$

THEORY AND METHOD

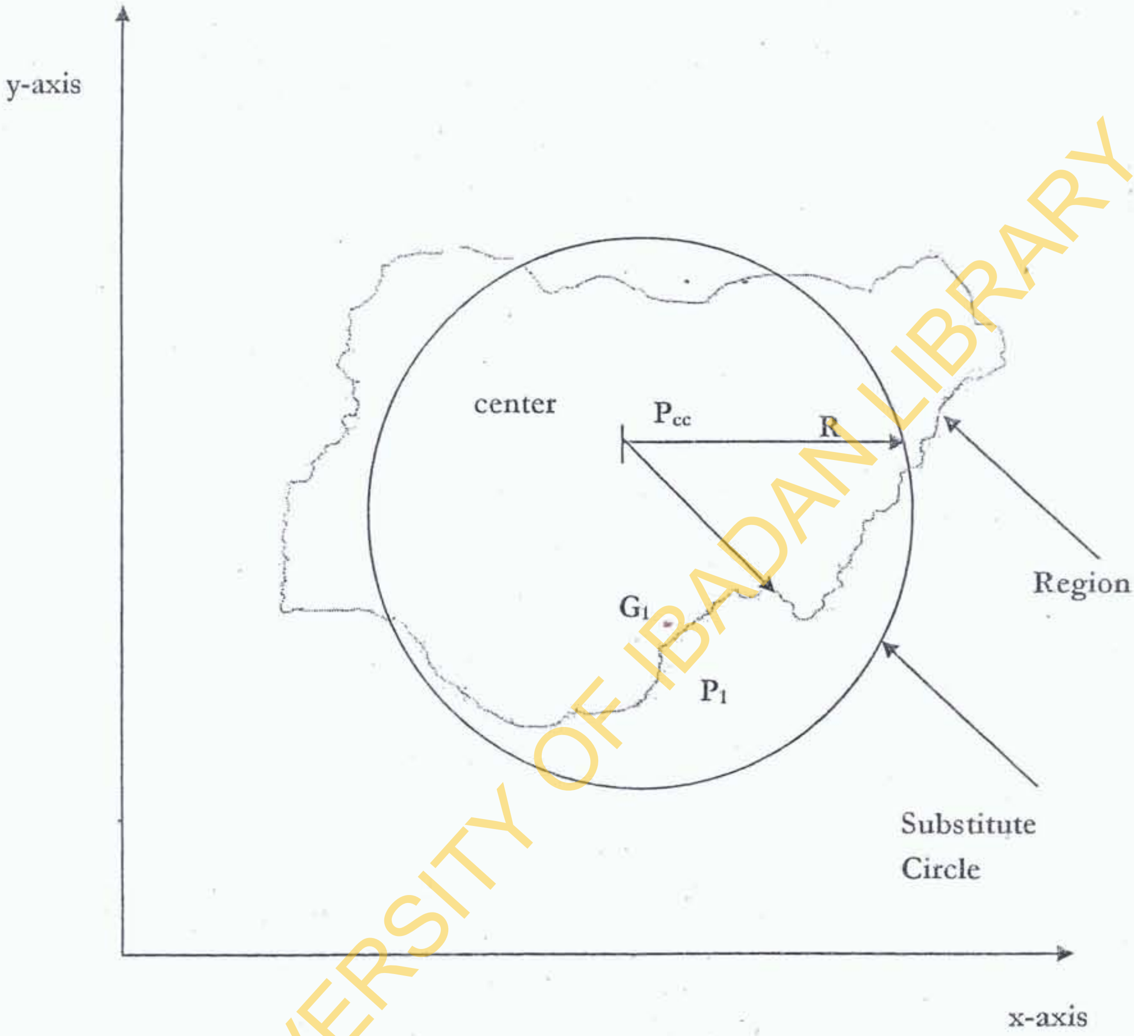

Figure 1: Arbitrary Designated Region Superimposed with Substitute Circle of radius (R)
Source: Senior Secondary Atlas, Balogun (2009).

Using equation (3) error square total will be:

$$\varepsilon^2_T = \sum_{i=1}^{i=N} (R^2 - G^2_i)^2 \tag{4}$$

Where N is the number of point Pi used to represent the irregular state boundary. Use equation (2) in equation (4) and let R_o be:

$$R_o = R^2 - x^2_c - y^2_c \tag{5}$$

The right hand side of equation (5) is now a function of 3-unknown constants: Ro, x_c and y_c.

