

**ANALYSIS OF PRE-CAST WAFFLE SLABS USING YIELD LINE  
AND RANKINE GRASHOFF THEORIES**

**BY**

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## ABSTRACT

Failure in pre-cast waffle slabs can be attributed to factors like analytical error, poor handling during transportation and placement which factors often lead to partial/total failure of slabs. The conventional method of analysing waffle slabs focuses on the ribs, while the slab portions at the top are avoided. This has led to under reinforcement and subsequent failure of the slab portions that are usually in direct contact with loads. In this study, a method that incorporates both the slab and rib portions in the analysis of pre-cast waffle slabs was therefore developed.

Yield Line and Rankine Grashoff Theories (YLRGT) were combined for the analysis of pre-cast waffle slab. Six physical models of waffle slab were developed, each having five replicates, with the following dimensions: W1 (1353 x 430 x 58 mm), W2 (900 x 300 x 50 mm), W3 (1085 x 430 x 58 mm), W4 (407 x 364 x 50 mm), W5 (1312 x 300 x 58 mm) and W6 (860 x 360 x 50mm). Solid slabs of the same size and number designated S1, S2, S3, S4, S5 and S6 served as control. These models were validated using the slabs by testing for failure loads, deflections and crack width. Each slab was subjected to incremental load of 1.0 kN until failure occurred. Maximum bending moments were obtained for slab and rib portions using YLRGT, a finite element based method called ETABS was also used to analyse the slabs and results obtained were subjected to statistical analysis using ANOVA at  $p=0.05$ .

The YLRGT analysis of the various physical models (slab portion, transverse and longitudinal ribs) yielded the following bending moments: W1 (5526.0, 34.5, 918.3) Nm, W2 (1122.0, 279.2, 36.5) Nm, W3 (2880.0, 27.2, 619.9) Nm, W4 (590.0, 171.9, 160.9) Nm, W5 (947.0, 37.0, 4.4) Nm and W6 (1276.0, 90.4, 36.2) Nm respectively. The ETABS combined both slab and ribs giving W1 (4729.0) Nm, W2 (581.0) Nm, W3 (3338.0) Nm, W4 (733.0) Nm, W5 (851.0) Nm and W6 (686.0) Nm. Deflections at failure for waffle slab were smaller compared to solid slabs: (W1 = 1.19 and S1 =3.56) mm, (W2 = 3.64 and S2 =9.28) mm, (W3 = 3.90 and S3 =7.44) mm, (W5 = 8.17 and S5 =12.18) mm, (W6 = 3.29 and S6 =3.89) mm with the exception of W4 (6.60 mm) and S4 (6.44mm), where deflection of waffle slab was higher than that of solid slab. Mean deflection of S1 was significantly higher than W1, while S2 was significantly higher than W2. Average crack width for waffle (0.48 mm) and solid slabs (0.99 mm) were significantly different. High crack width in solid slab indicated lower shear strength.

The Yield Line and Rankine Grashoff Theories have facilitated the accurate analysis of pre-cast waffle slabs by separating the slab and rib portions.

**Keywords:** Yield line theory, Rankine Grashoff theory, Waffle slabs, Crack width.

**Word Count: 464**

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## **DEDICATION**

Unto God, who was, who is and who is to come. To Him is all the Glory For Ever.

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## **CERTIFICATION**

I certify that this work was carried out by JOSEPH OLAWALE AKINYELE in the Department of Civil Engineering, University of Ibadan, Nigeria.

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## LIST OF SYMBOLS

The most commonly used symbols are listed below. Exceptions from the list may appear, but this will then be noted in the text in connection with the actual symbol.

$h$  = Height of a cross-section

$A$  = Area of a cross-section

$A_c$  = Area of a concrete cross-section

$A_s$  = Area of reinforcement close to the bottom face

$A_s'$  = Area of reinforcement close to the top face

$A_{sc}$  = Area of reinforcement in compression

$h_c$  = Distance from the bottom face to the centre of the bottom reinforcement

$h_c'$  = Distance from the top face to the centre of the top reinforcement

$y_o$  = Compression depth

$L$  = Length of an element

$L_x, L_y$  = Length of a slab in the  $x$  and  $y$  direction, respectively

$e$  = Eccentricity

$u$  = Deflection

$u_m$  = Deflection in the mid section

$x, y, z$  = Cartesian coordinates

$f_c$  = Compressive strength of concrete

$f_y$  = Yield strength of reinforcement

$\rho$  = Reinforcement ratio

$\Phi_0$  = Degree of Reinforcement

$\Phi_{0x}, \Phi_{0y}$  = Degree of Reinforcement in the  $x$  and  $y$  direction, respectively

