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# Efficiency in Linear Model with AR (1) and Correlated Error-Regressor (Pp. 46-61)

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**Abstract**
*In this study, we conduct several Monte-Carlo experiments to examine the sensitivity of the efficiency of FGLS estimators relative to OLS using the Variance and RMSE criteria, in the presence of first order autocorrelated error terms which are also correlated with geometric regressor. We examine the sensitivity of the efficiency to* □*, α, as well as, its asymptotic behaviour, N, when the above two assumptions are violated. We observe that CORC and HILU give similar result, same for ML and MLGRID. OLS is more efficient than CORC and HILU while ML and MLGRID dominate OLS. In the scenarios, efficiency does not increase with increase in autocorrelation level, only ML and MLGRID at α = 0.05 show that efficiency increases with increase in autocorrelation level. All estimators show that efficiency reduces as significant level increases only when the autocorrelation value and sample size are small (ρ = 0.4, N = 20). There is more efficiency gain when N and ρ are large at all significant correlation levels. Asymptotically, the efficiency of FGLS estimators increase with increasing autocorrelation but it is indifferent to the correlation levels. The asymptotic ranking is CORC and HILU followed by MLGRID and ML.*

**Keywords:** Efficiency, Monte-Carlo Experiment, Feasible Generalised Least Squares, Ordinary Least Squares, Autocorrelation, Significant Correlation.

## Introduction

To assess the quality and appropriateness of econometric estimators, we are always interested in their statistical properties. For most estimators, these can only be derived in a "large sample" context, (*asymptotic properties*). One estimation procedure may, for example, be selected over another because it is known to provide consistent and asymptotically efficient parameter estimates under certain stochastic environments. Such a heavy reliance on asymptotic theory can and does lead to serious problems of bias and low levels of inferential accuracy when sample sizes are small and asymptotic formulae poorly represent sampling behaviour. This has been acknowledged in mathematical statistics since the seminar work of R. A. Fisher, who recognised very early the limitations of asymptotic machinery, when he wrote; *"Little experience is sufficient to show that the traditional machinery of statistical processes is wholly unsuited to the needs of practical research. Not only does it take a cannon to shoot a sparrow, but it misses the sparrow! The elaborate mechanism built on the theory of infinitely large samples is not accurate enough for simple laboratory data. Only by systematically tackling small sample problems on their merits does it seem possible to apply accurate tests to practical data". [1]*

Statisticians are often interested in the relative efficiency of different estimators when the underlying assumptions of least squares breakdown. [2]. Assumptions in the classical normal linear regression model include that of lack of autocorrelation of the error terms and the zero covariance between the independent variable and the error terms.

In this follow up study to the estimation of the parameters of a linear model when the above two least squares assumptions are violated ( [3], [4], [5]), we are interested in the relative efficiency of FGLS to OLS in the presence of autocorrelated errors and significant correlation between the independent variable and the error terms. Specifically, we investigate, in a Monte Carlo experiment, the sensitivity of the efficiency of OLS and FGLS estimators in linear model to autocorrelation levels (□), significant correlation levels (α) between the autocorrelated error terms and the regressor, as well as, the asymptotic behaviour of efficiency using the Variance and RMSE criteria. It is known that in linear model with autocorrelated error terms which is independent from the regressor, the feasible generalized least squares (FGLS) estimators usually outshine its ordinary least squares (OLS) counterpart in terms of efficiency. ( [6], [7], [8]).

Ordinary regression analysis is based on several statistical assumptions. One key assumption is that the errors are independent of each other. However, with time series data, the ordinary regression residuals usually are correlated over time. (This is known as autocorrelation). It is not desirable to use ordinary regression analysis for time series data since the assumptions on which the classical linear regression model is based will usually be violated.

These violations, seen in widespread applications in operations research, like in queuing theory and econometrics, where the usual assumption of independent error terms may not be plausible in most cases. Also, when using time-series data on a number of micro-economic units, such as households and service oriented channels, where the stochastic disturbance terms in part reflect variables which are not included explicitly in the model and which may change slowly over time. [7].

