IMHOTEP

AFRICAN JOURNAL OF PURE AND APPLIED MATHEMATICS

Imhotep Mathematical Proceedings
Volume 4, Numéro 1, (2017),
pp. 27 - 33.

# Classifying a class of the fuzzy subgroups of the alternating groups $A_n$



**M.E. Ogiugo**
Department of Mathematics,
University of Ibadan, Ibadan,
Nigeria.
ekpenogiugo@gmail.com

**M. Enioluwafe**
Department of Mathematics,
University of Ibadan, Ibadan,
Nigeria.
michael.enioluwafe@gmail.com

**Abstract**

The aim of this paper is to classify the fuzzy subgroups of the alternating group. First, an equivalence relation on *the set of all fuzzy subgroups of a group $G$ is defined. Without any equivalence relation on fuzzy subgroups of group $G$, the number of fuzzy subgroups is infinite, even for the trivial group. Explicit formulae for the number of distinct fuzzy subgroup of finite alternating group are obtained in the particular case $n = 5$. Some inequalities satisfied by this number are also established for $n \geq 5$

**Proceedings of the $6^{th}$ annual workshop on CRyptography, Algebra and Geometry (CRAG-6), 15 - 17 June 2016, University of Bamenda, Bamenda, Cameroon.**

http://imhotep-journal.org/index.php/imhotep/
**Imhotep Mathematical Proceedings**

IMHOTEP

AFRICAN JOURNAL OF PURE AND APPLIED MATHEMATICS

**Imhotep Mathematical Proceedings**
Volume 4, Numéro 1, (2017),
pp. 27 - 33.



# Classifying a class of the fuzzy subgroups of the alternating groups $A_n$

**M.E. Ogiugo**
Department of Mathematics,
University of Ibadan, Ibadan,
Nigeria.
ndjeyas@yahoo.fr

**M. Enioluwafe**
Department of Mathematics,
University of Ibadan, Ibadan,
Nigeria.
hervekalachi@gmail.com

**Abstract**

The aim of this paper is to classify the fuzzy subgroups of the alternating group. First, an equivalence relation on *the set of all fuzzy subgroups of a group $G$ is defined. Without any equivalence relation on fuzzy subgroups of group $G$, the number of fuzzy subgroups is infinite, even for the trivial group. Explicit formulae for the number of distinct fuzzy subgroup of finite alternating group are obtained in the particular case $n = 5$. Some inequalities satisfied by this number are also established for $n \geq 5$

**Proceedings of the $6^{th}$ annual workshop on CRyptography, Algebra and Geometry (CRAG-6), 15 - 17 June 2016, University of Bamenda, Bamenda, Cameroon.**

http://imhotep-journal.org/index.php/imhotep/
**Imhotep Mathematical Proceedings**

IMHOTEP - Math. Proc. **4** (2017), 27–33
1608-9324/010027–26, DOI 13.1007/s00009-003-0000


**IMHOTEP Mathematical Proceedings**

# Classifying a class of the fuzzy subgroups of the alternating groups $A_n$

M.E. Ogiugo and M. Enioluwafe

**Abstract.** The aim of this paper is to classify the fuzzy subgroups of the alternating group. First, an equivalence relation on *the set of all fuzzy subgroups of a group $G$ is defined. Without any equivalence relation on fuzzy subgroups of group $G$, the number of fuzzy subgroups is infinite, even for the trivial group. Explicit formulae for the number of distinct fuzzy subgroup of finite alternating group are obtained in the particular case $n = 5$. Some inequalities satisfied by this number are also established for $n \geq 5$

**Keywords.** Fuzzy subgroups, chains of subgroups, maximal chains of subgroups,Alternating groups, symmetric groups, recurrence relations.

## I. Introduction

The concept of fuzzy sets was first introduced by Zadeh in 1965 (see[18]). The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy subgroups by Rosenfeld in 1971 (see[17]).The pioneering work of Zadeh on fuzzy subsets of a set and Rosenfeld on fuzzy subgroups of a group led to the fuzzification of algebraic structures.

One of the most important problems of fuzzy group theory is to classify the fuzzy subgroups of groups and to count all distinct fuzzy subgroups of finite groups. This topic has enjoyed a rapid development in the last few years . In our case the corresponding equivalence classes of fuzzy subgroups are closely connected to the chains of subgroups in $G$. The group structures can be classified by assigning equivalence classes to its fuzzy subgroups. As a guiding principle in determining the number of these classes, we first found the number of maximal chains of $G$. Note that an essential role in solving our counting problem is played again by the Inclusion-Exclusion Principle.

