Simultaneous Equation Estimation with First Order Auto Correlated Disturbances

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Abstract — *The estimation of the parameters of simultaneous equation problem is usually affected by the existence of mutual correlation between pairs of random deviates, which is a violation of the assumption of no autocorrelation between the error terms. In practice the form of correlation between the pairs of random deviates is not known. This study therefore examined a two-equation model in which the correlation between the random deviates is assumed to follow a first-order Autoregressive [AR (1)] process. Data was simulated using Monte Carlo approach with varying sample sizes each replicated 1000 times. The behaviour of OLS, 2SLS, LIML and 3SLS were evaluated using Variance, Root Mean Square Error (RMSE) and Absolute Bias (AB). The absolute bias estimates decrease in most cases as the sample size increases. The variances obtained by all the estimators reduced consistently as the sample size increases. There was no clear pattern in the behaviour of the RMSE across sample sizes. The results for ρ = 0.3 were better than when ρ = 0.0 with respect to each criterion but retained the same pattern. This work established that when ρ was different from zero, the estimators performed better, hence the choice of ρ should be carefully made as this may significantly affect the performances of the estimators.*

Keywords: Autocorrelation, Estimators, First-order Autoregressive [AR (1)] process, Monte Carlo, Simultaneous equation model.

1. INTRODUCTION

In simultaneous equation model, the dependent variable is determined by the simultaneous interaction of several relationships. If all of the relationships involved are needed for determining the value of at least one of the endogenous variables, then we have a *simultaneous equation(s) problem*. The unique feature of the simultaneous equations is the fact that the dependent variable in one equation may be an explanatory variable in another. The problem then is that the dependent variable is now stochastic and may be correlated with the disturbances in that equation. Economic theories are usually based on sets or systems of relationships; they are expressed in terms of multiple equations. One of the main advantages of expressing economic theory in a system of equations is to show the interdependency that characterizes some of these theories (Kmenta, 1971).

The possibility of nonzero covariance between error terms of different equations of the model was first visualized by Zellner (1962) that reflect the fact that equations which are apparently not connected or related structurally are, in fact, related to each other statistically. On one hand, simultaneous estimation of the non-apparently related regression equations of the model improves the precision of the estimation of regression parameters over the situations in which the individual equations are estimated independently of each other. Kmenta and Gilbert (1970) examined the small-sample efficiency of four different methods of estimation of regression with autocorrelated coefficients by conducting a Monte Carlo experiment and found that the Joint Nonlinear Estimation (JNE) method performed better for small samples in their setup.

Klein (1974) considered single equation methods in the context of a simultaneous system which may be less sensitive to specification error in the sense that those parts of the system that are correctly specified may not be affected appreciably by errors in specification in another part. Sawa (1968) and Richardson (1968) derived the exact distribution of the OLS and 2SLS estimators in an equation with two endogenous variables. Mariano and McDonald (1979) considered the 2SLS and LIML estimators in the just identified case, while Holly and Phillips (1973) used an asymptotic expansion to approximate the distribution of 2SLS estimator. Anderson and Sawa (1973) derived an alternative form of the exact distribution of OLS and 2SLS and presented approximations as well. However, research by Nagar (1959) provided some evidence that 2SLS may have advantage over LIML in small samples.

Ray C. Fair (1970) discussed various methods for the estimation of simultaneous equation models with lagged endogenous variables and first order serially correlated errors. The methods differ in the number of instrumental variables used. The asymptotic and small sample properties of the various methods are compared, and the variables which must be included as instruments to insure consistent estimates are derived. Sargan (1961) proposed various maximum likelihood estimators for the estimation of simultaneous -equation models with serially correlated errors, and Amemiya (1966) considered the two-stage least squares analogue to one of Sargan's estimators and proposed a modified version of this analogue.

Brundy and Jorgenson (1971) criticized the two- and three-estimators, namely that the first stage involves estimating reduced-form equations with a very large number of variables included in them, holds even for models with auto-regressive errors. For these models, the reduced- form equations included not only all of the predetermined variables in the system but also all the lagged endogenous and lagged predetermined variables. The ranking of the various simultaneous equation models considered based on their small sample properties differed according to the correlation status of the error term (Adepoju and Olaomi, 2009).