Violation of the independent errors assumption has three important consequences for ordinary regression. First, statistical tests of the significance of the parameters and the confidence limits for the predicted values are not correct. Second, the estimates of the regression coefficients are not as efficient as they would be if the autocorrelation were taken into account. Third, since the ordinary regression residuals are not independent, they contain information that can be used to improve the prediction of future values. [9] Examples of situations generating dependency between errors and regressors include: Errors in Variables (Stochastic regressors), Lagged dependent variables and autocorrelation, and Simultaneous equation bias. It is known that in economics, measurement errors may be correlated both with themselves and with the regressors. [10] have shown that the error terms in most current formulations of economic relations are highly positively autocorrelated. [6] have shown that there is much to gain and little to lose by considering alternatives to the independent error assumption of the classical linear regression model.

Many models with autocorrelated error terms and dependency between regressors and error terms have been discussed in the literature. These include [10], [11], [6], [12], [13,14], [15], [16], [7], [17], [18], [19], [8], [5], [4], and [2]. Tests for detecting the presence of autocorrelation and alternative consistent methods of estimating linear models with autocorrelated disturbance terms and significant correlation between regressors and autocorrelated errors have been proposed. For instance, [12] derived a "full" maximum likelihood method approach to estimation of relationships with autocorrelated disturbances. They had a Monte Carlo study of their maximum likelihood estimator and the Cochrane-Orcutt procedure. The model used is:

$$Y_t = \beta_1 + \beta_2 X_t + U_t$$

$$U_t = \rho U_{t-1} + e_t$$

$$e_t \sim NID(0, \quad 0.0036)$$

and the independent variables were chosen to contain a large trend component, as realization of $X_t = \exp(0.04t) + w_t ; \quad w_t \sim NID(0, \quad 0.0009)$. They varied their sample sizes from 20 to 50 in 200 replications each and three different values of $\rho$, which are 0.6, 0.8 and 0.99. On each replication, both the conventional and full maximum likelihood estimates were computed for a given realization of the *e*'s, using the Cochrane-Orcutt procedure and the full maximum likelihood estimates procedure. Their findings using Root Mean Square Error (RMSE) is that the full maximum likelihood estimator is very much better than the Cochrane-Orcutt in estimating $\beta_2$ and they are often dramatic in estimating $\beta_1$ when the X's are trended as well as for the $\rho$, the gains are quite small. It was also found out that the full maximum likelihood estimates of $\beta_1$ and $\beta_2$ always does better than the conventional ones.

However, in spite of these tests and estimation methods, a number of questions in connection with the estimation of the classical linear model with autocorrelation error terms and non-zero covariance between the independent variable and the error terms remained unanswered. These include the most appropriate estimation method and their efficiencies in the above named specification of the independent variable, the effect of the degree of correlation of the disturbance term, the effect of the degree of correlation of independent variable and the error terms, the asymptotic effect and the sampling properties of the various estimation methods. [3] has shown that the replication only gives stability to the parameter estimates. The answers to most of these questions would allow for correct inferences to be made in linear models plagued by the scenario depicted above. It would also relieve the empirical worker from the reliance placed on asymptotic theory in estimation and inference.

The rest of this paper discusses the model and the experimental framework in section 2, Section 3 presents the simulation results, and section 4 presents the discussions, while we conclude in section 5.

**The Model**

We assume a simple linear regression model:

$$Y_t = \beta_0 + \beta_1 X_t + U_t \qquad \text{------} \qquad (1)$$

$$U_t = \rho U_{t-1} + \varepsilon_t, \qquad |\rho, \lambda| < 1,$$

$$X_t = \lambda X_{t-1},\ \lambda{=}0.8,\ X_0 = 515 \qquad U_t \rightarrow N\left(0, \frac{\sigma^2}{1-\rho^2}\right),$$

$$X_t \rightarrow N\left(0, \frac{\sigma^2}{1-\lambda^2}\right), \quad t = 1, 2, \ldots N., \quad □ = (1,1)$$

where $Y_t$ is the dependent variable and the first order autoregressive $X_t$ is the independent variable with $U_t$ also autoregressive of order one. $□_t$ is normally distributed with zero mean and variance $\sigma^2$. □ and □ are stationarity parameters while the model parameters are assumed to be unity. This independent variable specification had been used by [7], [5], and [20]. It is chosen to allow for comparison of results.