Sulaiman and Abd Ghafur [11] have counted the number of fuzzy subgroups of symmetric group $S_2$, $S_3$ and alternating group $A_3$ . Sulaiman[10] have constructed the fuzzy subgroups of symmetric group $S_4$ using the Maximal chain method, while Tarnauceanu [16] have also computed the number of fuzzy subgroups of symmetric group $S_4$ by the Inclusion -Exclusion Principle.

The most familiar of the finite ($non-abelian$) simple groups are the alternating groups $A_n$ , which are subgroups of index 2 in the symmetric groups. The alternating group of degree $n$ is the only non- identity, proper normal subgroup of the symmetric group of degree n except

**Communication presentée au 6$^{ème}$ atelier annuel sur la CRyptographie, Algèbre et Géométrie (CRAG-6), 15 - 17 Juin 2016, Université de Bamenda, Bamenda, Cameroun / Paper presented at the 6$^{th}$ annual workshop on CRyptography, Algebra and Geometry (CRAG-6), 15 - 17 June 2016, University of Bamenda, Bamenda, Cameroon.**

when $n = 1, 2$, or 4. In cases $n \geq 2$ , then the alternating group itself is the identity, but in the case $n = 4$ , there is a second non- identity, proper, normal subgroup, the Klein four group. The normal subgroups of the symmetric groups on infinite sets include both the corresponding "alternating group" on the infinite set, as well as the subgroups indexed by infinite cardinals whose elements fix all but a certain cardinality of elements of the set. For instance, the symmetric group on a countably infinite set has a normal subgroup $S$ consisting of all those permutations which fix all but finitely many elements of the set. The elements of $S$ are each contained in a finite symmetric group, and so are either even or odd. The even elements of $S$ form a characteristic subgroup of $S$ called the alternating group , and are the only other non- identity, proper, normal subgroup of the symmetric group on a countably infinite set (*see*[2])

## II. Preliminaries

Let $G$ be a group with a multiplicative binary operation and identity $e$, and let $\mu\colon G \to [0, 1]$ be a fuzzy subset of $G$. Then $\mu$ is said to be a *fuzzy subgroup* of $G$ if (1) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$, and (2) $\mu(x^{-1}) \geq \mu(x)$ for all $x, y \in G$. The set $\{\mu(x)|x \in G\}$ is called the image of $\mu$ and is denoted by $\mu(G)$ . For each $\alpha \in \mu(G)$, the set $\mu_\alpha\colon = \{x \in G|\mu(x) \geq \alpha\}$ is called a level subset of $\mu$. It follows that $\mu$ is a fuzzy subgroup of $G$ if and only if its level subsets are either empty or subgroups of $G$. These subsets allow us to characterize the fuzzy subgroups of $G$ (see [3]).

Two fuzzy subgroups $\mu$ and $\nu$ of $G$ are equivalent, written as $\mu \sim \nu$, if $\mu(x) \geq \mu(y) \Leftrightarrow \nu(x) \geq \nu(y)$ for all $x, y \in G$. It follows that $\mu \sim \nu$ if and only if $\mu$ and $\nu$ have the same set of level Subgroups and two fuzzy subgroups $\mu, \nu$ of $G$ will be called distinct if $\mu \not\sim \nu$. (see[13]). Hence there exits a one-to-one correspondence between the collection of the equivalence classes of fuzzy subgroups of $G$ and the collection of chains of subgroups of $G$ which end in $G$. So,the problem of counting all distinct fuzzy subgroups of $G$ can be translated into a combinatorial problem on the subgroup lattice $L(G)$ of $G$ .This notion of equivalence relation was in [10,13,14, 16] in order to enumerate fuzzy subgroups of certain families of finite groups. There is another equivalence relation on the set of fuzzy subgroups used by Murali and Makamba [6, 7, 8, 9] in order to enumerate fuzzy subgroups of certain families of finite abelian groups. Some other different approaches to classify the fuzzy subgroups can be found in [ 4 ] and [ 5 ].
Most recent ,the problem of classifying the fuzzy subgroup of finite group $G$ by using a new equivalence relation $\approx$ on the lattice of all fuzzy subgroups of $G$, its definition has a consistent group theoretical foundation, by involving the knowledge of the automorphism group associated to $G$. The approach is motivated by the realization that in a theoretical study of fuzzy groups, fuzzy subgroups are distinguished by their level subgroups and not by their images in $[0, 1]$. Consequently, the study of some equivalence relations between the chains of level subgroups of fuzzy groups is very important. It can also lead to other significant results which are similar with the analogous results in classical group theory (see [15]). In this paper we follow the notion of the equivalence relation used in [15]. This equivalence relation generalizes that used in Murali' s papers [6] - [9]. It is also closely connected to the concept of level subgroup.
One next goal is to describe the method that will be used in counting the chains of subgroups of $G$. Let $M_1, M_2, ...M_k$ be the maximal subgroups of $G$ and denote by $g(G)$ (respectively by $h(G)$) the number of maximal chain of subgroups in $G$ (respectively the number of chains of subgroups of $G$ ending in $G$). The technique developed to obtain $g(G)$ is founded on the following simple remark: every maximal chain in $G$ contains a unique maximal subgroup of $G$. In this way, $g(G)$ and $g(M_i), i = 1, 2, \ldots, k$, are connected by the equality