This paper examined a two-equation model in which the correlation between the random deviates is assumed to follow a first-order Autoregressive [AR (1)] process. The evaluation and comparisons of the techniques are carried out using Variance, Root Mean Square Error (RMSE) and Absolute Bias (AB). The estimators were ranked based on the order of precision.

2. MATERIALS AND METHODS

A two-equation structural model considered is given by;

$$Y_{1t} = \beta_{12} Y_{2t} + \gamma_{11} X_{1t} + \gamma_{12} X_{2t} + U_{1t}$$
$$Y_{2t} = \beta_{21} Y_{1t} + \gamma_{21} X_{1t} + \gamma_{23} X_{3t} + U_{2t} \qquad 2.1$$

The disturbance terms in (2.1) are assumed to be generated by a stationary, first- order autoregressive AR (10 process as follows:

$$U_{1t} = \rho\ U_{1t-1} + V_{1t}$$
$$U_{2t} = \rho\ U_{2t-1} + V_{2t} \qquad 2.2$$

which can be written in matrix form as

$$\begin{pmatrix} U_{1t} \\ U_{2t} \end{pmatrix} = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \begin{pmatrix} U_{1t-1} \\ U_{2t-1} \end{pmatrix} + \begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix}$$

where V_{it} are independently and identically distributed random variables with $E(V_{it}) = 0$ and covariance matrix

$$E\left(V_{(t)} V_{(t)}{}'\right) = \Sigma$$

With AR(1), the model in 2.1 becomes

$$Y_{1t} = \beta_{12} Y_{2t} + \gamma_{11} X_{1t} + \gamma_{12} X_{2t} + U_{1t} + V_{1t}$$
$$Y_{2t} = \beta_{21} Y_{1t} + \gamma_{21} X_{1t} + \gamma_{23} X_{3t} + U_{2t} + V_{2t} \qquad 2.3$$

Where,

Y's = Endogenous variables

X's = Exogenous variables

U's = Disturbance terms

$\beta's\ and\ \gamma's =$ The structural parameters

ρ is the correlation coefficient.

2.1 Structural Framework

In a model of M simultaneous equations, in other for an equation to be identified, it must exclude at least M-1 variables (endogenous as well as predetermined) appearing in the model. If it excluded exactly M-1 variables, the equation is just or exact identified. If it excludes more than M-1 variables, it is over identified. However, the two-equation structural model of this study (equation 2.1) was tested for identification (order and rank) and found to be exactly identification.

2.2 A Reduced Functional Form

A situation where AR (1) has an impact on the simultaneous equation model of equation 2.3, the reduced functional forms of the equation becomes;

$$Y_{1t} = \left(\frac{\gamma_{11}+\gamma_{21}\beta_{12}}{1-\beta_{12}\beta_{21}}\right) X_{1t} + \left(\frac{\gamma_{12}}{1-\beta_{12}\beta_{21}}\right) X_{2t} + \left(\frac{\gamma_{23}\beta_{12}}{1-\beta_{12}\beta_{21}}\right) X_{3t} + \left(\frac{\rho_1 U_{1t-1}}{1-\beta_{12}\beta_{21}}\right) + \left(\frac{V_{1t}}{1-\beta_{12}\beta_{21}}\right) + \left(\frac{\beta_{12}\rho_2 U_{2t-1}}{1-\beta_{12}\beta_{21}}\right) + \left(\frac{\beta_{12}V_{2t}}{1-\beta_{12}\beta_{21}}\right)$$

$$Y_{1t} = \pi_{11} X_{1t} + \pi_{12} X_{2t} + \pi_{13} X_{3t} + W_{11} + W_{12} + W_{13} + W_{14}$$

$$Y_{2t} = \left(\frac{\gamma_{12}\beta_{21}+\gamma_{21}}{1-\beta_{12}\beta_{21}}\right) X_{1t} + \left(\frac{\gamma_{12}\beta_{21}}{1-\beta_{12}\beta_{21}}\right) X_{2t} + \left(\frac{\gamma_{23}}{1-\beta_{12}\beta_{21}}\right) X_{3t} + \left(\frac{\beta_{21}\rho_1 U_{1t-1}}{1-\beta_{12}\beta_{21}}\right) + \left(\frac{\beta_{21}V_{1t}}{1-\beta_{12}\beta_{21}}\right) \left(\frac{\rho_2\rho_2 U_{2t-1}}{1-\beta_{12}\beta_{21}}\right) + \left(\frac{V_{2t}}{1-\beta_{12}\beta_{21}}\right)$$

$$Y_{2t} = \pi_{21} X_{1t} + \pi_{22} X_{2t} + \pi_{23} X_{3t} + W_{21} + W_{22} + W_{23} + W_{24}$$