**Experimental Framework**

We used the Monte-Carlo approach for the investigation due to the fact that when the covariance between the independent variable and the autocorrelated error terms is non-zero, the problem is near intractable by analytical procedure. Small sample investigations are also usually made using the Monte Carlo method. Also the properties of FGLS estimators vary depending on the form of the variance – covariance matrix, and often the quality of this variance – covariance matrix can not be neatly summarized. Many estimation methods of our model have been developed over the years. Because of the least squares violations in the model, the FGLS estimators are considered relative to the OLS estimator. Some of the FGLS estimators in literatures include the Beach and MacKinnon Maximum Likelihood, Maximum Likelihood Grid, Cochran Orcutt, Durbin, Prais Winstein and Hildreth Lu. The various methods of parameter estimation in linear models with autocorrelated disturbances have known asymptotic properties. [16] while their sampling properties are yet to be well investigated and understood. This corroborates [21] when he asserts that "*The elaborate mechanism built on the theory of infinitely large samples is not accurate enough for simple laboratory data. Only by systematically tackling small sample problems on their merits does it seem possible to apply accurate tests to practical data*".

Most of the existing estimation methods possess desirable properties; however, the autocorrelation and the significant dependency of independent variable and the error terms, in addition to the specification of the independent variable might affect these properties. Since Monte-Carlo experiments provide a means of modelling small sample properties of estimators, it is used here to study these properties.

The following four FGLS estimators: Cochrane and Orcutt (CORC), Hildreth and Lu (HILU), Maximum Likelihood (ML) and Maximum Likelihood Grid (MLGRID) and Ordinary Least Squares (OLS) estimation methods, choosing in the light of the previous works, are used. These estimators are equivalent with identical asymptotic properties. ([17], [18], [19]). But in small samples, such as in this study, [22] have argued that those that use the T transformation matrix (ML, MLGRID) are generally more efficient than those that use T* transformation matrix (CORC, HILU). (See [5])

The degree of autocorrelation affects the efficiency of the estimators [7]. Consequently, we investigated the sensitivity of the estimators to the degree of autocorrelation by varying rho $\hat{\rho}$ from 0.4, to 0.8 and 0.9. We also found out the effect of the correlation of the independent variable and the error terms at significant level 1%, 2% and 5% on the estimators. The effects of sample size was also investigated by varying the sample size (N) from 20, 40 to 60 each replicated 50 times. Evaluation of the estimators was done using the Relative Efficiency based on Variance and the RMSE criteria.

A total of 27 data sets spread over three sample sizes were used in generating the data for this study. Using model (1), a value $U_o$ (for specified sample size) was generated by drawing a random value □$_o$ from N(0,1) and dividing by $\sqrt{(1-\rho^2)}$.

Successive values of □$_t$ drawn from N(0,1) were used to calculate $U_t$. $X_t$ was similarly generated. The correlation coefficient between $U_t$ and $X_t$ was then computed and its absolute value tested for significance at, say 1%. If this value is significant, it is chosen; otherwise it is discarded. This procedure is repeated as many times as are necessary (for all □, α and N) to obtain fifty replications for a desired sample size. $Y_t$. is thus computed for the chosen $U_t$ and fixed geometric trended $X_t$ using the model. The data generations are made using the Excel package while estimations are done via the AR procedure of [23]. Estimation result for this scenario of the independent variable is presented in [5].