To minimize error square total the following conditions must be satisfied:

$$\frac{\partial \varepsilon^2_T}{\partial R_o} = \frac{\partial \varepsilon^2_T}{\partial x_c} = \frac{\partial \varepsilon^2_T}{\partial y_c} = 0 \tag{6}$$

Implementation of equations (6) yields three (3) simultaneous equations in Ro, x_c and y_c as in equation (7).

$$\begin{bmatrix} N & 2\sum x_i & 2\sum y_i \\ \sum x_i & 2\sum x^2_i & 2\sum x_i y_i \\ \sum y_i & 2\sum x_i y_i & 2\sum y^2_i \end{bmatrix} \begin{Bmatrix} R_o \\ x_c \\ y_c \end{Bmatrix} = \begin{Bmatrix} \sum (x^2_i + y^2_i) \\ \sum (x^3_i + x_i y^2_i) \\ \sum (y^3_i + y_i x^2_i) \end{Bmatrix} \tag{7}$$

Scale map of Nigeria was obtained from Senior Secondary Atlas, Balogun (2009) and digitized using 3.5 mm by 3.5 mm square grids photocopied on transparency at room temperature. The integer value Cartesian coordinates obtained with the grids (see Table 1) was used in equations (7) to yield equation (8). Equation (8) was solved using Gauss Elimination method. Digitized data of Scale map of Nigeria as a case study was used as input to a FORTRAN 90 programme (Appendices 1 and 2) codes of an equivalent least squares method mathematical formulation.

RESULTS AND DISCUSSION

Results for this study are presented in tables and figures as follows:

Table 1: Cartesian coordinates of the Digitized Scale map of Nigeria in Grids

S/ No	Coordinates x	y	S/ No	Coordinates x	y	S/ No	Coordinates x	y	S/No	Coordinates x	y
1	16	1	26	13	7	51	38	11	76	42	18
2	17	1	27	28	7	52	39	11	77	2	19
3	18	1	28	29	7	53	40	11	78	43	19
4	19	1	29	11	8	54	2	12	79	2	20
5	20	1	30	12	8	55	32	12	80	43	20
6	15	2	31	28	8	56	33	12	81	44	20
7	16	2	32	29	8	57	34	12	82	2	21
8	21	2	33	10	9	58	37	12	83	3	21
9	22	2	34	11	9	59	40	12	84	45	21
10	23	2	35	29	9	60	2	13	85	4	22
11	24	2	36	30	9	61	33	13	86	45	22
12	25	2	37	2	10	62	34	13	87	4	23
13	26	2	38	3	10	63	35	13	88	45	23
14	14	3	39	4	10	64	36	13	89	46	23
15	15	3	40	5	10	65	40	13	90	4	24
16	26	3	41	6	10	66	41	13	91	5	24
17	14	4	42	7	10	67	2	14	92	47	24
18	27	4	43	8	10	68	41	14	93	5	25
19	28	4	44	9	10	69	2	15	94	6	25
20	13	5	45	10	10	70	42	15	95	47	25
21	28	5	46	30	10	71	2	16	96	6	26
22	13	6	47	31	10	72	42	16	97	47	26
23	28	6	48	2	11	73	2	17	98	48	26
24	29	6	49	31	11	74	42	17	99	6	27
25	12	7	50	32	11	75	2	18	100	48	27

Table 1: Cartesian coordinates of the Digitized Scale map of Nigeria in Grids (Contd.)

S/No	Coordinates x	y	S/No	Coordinates x	y	S/No	Coordinates x	y	S/No	Coordinates x	y
101	48	28	126	51	37	151	50	39	176	18	41
102	6	29	127	8	38	152	8	40	177	19	41
103	48	29	128	20	38	153	18	40	178	45	41
104	49	29	129	27	38	154	19	40	179	46	41
105	6	30	130	28	38	155	23	40	180	47	41
106	49	30	131	29	38	156	24	40	181	48	41
107	50	30	132	30	38	157	25	40	182	49	41
108	5	31	133	31	38	158	34	40	183	13	42
109	50	31	134	32	38	159	35	40	184	14	42
110	51	31	135	51	38	160	36	40	185	15	42
111	52	31	136	8	39	161	37	40	186	16	42
112	5	32	137	19	39	162	38	40	Centre	26	24
113	53	32	138	20	39	163	39	40			
114	6	33	139	21	39	164	40	40			
115	53	33	140	22	39	165	44	40			
116	6	34	141	23	39	166	45	40			
117	53	34	142	25	39	167	49	40			
118	6	35	143	26	39	168	50	40			
119	52	35	144	27	39	169	8	41			
120	53	35	145	33	39	170	9	41			
121	6	36	146	40	39	171	10	41			
122	51	36	147	41	39	172	11	41			
123	52	36	148	42	39	173	12	41			
124	6	37	149	43	39	174	13	41			
125	7	37	150	44	39	175	17	41			