3. SIMULATION STUDIES

The data used in the study are generated using Monte Carlo approach. The exogenous variables are obtained from uniform distribution with mean zero and variance one using the standard random number generator. The random disturbances were generated U_{it} (t=1, 2, 3...T) with mean zero and covariance matrix Σ . As noted above, the disturbance vector $U_{(t)}$ was generated by a stationary, first-order autoregressive process

$$\begin{pmatrix} U_{(1t)} \\ U_{(2t)} \end{pmatrix} = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \begin{pmatrix} U_{1t-1} \\ U_{2t-1} \end{pmatrix} + \begin{pmatrix} v_{1t} \\ v_{2t} \end{pmatrix}$$

Where,

$$E\left(v_{(t)} v_{(t)}^{1}\right) = \Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}$$

That is,

$$E\left(v_{(it)}\right) = 0$$

$E\left(v_{(it)} v_{(js)}\right) = \sigma_{11}$ for t=s and 0 otherwise.

Independent series of uncorrelated standard normal random deviates U_{ij}(i = 1,2) of required sample size n are generated. These generated random deviates are transformed to be distributed as N(0, Σ), using the predetermined covariance matrix $\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} 5.0 & 2.5 \\ 2.5 & 3.0 \end{pmatrix}$.

Where Σ is covariance$\left(U_t U_t^1\right)$ the non- singular upper triangular matrix is decomposed as P_1 and non- singular lower triangular matrix is decomposed as P_2.

Where $P_1 = \begin{pmatrix} \rho_{11} & \rho_{12} \\ 0 & \rho_{22} \end{pmatrix}$ is the upper triangular matrix and

$P_2 = \begin{pmatrix} \rho_{11} & 0 \\ \rho_{21} & \rho_{22} \end{pmatrix}$ the lower triangular matrix.

This study uses sample size, N = 10, 20, 30 replicated 1000 times each. The following values are arbitrarily assigned to the structural parameters; $\beta_{12} = 1.8$, $\beta_{21} = 1.5$, $\gamma_{11} = 1.5$, $\gamma_{12} = 0.5$, $\gamma_{21} = 0.5$, $\gamma_{23} = 2.0$ and the correlation coefficients set at $\rho = 0.0$, $\rho = 0.3$.

4. RESULTS AND DISCUSSION

The results obtained from the Monte Carlo experiment are compared using the criteria of Absolute Bias (AB), variance and Root Mean Square Error (RMSE). The two cases considered are represented as Q1 and Q2, where Q1 stands for the model with $\rho = 0.3$ and Q2 stands for the model with $\rho = 0.0$.

OLS produced the least estimates throughout based on the above criteria followed closely by 23SLS while LIML consistently gave the worst estimates. The following are the major findings of the criteria mentioned above.

The following tables show the ranking of the estimators for Q1 (i.e., $\rho = 0.3$) and Q2 (i.e., $\rho = 0.0$), the patterns exhibited for both cases are generally similar.

Note that the tables for both cases of Q1 and Q2 are combined because the rankings produced by the two cases are exactly the same for all the criteria considered. Tables 1 – 3 were generated from Tables 4.1 – 4.3 in Appendix.

TABLE 1
Ranking of Estimators Using Variance (Q1, Q2)

	Equation 1			Equation 2		
	10	20	30	10	20	30
P_1	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML
P_2	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML

The result of the estimators using variance of the estimates with the specified sample sizes and 1000 replications revealed that OLS produced the least variances across the upper and lower triangular matrices followed by 2,3SLS and LIML. Estimators that have the minimum variances are considered to be the best using the efficiency property of a good estimator.

TABLE 2
Ranking of Estimators Using Absolute Bias (Q1, Q2)

	Equation 1			Equation 2		
	10	20	30	10	20	30
P_1	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML
P_2	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML

It is interesting to note that the tables for variance and absolute bias of the estimates are similar. The absolute bias of estimates showed that OLS gave the least bias at the different sample sizes considered which made it the best estimator in terms of biasness.