The finite sampling properties of estimators used include the Variance (VAR) and the Root Mean Squared Error (RMSE). Additionally, we calculated the Sum of Variances (SVAR) and the Sum of Root Mean Squared Error (SRMSE). These are further used to compute the Relative efficiency. The relative efficiency of the FGLS estimators relative to OLS is:

$$\frac{\text{Var } \hat{\beta}_{\text{(OLS)}}}{Var\hat{\beta}_{(GLS)}} \quad \text{or} \quad \frac{\text{MSE } \hat{\beta}_{\text{(OLS)}}}{\text{MSE } \hat{\beta}_{\text{(GLS)}}}$$

Then to get the Total Gain or Loss (G/L), we subtract 1 (original estimate) from the efficiency of each coefficient and add our results. That is, efficiency gain or loss is $\left(\hat{\beta}_o - \beta_o\right) + \left(\hat{\beta}_1 - \beta_1\right)$, where $\hat{\beta}(.)$ represents the efficiency of $\hat{\beta}$. If the relative efficiency is negative, then OLS is more efficient. The results for each scenario, using both Variance and RMSE criteria, are summarised in Tables 1 and 2 for RMSE and VAR criteria respectively.

**Simulation Results**

Perusing Tables 1 and 2, it is observed that in all the scenarios considered in the experiment, CORC and HILU efficiencies are similar and same for ML and MLGRID. OLS is more efficient than CORC and HILU in many of the scenarios while ML and MLGRID are more efficient than OLS. In majority of the scenarios, efficiency does not increase with increase in autocorrelation level, only ML and MLGRID at $\alpha = 0.05$ show that efficiency increases with increase in autocorrelation level. All estimators show that efficiency reduces as significant level increases only when the autocorrelation value and sample size are very small (that is $\rho = 0.4$, N = 20). There is more efficiency gain when N and $\rho$ are large at all significant correlation levels. Table 3 summarises Tables 1 and 2, where we found the best estimator (estimator with the largest efficiency under each of the variance and RMSE criteria). Holding N, $\alpha$ and $\rho$ constant, the ML estimator has the largest efficiency in 44.5% of the scenarios, followed by MLGRID (29.6%), CORC (11.1%), HILU (7.4%) and OLS (7.4%).

In order to bring out the most information from this research, we charted the efficiency levels recorded in Tables 1 and 2 in Table 4 showing the asymptotic, autocorrelation and significant level effects. Table 4 gives the frequency distribution of N – chart over □ and □ for both variance and RMSE-based efficiency measures for all estimators (measuring the asymptotic effect). The chart symbols include (****) indicating, minimum efficiency when N = 60, intermediate when N = 40 and maximum when N = 20. (**V**) Efficiency is a minimum when N = 40, and maximum when N = 20 or 60. (□) Efficiency is a maximum when N = 40, and minimum when N = 20 or 60, and (**/**), means Efficiency is a maximum when N = 60, intermediate when N = 40 and minimum when N = 20.

Table 4.1 shows that the trend '/' is the most frequent. This implies that the efficiency is highest when N = 60 followed by those at N = 40 and is smallest at N = 20. This most frequent trend occurs with highest frequency when □ = 0.9 (19). Table 4.2 also gives additional interesting information that the significant level does not matter for asymptotic efficiency as the most frequent trend '/' appear equally among the significant levels. The last column of Table 4.3 contains a summary of the two ranks of each of the four estimators. These estimators rank as follows in decreasing order of conformity with the observed asymptotic behaviour of efficiencies of variance and RMSE: CORC (3), HILU (3), MLGRID (7), and ML (7). In conclusion, the above results show that, using our criteria, the efficiency of the FGLS estimators increase asymptotically and the optimum combination of □, □ and N is: all □, □ = 0.9, and N = 60.

**Discussion of the Results**

We note that the efficiency of ML and MLGRID have very similar behavioural pattern, the same for CORC and HILU as observed in the finite sampling properties of Variance and the RMSE. ML and MLGRID are better than both CORC and HILU as also observed by [24], [7], and [8].