$$g(G) = \sum_{i=1}^{k} g(M_i) \tag{1}$$

For finite cyclic groups , this equality leads to the well- known formula

$$g(Z_n) = \begin{pmatrix} m_1 + m_2 + \cdots + m_s \\ m_1, m_2, \cdots, m_s \end{pmatrix} = \frac{(m_1 + m_2 + \cdots + m_s)!}{m_1! m_2! \cdots m_s!} \quad (2)$$

In order to compute the number of all distinct fuzzy subgroups of a finite $G$ which is denoted by $h(G)$, we shall apply the inclusion-Exclusion Principle. (see[15])

$$h(G) = 2\left(\sum_{i=1}^{k} h(M_i) - \sum_{1\le i_1<i_2\le k} h(M_{i_1}\cap M_{i_2}) + ... + (-1)^{k-1} h(\bigcap_{i=1}^{k} M_i)\right) \quad (3)$$

## III. Main Results

### III.1. The cases $n = 3, 4$ and 5

Our problem is very simple for $n = 3$, since the Alternating group $A_3$ is isomorphic to Cyclic group of order 3. In the number of all distinct fuzzy subgroups of $A_3$ is

$$h(A_3) = h(C_3) = 2 \quad (4)$$

The alternating group $A_4$ possesses five maximal subgroups, one isomorphic with $D_4$ and four isomorphic to $C_3$. Therefore, we have

$$g(A_4) = g(D_4) + 4g(C_3) = 7$$

The number of all distinct fuzzy subgroups of alternating group $A_4$ was computed using the Inclusion-Exclusion Principle $(eqn3)$

$$h(A_4) = 24 \quad (5)$$

In order to compute the number of all distinct fuzzy subgroups of alternating group $A_5$, we need to describe its maximal subgroups structure. It is well-known the simplicity of alternating groups i.e., for $n \ge 5$, the alternating group $A_n$ is simple.

**Definition[1]:** A group $G$ is simple if $G$ has no normal subgroups other than $\{1\}$ and $G$ itself. Such groups have remarkable properties.

By Lagrange's Theorem, any group of prime order is simple. All other simple groups are non-abelian. These small simple groups belong to two families $A_n$ and $PSL_2(q)$ for $n \ge 5$ and $q \ge 5$ a prime power.The alternating group $A_5$ exhibits exceptional isomorphisms:

$$A_5 \cong PSL_2(4) \cong PSL_2(5)$$

The alternating group $A_5$ possesses 21 maximal subgroups, of which 6 are isomorphic to $D_{10}$, 10 are isomorphic to $S_3$ and 5 are isomorphic to $A_4$. So that we get:

$$g(A_5) = \sum_{i=1}^{21} g(M_i) = 10g(S_3) + 6g(D_{10}) + 5g(A_4)$$

$$g(A_5) = 10(4) + 6(8) + 5(7)$$

$$g(A_5) = 123$$

**Lemma 1**. The number $g(A_5)$ of all the maximal chains of subgroups of the alternating group $A_5$ is 123

We deduce that a lower bound for $g(A_n)$ , where $n \ge 5$ is arbitrary.