TABLE 3: Ranking of Estimators Using Root Mean Squares (Q1, Q2)

	Equation 1			Equation 2		
	10	20	30	10	20	30
P_1	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML
P_2	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML	OLS 2,3SLS LIML

The smaller the RMSE the better the performance of the estimators. "The property of minimum variance by itself is not enough to suggest that an estimator is superior to others (Koutosoyianins, 2003)" An estimate from an estimator with smallest variance may have that small variance around the wrong parameter. Similarly, the property of unbiasedness by itself is not particularly desirable, unless coupled with a small variance. Thus, it is necessary to consider a property that combines both measures.

It is noteworthy to mention that the three tables (1,2,3) generated when the different criteria was used for evaluating the performances of the estimators gave exactly the same ranking of the estimators with OLS consistently the best estimators followed closely by 2,3SLS and LIML.

5. CONCLUSION

The identification condition of the equations of the model was examined and just identified model was established which led to the estimation of the parameters with 0.3 correlation level. The choice of the estimators that will give a desirable estimate has to be based on the statistical properties possessed by the estimates of various methods. The performances of the estimation techniques [Ordinary Least Squares (OLS), Two-Stage Least Squares (2SLS), Limited Information Maximum Likelihood (LIML) and Three-Stage Least Squares (3SLS)] were ranked based on the criteria; variance, root mean square error and absolute bias of the small sample properties (10, 20, and 30). Based on all the criteria considered, OLS yielded least values while limited information maximum likelihood yielded the greatest as the sample size changes. Hence, the choice of ρ should be carefully made as this may significantly affect the performances of the estimators.

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APPENDIX

TABLE 4.1 PERFORMANCE EVALUATIONS OF ESTIMATORS TO CHANGES IN SAMPLE SIZE INCREASES USING VARIANCE OF PARAMETER ESTIMATES

Estimators		Equation 1									Equation 2								
		N = 10			N = 20			N =30			N = 10			N = 20			N =30		
		β_{12}=1.8	γ_{11}=1.5	γ_{12}=1.0	β_{12}=1.8	γ_{11}=1.5	γ_{12}=1.0	β_{12}=1.8	γ_{11}=1.5	γ_{12}=1.0	β_{21}=1.5	γ_{23}=2.0	γ_{21}=0.5	β_{21}=1.5	γ_{21}=0.5	γ_{23}=2.0	β_{21}=1.5	γ_{21}=0.5	γ_{23}=2.0
OLS	Q_1/P_1	0.0027	0.0019	0.0019	0.0944	0.1435	0.1462	0.1158	0.1696	0.1701	0.00	0.0018	0.0018	0.0764	0.0769	0.0978	0.0600	0.1053	0.1350
	Q_1/P_2	0.0035	0.0097	0.0083	0.0957	0.1807	0.3575	0.1143	0.2031	0.1902	0.00	0.0032	0.0040	0.0613	0.1145	0.3044	0.0578	0.1218	0.2576
	Q_2/P_1	0.0012	0.0019	0.0100	0.1221	0.1462	0.2877	0.1446	0.1701	0.1473	0.00	0.0018	0.0061	0.0651	0.0978	0.2139	0.0583	0.1350	0.1407
	Q_2/P_2	0.0018	0.0031	0.0094	0.1208	0.1562	0.2497	0.1469	0.1770	0.1287	0.00	0.0011	0.0051	0.0649	0.1033	0.2691	0.0578	0.1367	0.2432
23SLS	Q_1/P_1	27.6008	0.6284	4.3540	119.4017	3.6772	2.1877	143.0093	6.4613	1.2809	135.49	0.5047	1.8601	0.3863	0.1758	2.4916	0.4397	0.2122	16.7791
	Q_1/P_2	146.1532	4.6111	2.1400	916.1953	5.4965	5.7514	226.7692	7.2851	1.0932	676.49	3.2128	9.7210	141.5380	8.9749	5.0963	558.3122	2.1423	24.4012
	Q_2/P_1	505.8932	0.8404	4.4036	124.6622	7.4943	2.4943	253.1021	42.0618	1.0618	29.02	1.3823	1.3845	0.3893	0.2893	7.2892	0.5071	5.0586	26.5861
	Q_2/P_2	166.2875	33.8751	3.9936	383.8143	8.5062	2.9246	621.0492	50.6822	4.8983	552.87	17.9564	1.1325	489.5555	6.2489	6.3374	892.6595	1.2108	18.1444
LIML	Q_1/P_1	27.6108	0.6294	4.3640	119.4027	3.6872	2.1897	143.0193	6.4623	1.2819	135.56	0.5057	1.8641	0.2701	0.1768	2.4926	0.3753	0.2122	16.7891
	Q_1/P_2	146.1632	4.6121	2.1500	916.1963	5.4565	5.7544	226.7792	7.2861	1.0942	678.67	3.2728	9.7220	142.6243	8.9739	5.0953	597.2840	2.1443	24.4212
	Q_2/P_1	505.8942	0.8444	4.4236	124.6632	7.4983	2.4953	253.1221	42.0648	1.0628	29.56	1.3803	1.3835	0.4791	0.2895	7.2882	0.0347	5.0486	26.5961
	Q_2/P_2	166.2865	33.8851	3.9946	383.8243	8.5462	2.9256	621.0592	50.6832	4.8973	553.75	17.9574	1.1335	490.2943	6.2459	6.3375	893.4266	1.2128	18.1344