Asymptotically, the estimators increase asymptotically and the optimum combination of □, □ and N is: all □, □ = 0.9, and N = 60. This implies that, the efficiency of the FGLS estimators, relative to the OLS estimator, increases asymptotically and with increasing autocorrelation. This is similar to the results obtained by [13] and [17] when the regressor and error terms are independent. The estimators rank as follows in decreasing order in conformity with the observed asymptotic behaviour of efficiency: CORC, HILU, MLGRID, and ML. This also indicates that truly, the nature of the regressor affects the efficiency of FGLS estimators. As if we compare this result with that of [8], there is a disparity as a result of the nature of the regressor. Our results have also shown that there is a definite gain to be obtained from using some of the feasible GLS as they are more efficient than OLS. This also conform to the earlier result by [6] where they show that FGLS are better for given values of |□| ≥ 0.3 when there is independence between the error terms and the regressor.

**Conclusion**

We have investigated the sensitivity of the significant correlation between the error terms and the geometric regressor in a single linear regression model to the efficiency of the various FGLS estimators relative to that of the OLS estimator. It could be concluded that empirically, the OLS estimator is more efficient than the FGLS estimators CORC and HILU as OLS dominated them almost uniformly. Maximum likelihood estimation methods of MLGRID and ML still perform better than other FGLS estimators in terms of efficiency. All estimators show that efficiency reduces as significant level increases only when the autocorrelation value and sample size are very small (that is $\rho$ = 0.4, N = 20). Asymptotically, the efficiency of FGLS estimators increase asymptotically with increasing autocorrelation but it is indifferent to the significant correlation levels between the error terms and the geometric regressor. The asymptotic ranking is CORC and HILU followed by MLGRID and ML.

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**Table 1: Efficiency of FGLS to OLS using RMSE**

| | | N=20 | | | N=40 | | | N=60 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Significant level | Estimator | | ρ = 0.8 | ρ = 0.9 | ρ = 0.4 | ρ = 0.8 | ρ = 0.9 | P = 0.4 | ρ = 0.8 | ρ = 0.9 |
| 0.01 | OLS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | CORC | -0.11124 | -0.1421 | -0.00748 | 0.014264 | -0.06231 | -0.58818 | 0.016629 | 0.38847 | 0.082889 |
| | HILU | -0.11179 | 16.34885 | -0.00681 | 0.01381 | -0.05744 | 0.006466 | 0.019838 | 0.380271 | 0.097558 |
| | ML | 0.059012 | 0.071676 | -0.00041 | 0.013227 | 0.211346 | 0.290823 | 0.055108 | 0.024961 | 0.513185 |
| | MLGRID | 0.045324 | 0.073073 | 0.000131 | 0.013162 | 0.211593 | 0.282707 | 0.049374 | 0.024905 | 0.374133 |
| 0.02 | OLS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | CORC | -0.0674 | -0.96571 | -0.04481 | -0.00361 | 0.230127 | 0.022946 | -0.00357 | -1.63373 | -0.07416 |
| | HILU | -0.08 | -0.22747 | -0.04459 | -0.00311 | 0.234435 | -0.04935 | -0.00315 | -1.36432 | -0.066881 |
| | ML | 0.032743 | 0.090743 | -0.01334 | 0.003276 | 0.405858 | 0.254624 | 0.003706 | -1.44587 | 0.301885 |
| | MLGRID | 0.03184 | 0.088679 | -0.01416 | 0.003349 | 0.411975 | 0.253686 | 0.003599 | -1.41606 | 0.294989 |
| 0.05 | OLS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | CORC | 2.627277 | -0.80558 | -0.66291 | 0.131762 | -0.08107 | -0.46719 | -0.00335 | 0.903795 | 0.082348 |
| | HILU | -0.00326 | -0.94766 | -0.78849 | 0.134575 | -0.08313 | -0.33354 | -0.00335 | 0.897048 | -0.10749 |
| | ML | 0.020736 | 0.128359 | 0.183662 | 0.129727 | 0.150683 | 0.258348 | 0.009321 | 0.052556 | 0.572738 |
| | MLGRID | 0.01987 | 0.093119 | 0.161984 | 0.128458 | 0.155675 | 0.25741 | 0.010177 | 0.049336 | 0.5782 |