$$\begin{pmatrix} 186 & 9678 & 8914 \\ 4839 & 350742 & 250648 \\ 4457 & 250648 & 287138 \end{pmatrix} \begin{Bmatrix} R_o \\ x_c \\ y_c \end{Bmatrix} = \begin{Bmatrix} 318940 \\ 11316627 \\ 9926765 \end{Bmatrix} \tag{8}$$

Solving for the three unknown parameters in equation (8) yielded respectively R_o =-804.21 grids, x_c=25.87 grids, and y_c=24.47 grids. In addition using equation (5) yields best representative radius (R =21.54 grids)

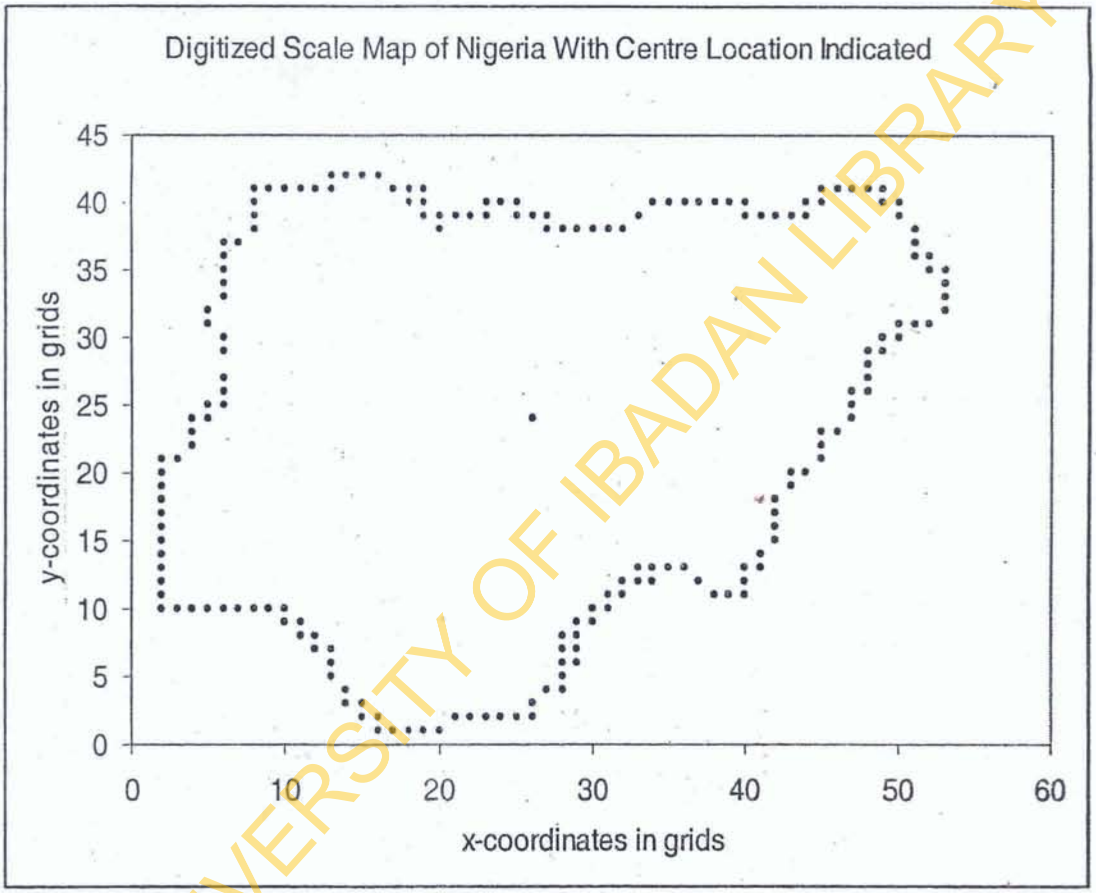

Figure 2: Digitized Scale map of Nigeria

Referring to Table 1 and Figure 2 political capital city coordinate location (i.e. (26, 24) in grids) prescribed fall to within the regional location of Abuja by visual assessment. It is important to note that Abuja replaced Lagos as the political capital of Nigeria in 1991, a new city, built over a decade [5, 6] for 'equal access' to all Nigerians.