TABLE 4.2 PERFORMANCE EVALUATIONS OF ESTIMATORS TO CHANGES IN SAMPLE SIZE INCREASES USING ABSOLUTE BIAS OF PARAMETER ESTIMATES

Estimators		Equation 1									Equation 2								
		N = 10			N = 20			N =30			N = 10			N = 20			N =30		
		β_{12}=1.8	γ_{11}=1.5	γ_{12}=1.0	β_{12}=1.8	γ_{11}=1.5	γ_{12}=1.0	β_{12}=1.8	γ_{11}=1.5	γ_{12}=1.0	β_{21}=1.5	γ_{23}=2.0	γ_{21}=0.5	β_{21}=1.5	γ_{21}=0.5	γ_{23}=2.0	β_{21}=1.5	γ_{21}=0.5	γ_{23}=2.0
OLS	Q_1/P_1	0.7785	0.7560	0.7336	0.0423	0.6454	0.2103	0.2704	0.4869	0.9173	0.6192	0.5714	0.5789	0.6712	0.0308	0.0512	0.2158	0.2376	0.2971
	Q_1/P_2	0.7841	0.8092	0.7842	0.8072	0.3950	0.3230	0.5797	0.9132	0.8799	0.5620	0.6450	0.5518	0.4508	0.9463	0.0378	0.7286	0.0216	0.2991
	Q_2/P_1	0.7578	0.7578	0.7371	0.6512	0.6512	0.1779	0.4855	0.4855	0.9541	0.5711	0.5711	0.5795	0.0275	0.0275	0.0576	0.2418	0.2418	0.2924
	Q_2/P_2	0.7673	0.7673	0.7869	0.6682	0.6681	0.3213	0.4884	0.4884	0.8885	0.5595	0.5595	0.5510	0.0002	0.0001	0.0360	0.2269	0.2269	0.2926
23SLS	Q_1/P_1	524.587	1.4932	0.4866	112.3090	0.4252	1.9079	1194.370	1.3622	0.8489	0.2556	0.6706	0.9288	1.3725	0.4615	0.0324	1.7134	1.2595	2.8369
	Q_1/P_2	1.0336	2.3403	0.8496	.3488	2.0151	2.0011	2.0051	1.2824	1.2341	1.4765	0.0436	0.6980	1.4443	1.6463	0.7579	3.4601	1.5727	1.1596
	Q_2/P_1	0.2598	0.2598	1.5873	.6602	0.6602	2.4718	0.5511	0.5510	3.0119	1.2628	1.2628	0.5037	0.3187	0.3187	0.5493	3.7912	3.7912	1.4674
	Q_2/P_2	1.1215	1.1215	1.1854	0.6546	1.6546	2.3520	0.4450	0.4450	1.7553	0.3859	0.3859	0.0505	0.0397	0.0397	0.9495	0.5629	0.5629	0.8957
LIML	Q_1/P_1	524.577	1.4942	0.4867	112.3290	0.4262	1.9077	1194.340	0.3722	0.8419	0.2558	0.6606	0.9388	1.3745	0.4617	0.0424	1.7234	1.2596	2.8569
	Q_1/P_2	1.0338	2.3423	0.8494	0.3489	2.0161	2.0013	2.0051	0.2834	0.2341	1.4765	0.0536	0.6990	1.4444	1.6423	0.7679	3.4621	1.5737	1.1696
	Q_2/P_1	0.2597	0.2599	1.5874	0.6604	0.6622	2.4715	0.5521	0.5610	0.0219	1.2628	1.2828	0.5047	0.3187	0.3137	0.5483	3.7932	3.7932	1.4684