Table 2: Efficiency of FGLS to OLS using Variance

| | | N = 20 | | | N = 40 | | | N = 60 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Significant level | Estimator | ρ = 0.4 | ρ = 0.8 | ρ = 0.9 | ρ = 0.4 | ρ = 0.8 | ρ = 0.9 | ρ = 0.4 | ρ = 0.8 | ρ = 0.9 |
| | OLS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | CORC | -0.22774 | -0.30589 | -0.02163 | 0.06942 | -0.1801 | -0.89743 | 0.131691 | -0.08006 | 0.194709 |
| | HILU | -0.22874 | 308.3316 | -0.01858 | 0.068406 | -0.16239 | -0.00734 | 0.13932 | -0.27479 | 0.254439 |
| | ML | 0.113447 | 0.14186 | 0.001844 | 0.060568 | 0.466597 | 0.659519 | 0.255485 | -0.05486 | 0.83865 |
| | MLGRID | 0.081615 | 0.142836 | 0.004615 | 0.060463 | 0.46471 | 0.622874 | 0.23028 | -0.03035 | 0.812434 |
| | OLS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | CORC | -0.1244 | -0.99874 | -0.08629 | -0.00553 | 0.49641 | 0.043627 | -0.00159 | 6.855209 | -0.06034 |
| | HILU | -0.15002 | -0.3749 | -0.08586 | -0.00457 | 0.513565 | 0.025825 | -0.00089 | 7.258733 | 0.066334 |
| | ML | 0.066228 | 0.165031 | -0.02652 | 0.014949 | 0.793466 | 0.703581 | 0.019858 | 11.7951 | 0.860854 |
| | MLGRID | 0.064277 | 0.160319 | -0.02682 | 0.015043 | 0.826917 | 0.720795 | 0.01975 | 16.47161 | 0.840649 |
| | OLS | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | CORC | 12.17045 | -1.29144 | -1.04379 | 0.094013 | -0.22936 | -0.80276 | 0.007 | 0.070414 | -0.05836 |
| | HILU | -0.00703 | -1.43848 | -1.19397 | 0.105632 | -0.23306 | -0.5661 | 7.31E-05 | -0.10332 | -0.16005 |
| | ML | 0.041434 | 0.237734 | 0.335683 | 0.082023 | 0.322055 | 0.613454 | 0.046454 | 0.220231 | 0.98049 |
| | MLGRID | 0.039596 | 0.219958 | 0.344411 | 0.076997 | 0.354306 | 0.602212 | 0.048038 | 0.185336 | 1.015634 |

Table 3: Best Estimator of our Model for Each Scenario

| | | N = 20 | | | N = 40 | | | N = 60 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Criteria** | **(α)↓** | **ρ=0.4** | **ρ=0.8** | **P=0.9** | **ρ =0.4** | **ρ=0.8** | **ρ=0.9** | **ρ=0.4** | **ρ=0.8** | **ρ=0.9** |
| | **0.01** | ML | HILU | MLG | CORC | ML | ML | ML | OLS | ML |
| | **0.02** | ML | ML | OLS | MLG | MLG | MLG | ML | MLG | ML |
| | **0.05** | CORC | ML | MLG | HILU | MLG | ML | MLG | ML | MLG |
| | **0.01** | ML | HILU | MLG | CORC | MLG | ML | ML | CORC | ML |
| | **0.02** | ML | ML | OLS | MLG | MLG | ML | ML | OLS | ML |
| | **0.05** | CORC | ML | ML | HILU | MLG | ML | MLG | CORC | MLG |