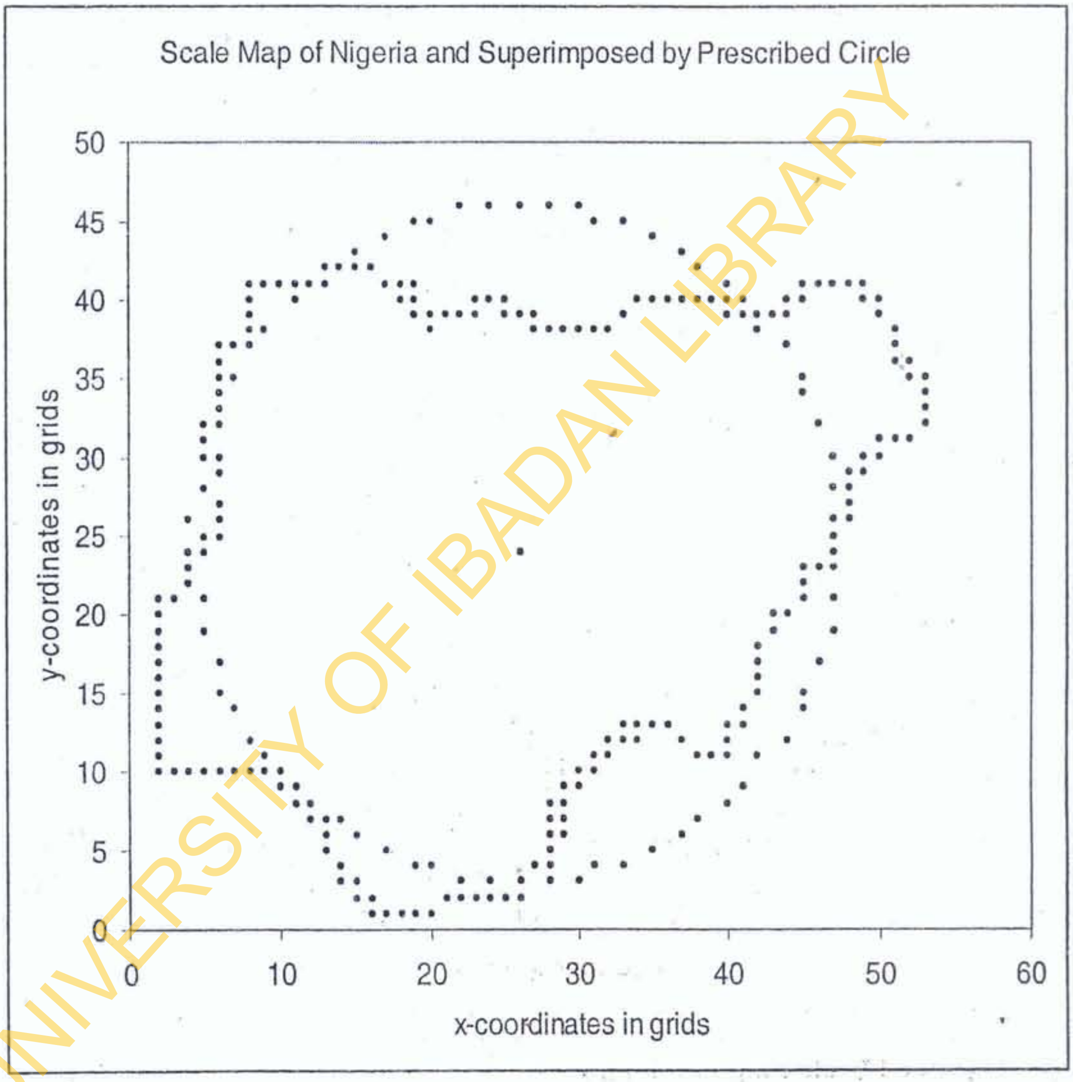

Figure 3: Digitized Scale Map of Nigeria Superimposed by Prescribed Circle of Radius 22 grids

Estimate of Land Area Cover and Boundary Perimeter

Best circle substitute for scale map of Nigeria refers. The circle radius was estimated as 21.54 grids. For a grid of size 3.5mm the circle radius is $21.54 \times 3.5 = 75.39\,mm$ or 7.5 cm.

Referring to the scale map of Nigeria 4.2cm is equivalent to 300 km, therefore the circle radius is $\frac{7.5 \times 300}{4.2} \approx 536\,km$.