**Proposition 1**. For $n \ge 5$, the number $g(A_n)$ of all maximal chains of subgroups of the alternating group $A_n$ satisfies the following inequality:

$$g(A_n) \ge g(S_{n-2}) + ng(A_{n-1}) \quad (6)$$

For instance the inequality (6) derived the lower bounds for $n = 5, 6$ as below:

$$g(A_5) \geq g(S_3) + ng(A_4)$$

$$g(A_5) \geq 4 + 5(7)$$

$$g(A_5) \geq 39$$

For $n = 6$

$$g(A_6) \geq g(S_4) + ng(A_5)$$

$$g(A_6) \geq 44 + 6(123)$$

$$g(A_6) \geq 782$$

**Maximal Subgroup Structure of $A_5$**

| Maximal Subgroups | Generating sets | Order | Number |
|---|---|---|---|
| $S_3$ | $\langle(1,2,3),(1,2)(4,5)\rangle$ | 6 | 10 |
| $A_4$ | $\langle(1,2,3),(2,3,4)\rangle$ | 12 | 5 |
| $D_{10}$ | $\langle(2,4)(3,5),(1,2,3,5,4)\rangle$ | 10 | 6 |

There exist 10 such maximal subgroups, all isomorphic to $S_3$ :

1. , $M_1 = \langle(1,2,3),(1,2)(4,5)\rangle$
2. $M_2 = \langle(1,2,4),(1,2)(3,5)\rangle$,
3. $M_3 = \langle(1,2,5),(1,2)(3,4)\rangle$,
4. $M_4 = \langle(1,3,4),(1,3)(2,5)\rangle$,
5. $M_5 = \langle(1,3,5),(1,3)(2,4)\rangle$,
6. $M_6 = \langle(1,4,5),(1,4)(2,3)\rangle$,
7. $M_7 = \langle(2,3,4),(1,5)(2,3)\rangle$,
8. $M_8 = \langle(2,3,5),(1,4)(2,3)\rangle$,
9. $M_9 = \langle(2,4,5),(1,3)(2,4)\rangle$,
10. $M_{10} = \langle(3,4,5),(1,2)(3,4)\rangle$,

There exist 5 such maximal subgroups, all isomorphic to $A_4$ ;

1. $M_{11} = \langle(1,2,3),(2,3,4)\rangle$,
2. $M_{12} = \langle(1,2,5),(1,2,3)\rangle$,
3. $M_{13} = \langle(1,4,5),(1,2,5)\rangle$,
4. $M_{14} = \langle(3,4,5),(1,4,5)\rangle$,
5. $M_{15} = \langle(2,3,4),(3,4,5)\rangle$,

There exist 6 such maximal subgroups, all isomorphic to $D_{10}$ ;

1. $M_{16} = \langle(2,4)(3,5),(1,2,3,5,4)\rangle$,
2. $M_{17} = \langle(2,5)(3,4),(1,2,4,3,5)\rangle$,
3. $M_{18} = \langle(2,3)(4,5),(1,2,5,4,3)\rangle$,
4. $M_{19} = \langle(2,4)(3,5),(1,3,2,4,5)\rangle$,
5. $M_{20} = \langle(2,5)(3,4),(1,3,5,2,4)\rangle$,
6. $M_{21} = \langle(2,3)(4,5),(1,4,3,2,5)\rangle$,

## IV. Counting the number of fuzzy subgroups of alternating group $A_5$

The problem of counting all distinct fuzzy subgroups of $G$ can be translated into combinatorial problem on the subgroup lattice $L(G)$ of G: finding the number of all chain of subgroups of $G$

that terminates in $G$. Clearly, we obtain that in any group with at least two elements there are more distinct fuzzy subgroup than subgroups.
Since the maximal subgroups $M_i$ of $A_5$ and their intersections have been precisely determined with the help of the computational group theory system GAP.

This has been used in[12, 15] to obtain explicit formulas of $h(D_{2n})$ for some classes of positive integers $n$. Recall here that

$$h(D_{2n}) = \frac{2^{m_1}}{p_1 - 1}\left(P_1^{m_1+1} + p_1 - 2\right)$$

if

$$n = p_1^{m_1}$$

and, in particular , $h(D_4) = 8$, $h(D_6) \cong h(S_3) = 10$ , $h(D_8) = 32$ , $h(D_{10}) = 68$
$h(C_2 \times C_2) \cong h(C_4) \cong h(D_4) = 8$
$h(A_3) \cong h(C_3) = 2$.
It follows that :

$$M_{i_1} \cap M_{i_2} \cap \cdots \cap M_{i_r} = \{e\}, \text{ for all } r \geq 8 \text{ and all } 0 \leq i_1 < i_2 < \cdots < i_r \leq 21$$

$$c_r = (-1)^{r-1} \sum_{0 \leq i_1 < i_2 \cdots < i_r \leq 21} h(M_{i_1} \cap M_{i_2} \cap \cdots \cap M_{i_r})$$