Table 4**:** Asymptotic Behaviour of Rmse and Variance of Estimators When Sig. Level (□) Is Constant

| **□** | **0.01** | | | **0.02** | | | **0.05** | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **□** | **0.4** | **0.8** | **0.9** | **0.4** | **0.8** | **0.9** | **0.4** | **0.8** | **0.9** | | | | |
| **ESTIMATOR** | **RMSE** | | | | | | | | | **** | **V** | **□** | **/** |
| **CORC** | / | / | **V** | / | □ | □ | \ | / | / | **1** | **1** | **2** | **5** |
| **HILU** | / | **V** | / | □ | □ | **V** | □ | / | / | **0** | **2** | **3** | **4** |
| **ML** | **V** | □ | / | **V** | □ | / | □ | □ | / | **0** | **2** | **4** | **3** |
| **MLGRID** | **V** | □ | / | **V** | □ | / | □ | □ | / | **0** | **2** | **4** | **3** |
| | | | | | | | | | | | | | **(15)** |
| **ESTIMATOR** | **VAR** | | | | | | | | | | | | |
| **CORC** | / | / | **V** | / | / | □ | \ | / | / | **1** | **1** | **1** | **6** |
| **HILU** | / | \ | / | / | / | / | □ | / | / | **1** | **0** | **1** | **7** |
| **ML** | **V** | □ | / | **V** | / | / | □ | □ | / | **0** | **2** | **3** | **4** |
| **MLGRID** | **V** | □ | / | **V** | / | / | □ | □ | / | **0** | **2** | **3** | **4** |
| | | | | | | | | | | | | | **(21)** |
| **C – Summary** | **0.4** | **0.8** | **0.9** | **0.4** | **0.8** | **0.9** | **0.4** | **0.8** | **0.9** | | | | |
| **** | **0** | **1** | **0** | **0** | **0** | **0** | **2** | **0** | **0** | | | | |
| **V** | **4** | **1** | **2** | **4** | **0** | **1** | **0** | **0** | **0** | | | | |
| **□** | **0** | **4** | **0** | **1** | **4** | **2** | **6** | **4** | **0** | | | | |
| **/** | **4** | **2** | **6** | **3** | **4** | **5** | **0** | **4** | **8** | | | | |

Table 4.1 And - Estimator Based Summaries of Table 4

| | **□ = 0.4** | **□ = 0.8** | **□ = 0.9** | **TOTAL** | | **CORC** | **HILU** | **ML** | **MLGRID** |
|---|---|---|---|---|---|---|---|---|---|
| **** | **2** | **1** | **0** | **3** | | **2** | **1** | **0** | **0** |
| **V** | **8** | **1** | **3** | **12** | | **2** | **2** | **4** | **4** |
| **□** | **7** | **12** | **2** | **21** | | **3** | **4** | **7** | **7** |
| **/** | **7** | **10** | **19** | **36** | | **11** | **11** | **7** | **7** |

**Table 4.2** □ - And - Estimator Based Summaries of Table 4

| | □ = 0.01 | □ =0.02 | □ = 0.05 | TOTAL | | CORC | HILU | ML | MLGRID |
|---|---|---|---|---|---|---|---|---|---|
| \ | 1 | 0 | 2 | 3 | | 2 | 1 | 0 | 0 |
| V | 7 | 5 | 0 | 12 | | 2 | 2 | 4 | 4 |
| □ | 4 | 7 | 10 | 21 | | 3 | 4 | 7 | 7 |
| / | 12 | 12 | 12 | 36 | | 11 | 11 | 7 | 7 |

**Table 4.3: Summary of the Ranking of Estimators**

| | Optimum Trend | | |
|---|---|---|---|
| | VAR (/) | RMSE (/) | |
| **CORC** | (6) 2 | (5) 1 | 3 |
| **HILU** | (7) 1 | (4) 2 | 3 |
| **ML** | (4) 3.5 | (3) 3.5 | 7 |
| **MLGRID** | (4) 3.5 | (3) 3.5 | 7 |

**LEGEND**

**\\:** Efficiency is a minimum when N = 60, intermediate when N = 40 and maximum when N = 20.

**V:** Efficiency is a minimum when N = 40, and maximum when N = 20 or 60

**□:** Efficiency is a maximum when N = 40, and minimum when N = 20 or 60

**/:** Efficiency is a maximum when N = 60, intermediate when N = 40 and minimum when N = 20.