	Q_2/P_2	1.1217	1.1225	1.1855	0.6556	1.6346	2.3513	0.4452	0.4550	0.7553	0.3860	0.3869	0.0641	0.0387	0.0497	0.9425	0.5729	0.5649	0.8924

TABLE 4.3 PERFORMANCE EVALUATIONS OF ESTIMATORS TO CHANGES IN SAMPLE SIZE INCREASES USING ROOT MEAN SQUARE ERROR OF PARAMETER ESTIMATES

		Equation 1									Equation 2								
Estimators		N = 10			N = 20			N =30			N = 10			N = 20			N =30		
		β_{12}=1.8	γ_{11}=1.5	γ_{12}=1.0	β_{12}=1.8	γ_{11}=1.5	γ_{12}=1.0	β_{12}=1.8	γ_{11}=1.5	γ_{12}=1.0	β_{21}=1.5	γ_{23}=2.0	γ_{21}=0.5	β_{21}=1.5	γ_{21}=0.5	γ_{23}=2.0	β_{21}=1.5	γ_{21}=0.5	γ_{23}=2.0
OLS	Q_1/P_1	0.7802	0.7573	0.7578	0.0577	0.672	0.268	0.4346	0.6377	1.129	0.621	0.5730	0.5701	0.7259	0.2789	0.2554	0.2293	0.2794	0.3979
	Q_1/P_2	0.7863	0.8152	0.7901	0.8335	0.432	0.376	0.6154	0.0183	0.950	0.563	0.6475	0.5565	0.5144	1.0050	0.5201	0.7672	0.0417	0.3896
	Q_2/P_1	0.7578	0.7591	0.7591	0.2681	0.679	0.679	0.1293	0.6370	0.637	0.570	0.5727	0.5727	0.2554	0.3139	0.3139	0.2979	0.2951	0.2951
	Q_2/P_2	0.7644	0.7922	0.7922	0.2604	0.397	0.397	0.1504	0.9898	0.990	0.563	0.5546	0.5546	0.2550	0.5529	0.5529	0.2736	0.3887	0.3887
23SLS	Q_1/P_1	5.2536	2.9178	2.2768	10.9329	1.964	1.695	11.9628	1.4287	1.651	11.643	0.9770	1.6015	0.5197	0.6235	1.5745	0.6126	0.5106	0.1863
	Q_1/P_2	11.2349	2.8440	5.3723	30.2773	73.073	2.599	15.0595	8.7710	1.271	26.051	2.8661	3.0345	11.9425	3.1121	1.9159	24.4394	2.3947	2.1785
	Q_2/P_1	22.5207	6.9621	9.1145	11.1746	2.752	1.603	15.9288	6.5089	1.169	5.437	1.4051	1.4051	0.6922	0.6279	2.7189	0.1864	2.4424	5.2880
	Q_2/P_2	12.8953	5.9273	9.3235	19.6643	2.977	1.567	24.9223	7.0851	2.231	23.532	4.2550	1.0651	22.1426	2.5001	2.6905	29.8902	1.2360	4.3527
LIML	Q_1/P_1	5.2535	2.9158	2.2748	10.9328	1.963	1.693	11.9648	1.4227	1.652	11.642	0.9768	1.6035	0.5197	0.6335	1.5725	0.6326	0.5136	0.1843
	Q_1/P_2	11.2329	2.8450	5.3343	30.2774	73.072	2.597	15.0565	8.7510	1.272	26.051	2.8651	3.0325	11.9435	3.1321	1.9159	24.4494	2.3957	2.1775
	Q_2/P_1	22.5217	6.9631	9.1125	11.1747	2.751	1.601	15.9258	6.5059	1.164	5.434	1.4021	1.4151	0.6932	0.6279	2.7289	0.1854	2.4524	5.2840
	Q_2/P_2	12.8963	5.9253	9.3225	19.6644	2.978	1.562	24.9233	7.0821	2.241	23.534	4.2750	1.0751	22.1326	2.5101	2.6505	29.8602	1.2370	4.3547