We have

$$c_1 = (10h(S_3)) + 6h(D_{10}) + 5h(A_4)) = 304$$
$$c_2 = -(30h(A_e) + 30h(C_3) + 150h(C_2)) = -390$$
$$c_3 = \binom{21}{3} - 160 + 10h(C_3) + 150h(C_2) = 1490$$
$$c_4 = -\binom{21}{4} - 75 + 75h(C_2) = -6060$$

$$c_5 = \binom{21}{5} - 15 + 15h(C_2) = 20364 \qquad c_6 = -\binom{21}{6} = -54264$$
$$c_7 = \binom{21}{7} = 116280 \qquad c_8 = -\binom{21}{8} = -203490$$
$$c_9 = \binom{21}{9} = 293930 \qquad c_{10} = -\binom{21}{10} = -352716$$
$$c_{11} = \binom{21}{11} = 352716 \qquad c_{12} = -\binom{21}{12} = -293930$$
$$c_{13} = \binom{21}{13} = 203490 \qquad c_{14} = -\binom{21}{14} = -116280$$
$$c_{15} = \binom{21}{15} = 54264 \qquad c_{16} = -\binom{21}{16} = -20349$$
$$c_{17} = \binom{21}{17} = 5785 \qquad c_{18} = -\binom{21}{18} = -1330$$
$$c_{19} = \binom{21}{19} = 210 \qquad c_{20} = -\binom{21}{20} = -21$$
$$c_{21} = \binom{21}{21} = 1$$
$$h(A_5) = 2\sum_{r=1}^{21} c_r = 408$$

**Theorem 4** The number $h(A_5)$ of all distinct fuzzy subgroups of the alternating group $A_5$ is 408

## V. An Upper Bound for $h(A_n)$, $n \geq 5$

Finally, for the cases of $n = 3, 4, 5$ considered, the number of distinct fuzzy subgroups of the alternating group was computed by above-mentioned method (Inclusion-Exclusion Principle) and group Isomorphism for the case of $n = 3$. The alternating group of degree n is very difficult to describe all its maximal subgroups when $n$ becomes large and so the method of computing the number of all distinct fuzzy subgroups will be tedious . It necessitate develop a lower bound and upper bound for the number of all distinct fuzzy subgroups for larger classes of $n$. In order of estimate (3), the group isomorphism for the maximal subgroups and their direct computation of intersections.
Studying the classification of certain maximal subgroups of the alternating groups, we can easily see the only case where $A_m$ has a maximal subgroup that is isomorphic to a symmetric group $S_n$ that acts transitively but imprimitively on $\{1, \ldots, m\}$ is when $(n, m) = (4, 6)$ . Also, $A_m$ has a maximal subgroup that is isomorphic to $A_n$ , which does act transitively on $\{1, \ldots, m\}$ that is $(n, m) = (5, 6)$. The subgroup structure of $A_n$ and $S_n$ is described by the O'Nan-Scott Theorem. It happens that classification of maximal subgroups is convenient in terms of concept of transitivity (see [1]). Since our method (Inclusion and Exclusion Principle) of the computing the number of fuzzy subgroups of $A_n$ is based on the direct calculation of intersection of their maximal subgroups. This allows us to deduce the following inequalities :
**Theorem 5** . For $n \geq 5$, the upper bound for the number $h(A_n)$ of all distinct fuzzy subgroups of the alternating group $A_n$ satisfies the following inequality:

$$h(A_n) \leq 2\left(\sum_{r=0}^{n-2}(-1)^r \binom{n}{r+2} h(A_{n-r-1}) + \sum_{r=0}^{n-2}\binom{n}{r+2} h(S_{n-r-2}) + \sum_{r=0}^{n-2}\binom{n}{r+2} h(D_{2r+4})\right) \quad (7)$$