Therefore true estimated land area cover of Nigeria at substitute circle radius of 536 km is

$$\pi R^2 = \pi \times 536 \times 536 \approx 902931\,sq\,km$$

In here $\pi = \frac{22}{7}$ and radius (R) is 536 km. If 923768 sq km quoted land area cover of Nigeria in the Microsoft Encarta Premium (2009) is taken as standard the relative percentage under estimation error due to the present estimate is 2.3%.

Similarly the estimated perimeter of scale map of Nigeria is $2\pi R = 2 \times \pi \times 536 \approx 3369\,km$.

The coastline (853 km) is therefore 25.3% of the estimated perimeter using the Microsoft Encarta Premium (2009) figure as standard.

CONCLUSION

This study has shown that mathematics can help in taking good decision about political capital city location or relocation economically, timely, accurately, and reliably. In addition developmental activities with its attendant challenges involving engineers will implicitly suffer little or no delay thereby broadening knowledge bank at a faster rate. It is recommended that this programme can be used for citing of a central capital city or for relocating an existing capital city to a central location.

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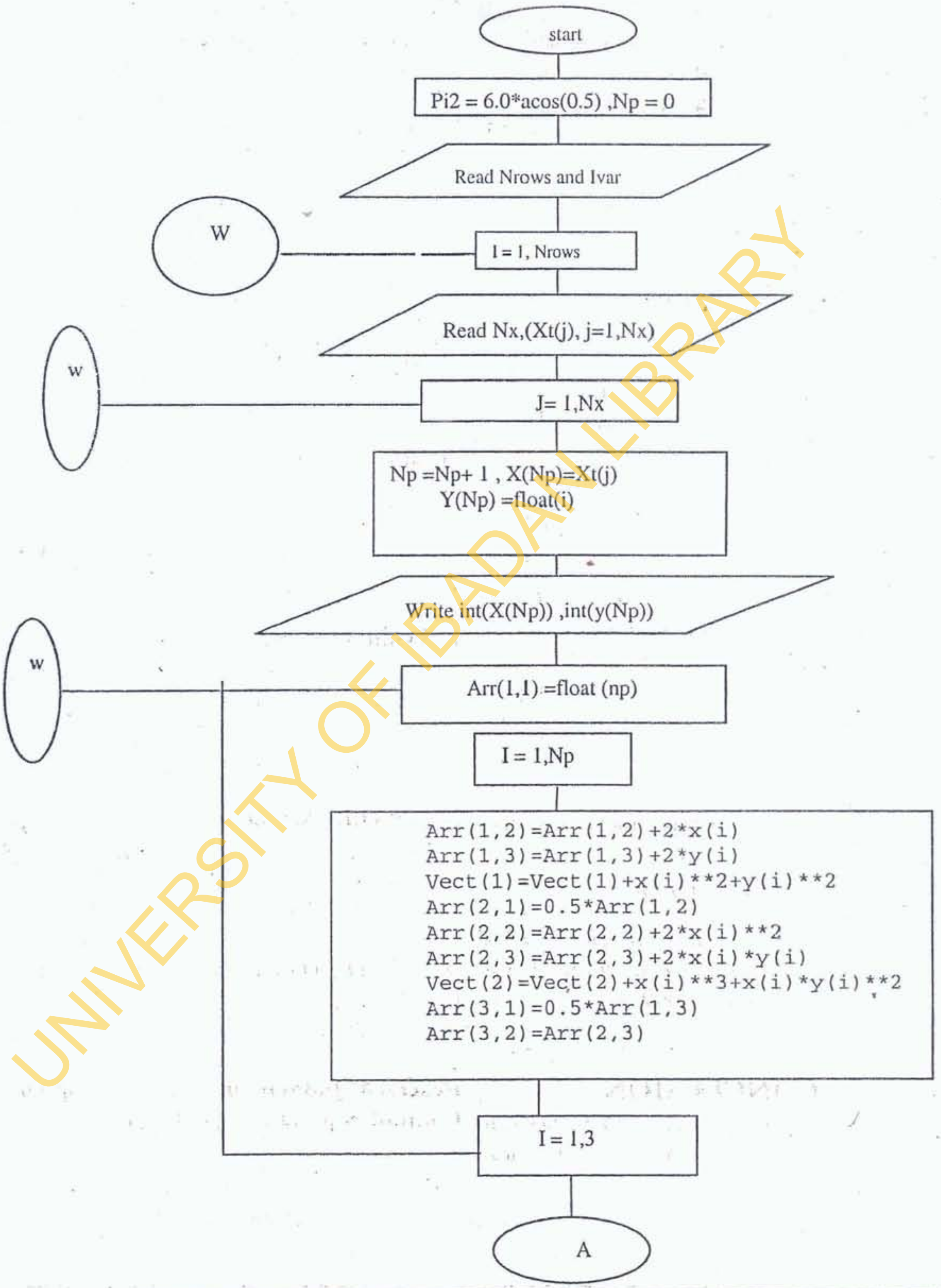
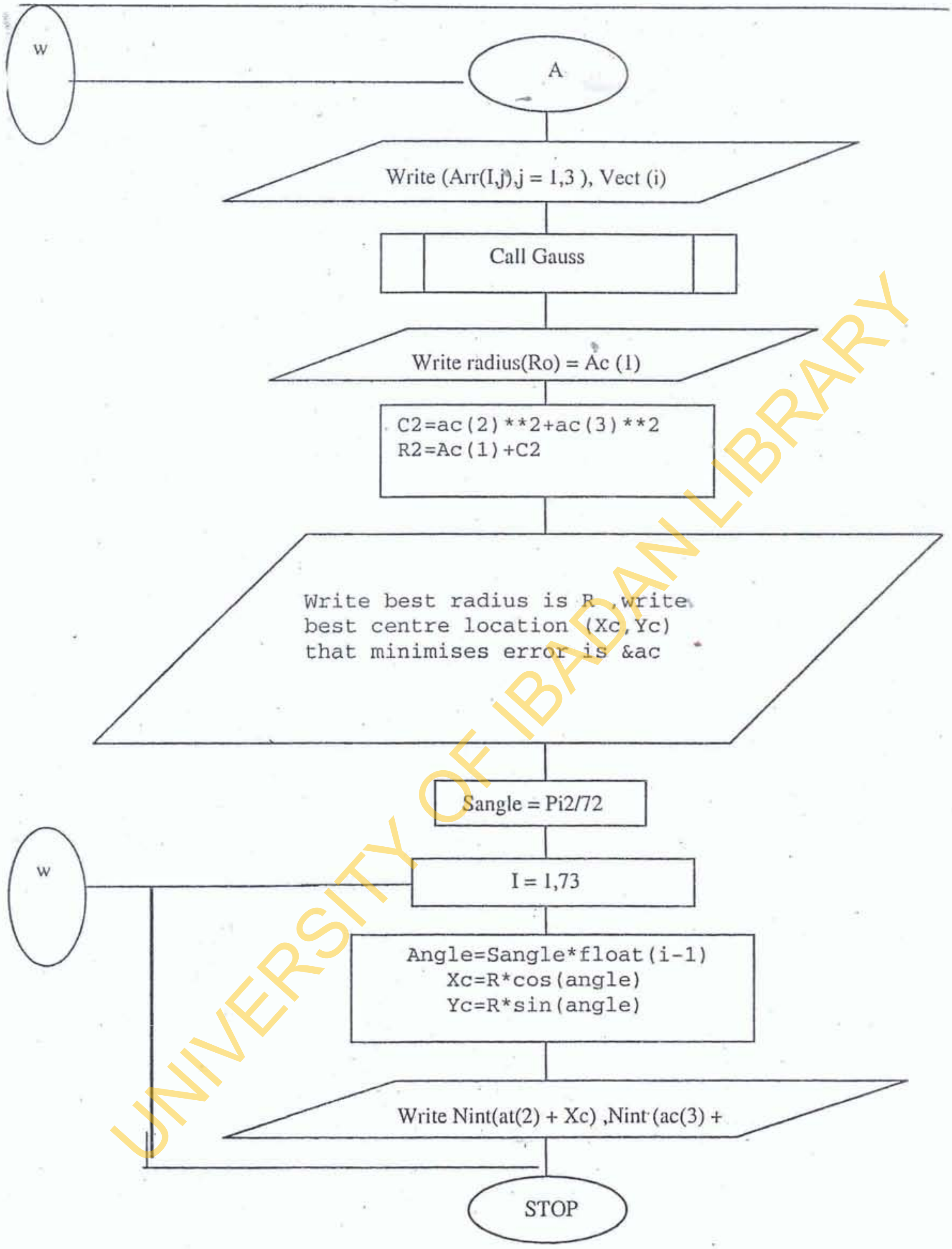
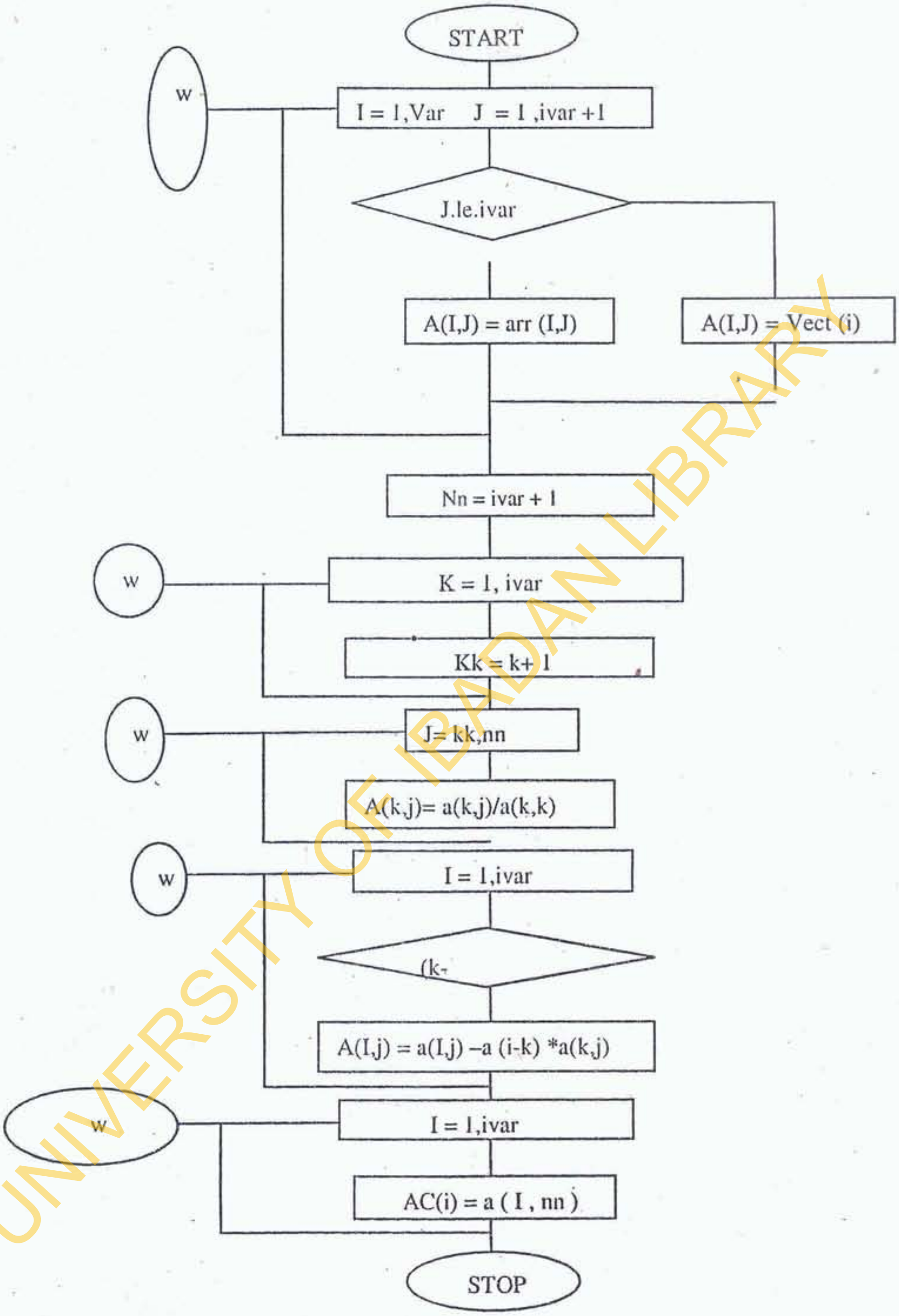
Appendix-I: Flow Chart Used to Generate the Source Codes in Appendix-II