For example $n = 5$, then we have that (7) becomes

$$h(A_5) \leq 592 + 10h(A_4)$$

which implies that

$$h(A_5) < 602 \quad (8)$$

For $n = 6$, and a upper bound for $h(A_6)$ becomes

$$h(A_6) \leq 6468 + 15h(A_5)$$

which leads to

$$h(A_6) < 6483 \quad (9)$$

## VI. Conclusion

The study concerning the classification of the fuzzy subgroups of (finite) groups is a significant aspect of fuzzy group theory. The problem of counting the number of distinct fuzzy subgroups relative to the the notion of the equivalence relation. Without any equivalence relation on fuzzy subgroups of finite group, the number of fuzzy subgroups is infinite, even for the trivial group. These equivalence relations provide settings for classifying the fuzzy subgroups of finite groups. The group structures can be classified by the assigning equivalence classes to its fuzzy subgroups. This will surely constitute the subject of further research on the classification of the fuzzy subgroups of finite symmetric groups.

### VI.1. Further Research

- Establishing some explicit formulas for $g(S_n)$ and $h(S_n)$ for other $n \geq 5$
- Establishing some explicit formulas for $g(A_n)$ and $h(A_n)$ for other $n \geq 5$
- Develop a method to find some upper bounds for $g(S_n)$ and $h(S_n)$, $n \geq 5$

## References

[1] Bret Benesh, *A classification of certain maximal subgroups of alternating groups*, AMS Special Session on Computational Group Theory, March 3-4, 2007, Davidson, North Carolina, 2008, 21-26

[2] Dixon, P. J.; Mortimer B. *Permutation group.* Graduate Texts in Mathematics , Springer-Verlag, Berlin(1996)

[3] Gratzer, G., *General Lattice*, Academic Press, New York, 1978.

[4] Mashinichi, M., Mukaidono, M., *A classification of fuzzy subgroups*, Ninth Fuzzy System Symposium, Sapporo, Japan, 1992, 649- 652.

[5] Mashinichi, M., Mukaidono, M., *A classification of fuzzy subgroups* , Research Report Of Meiji Univ. **9**(1993), 31- 36.

[6] Murali, V., Makamba, B. B., *On an equivalence of fuzzy subgroups*, I, Fuzzy Sets and Systems **123**(2001), 259-264.

[7] Murali, V., Makamba, B. B., *On an equivalence of fuzzy subgroups*, II, Fuzzy Sets and Systems **136** (2003), 93- 104

[8] Murali, V., Makamba, B. B., *On an equivalence of fuzzy subgroups*, III, Int. J. Math. Sci. **36** (2003), 2303- 2313.

[9] Murali, V., Makamba, B. B., *Fuzzy subgroups of finite abelian groups*, FJMS **14**(2004), 113-125

[10] Sulaiman, R., *Constructing Fuzzy Subgroups of Symmetric group $S_4$*, Int. J. Alg., **6**(2012), 23- 28.

[11] Sulaiman, R., Abd Ghafur, A., *Counting Fuzzy Subgroups of Symmetric groups $S_2, S_3$ and Alternating group $A_4$* , J. Qual. Meas. Anal., **6**(2010), 57-63.

[12] Suzuki, M., *Group theory*, I ,II, Springer Verlag, Berlin, 1982-1986.

[13] Tarnauceanu, M., *Classifying fuzzy subgroups of finite nonabelian groups*, Iran. J. Fuzzy Syst. **9** (2012), 33 - 43.

[14] Tarnauceanu, M., Bentea, L., *On the number of fuzzy subgroups of finite abelian groups*, Fuzzy Sets and Systems **159**(2008), 1084- - 1096.

[15] Tarnauceanu, M., *A new Equivalence relation to classify the fuzzy subgrioups of finite groups*, Fuzzy Sets and Syst. **289** (2016), 113 - 121.

[16] Tarnauceanu, M., *On the number of fuzzy subgroups of finite symmetric groups*, J. multi-val. logic soft comp. **21** (2013), 201 - 213.

[17] Rosenfeld, A., *fuzzy subgroups* , J. Math. Anal. Appl., **35** (1971), 512-517.

[18] Zadeh L.A. *Fuzzy sets*, Inform. and Control, **8**(1965), 338-353.

M.E. Ogiugo
e-mail: ekpenogiugo@gmail.com
Department of Mathematics, University of Ibadan, Ibadan, Nigeria.

M. Enioluwafe
e-mail: michael.enioluwafe@gmail.com
Department of Mathematics, University of Ibadan, Ibadan, Nigeria.