W
A
Write (Arr(I,j),j = 1,3), Vect (i)
Call Gauss
Write radius(Ro) = Ac (1)
C2=ac(2)**2+ac(3)**2
R2=Ac(1)+C2
Write best radius is R ,write best centre location (Xc,Yc) that minimises error is &ac
Sangle = Pi2/72
W
I = 1,73
Angle=Sangle*float(i-1)
Xc=R*cos(angle)
Yc=R*sin(angle)
Write Nint(at(2) + Xc) ,Nint (ac(3) +
STOP


START
w
I = 1,Var J = 1 ,ivar +1
J.le.ivar
A(I,J) = arr (I,J)
A(I,J) = Vect (i)
Nn = ivar + 1
w
K = 1, ivar
Kk = k+ 1
w
J= kk,nn
A(k,j)= a(k,j)/a(k,k)
w
I = 1,ivar
(k-
A(I,j) = a(I,j) –a (i-k) *a(k,j)
w
I = 1,ivar
AC(i) = a (I , nn)
STOP

Appendix II: FORTRAN Source Codes for solving centre location coordinate of Designated Region Such as Scale Map of Nigeria

```
C... This programme uses the cartesian coordinates of the boundary
C....of any designated area to formulate 3-simultaneous linear equations
C....involving the best centre coordinate of a substitute circular
C....boundary and the Radius.
C....The resulting equations was solved using Gauss Elimination Algorithm.
      Implicit real *8(a-h,o-z)
        Common Arr(3,3),Vect(3),Ac(3),Ivar
        Dimension X(2000),Y(2000),Xt(200)
        Open(unit=1,file='PoliticsMaths',Status='old')
        Open(unit=2,file='PoliticsMaths.out')
        Open(unit=3,file='PolitMathImage.out')
        Pi2=6.0*acos(0.5)
C... Read Input Parameters from file (unit=1)
C....as follow:
        Np=0
        Read(1,*)Nrows,Ivar
        Do 10 i=1,Nrows
        Read(1,*)Nx,(Xt(j),j=1,Nx)
        Do 10 j=1,Nx
        Np=Np+1
        X(Np)=Xt(j)
        Y(Np)=float(i)
        Write(3,30)Int(x(Np)),Int(y(Np))
  10 Continue
C....Prepare Array Entries as follow
        Arr(1,1)=float(np)
        Do 20 i=1,Np
        Arr(1,2)=Arr(1,2)+2*x(i)
        Arr(1,3)=Arr(1,3)+2*y(i)
        Vect(1)=Vect(1)+x(i)**2+y(i)**2
        Arr(2,1)=0.5*Arr(1,2)
        Arr(2,2)=Arr(2,2)+2*x(i)**2
        Arr(2,3)=Arr(2,3)+2*x(i)*y(i)
        Vect(2)=Vect(2)+x(i)**3+x(i)*y(i)**2
        Arr(3,1)=0.5*Arr(1,3)
      Arr(3,2)=Arr(2,3)
        Arr(3,3)=Arr(3,3)+2*y(i)**2
        Vect(3)=Vect(3)+y(i)**3+y(i)*x(i)**2
  20 Continue
        Do 40 i=1,3
        Write(2,35)(Arr(i,j),j=1,3),Vect(i)
```

```
  40 Continue
  35 Format(4(f12.2,2x))

C....Call Subroutine Gauss for Solution of Resulting 3-simultaneous equations
        Call Gauss
        Write(*,*)'Radius (Ro)=',Ac(1)
        C2=ac(2)**2+ac(3)**2
        R2=Ac(1)+C2
        R=Sqrt(R2)
        Write(*,*)'Best Radius (R) is =', R
        Write(*,*)
        Write(*,*)'Best Centre Location (xc,yc) that Minimise error is =',
        &ac(2),ac(3)
        Write(3,*)
        Write(3,30)Nint(ac(2)),Nint(ac(3))
        Write(3,*)
        Sangle=pi2/72
        Do 50 i=1,73
        Angle=Sangle*float(i-1)
        Xc=R*cos(angle)
        Yc=R*sin(angle)
        Write(3,30)Nint(ac(2)+xc),Nint(ac(3)+yc)
 50     Continue
 30  Format(2(i3,2x))
        Stop
        End
C...Subroutine for Implementing Gauss Elimination Algorithm.
        Subroutine Gauss
        Implicit real *8(a-h,o-z)
        Common Arr(3,3),Vect(3),Ac(3),Ivar
        Dimension A(3,4)
        Do 10 i=1,ivar
        Do 10 j=1,ivar+1
        If(j.le.ivar)then
        A(i,j)=arr(i,j)
        Else
        A(i,j)=Vect(i)
        Endif
  10    Continue
    Nn=ivar+1
        Do 20 k=1,ivar
        kk=k+1
        Do 20 j=kk,nn
        A(k,j)=a(k,j)/a(k,k)
```

```
      Do 20 i=1,ivar
      If(k-i)30,20,30
30 A(i,j)=a(i,j)-a(i,k)*a(k,j)
20 Continue
      Do 40 i=1,ivar
      Ac(i)=a(i,nn)
40    Continue
      Return
      End
```