$p$  = Line load, uniform load per unit length

$q$  = Surface load, uniform load per area unit

$m_p$  = yield moment in pure bending

$m_{px}, m_{py}$  = yield moment in pure bending in the  $x$  and  $y$  direction, respectively

$m_f$  = yield moment for a given axial load

$m_{fx}, m_{fy}$  = yield moment for a given axial load in the  $x$  and  $y$  direction, respectively

$n$  = Axial load per unit length

$n_x, n_y$  = Axial load per unit length in the  $x$  and  $y$  direction, respectively

$W_i, W_e$  = internal and external work, respectively

$W_c, W_s$  = concrete and reinforcement contribution to the dissipation, respectively

$M$  = mass

$T$  = time

$N$  = load acting within a particular region (kN)

$\sigma$  = stress

$\delta$  = vertical displacement of the load  $N$  on each region expressed as a fraction of Unity (m)

$\delta$  = unit weight

$m$  = ultimate design moment of resistant for the slab (kNm)

$\theta$  = rotation of the region about its axis of rotation (m/m)

$a_r$  = reduced short span dimension

$b_r$  = reduced long span dimension

$k$  = a constant = 1

$\nu$  = a variable =  $0 \leq 0.5$

$i$  = the fixity ratio at supports, e.g.  $i_1, i_2$ .

If :  $i_1 = i_2 = 1$ , that support is a continuous support

And  $i_1 = i_2 = 0$  that support is a simple support.

$a_1$  and  $b_1$  = are the spacing of the ribs on the short and long span respectively.

$q$  = total load per unit area

$q_1$  and  $q_2$  = the load shared on the short and long span respectively

$a$  = Shorter dimension of grid

$b$  = Longer dimension of grid

$M_x$  and  $M_y$  = moments on the short and long span respectively

$Q_x$  and  $Q_y$  = the shear forces on the short and long span respectively.

$L_p$  = Diameter of bars in the prototype

$L_m$  = Diameter of bars in the model

$\delta_p$  = Characteristic strength of steel used in the prototype

$\delta_m$  = Characteristic strength of steel used in the model

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# CHAPTER ONE

## INTRODUCTION

### 1.1 Background

Reinforced concrete slabs are one of the most common structural engineering elements. They are used as floors and roofs, to carry vertical loads in structures such as buildings and bridges. A slab is part of a reinforced concrete structure which is often subjected to bending (tensile or compressive) but in rare cases, subjected to shear, such as a bridge deck. In most cases, slabs are horizontal members but they can be used as vertical members, such as walls, to infill panels, side to drains and sewers appurtenances (Oyenuga, 2001).

The various types of slabs include:

- (a) Solid slab,
- (b) Ribbed slab,
- (c) Flat slab, and
- (d) Waffle slab.

The type to be preferred may depend on:

- (i) span of the slab,
- (ii) use of the space which may determine the span,
- (iii) load to be carried, and
- (iv) architectural aesthetics that is required.

Most of the researches carried out on yield line theory have basically been on solid slabs with little or no research on waffle slabs. According to Mosalam and Naito (2002), a limited number of experimental studies exist in the literature concerning waffle slabs.

Waffle slab has its genesis in a rather thick solid-slab floor from which the bottom layer concrete in tension is partially replaced by their ribs along orthogonal directions. The ribs are reinforced with steel to resist flexural tensile stresses. The dimensions and spacing of ribs are decided in a manner so as to achieve better load distribution without requiring the shear reinforcement (Prasad *et al.* 2005).

Waffle slabs are generally employed in large span slabs, as spans become larger still, the required slab thickness for the flat plate and flat slab increases to the point where the slab may be unable to carry its own weight. A solution to this is to provide thickness so that reinforcement can be placed in a member at greater depth, but remove concrete from regions of the slab not required for strength (Konda, 2003). It is an extension of the ribbed floor slab in which the slab is ribbed in two directions. Hence, an inverted pot-like hollow is formed which serves as the ceiling for the floor below, (Figure 1.1.). Waffle slabs are all concrete. The inverted pot-like shape is formed through the use of a special mould. When compared with the conventional solid flat slab construction, waffle slabs allow a considerable reduction in dead load; can support heavy loads over a long span (Daniel and Onur, 2005). It is commonly used in parking garages of tall buildings with ramps and also in industrial facilities and warehouses (Buildings, 2005) and meet fire proofing requirements (Sadusky, 2004; Kenichi and Ai, 2005). Also, it has the advantage of medium to long span, light weight, economical in material usage and profiles may be expressed architecturally or used for heat transfer (The concrete centre, 2006). This slab is usually employed for architectural and structural reasons for large rooms such as auditoria, vestibules, theatre halls, show rooms of shops where column-free-space is often the main requirement (Krishna Raju, 1988).

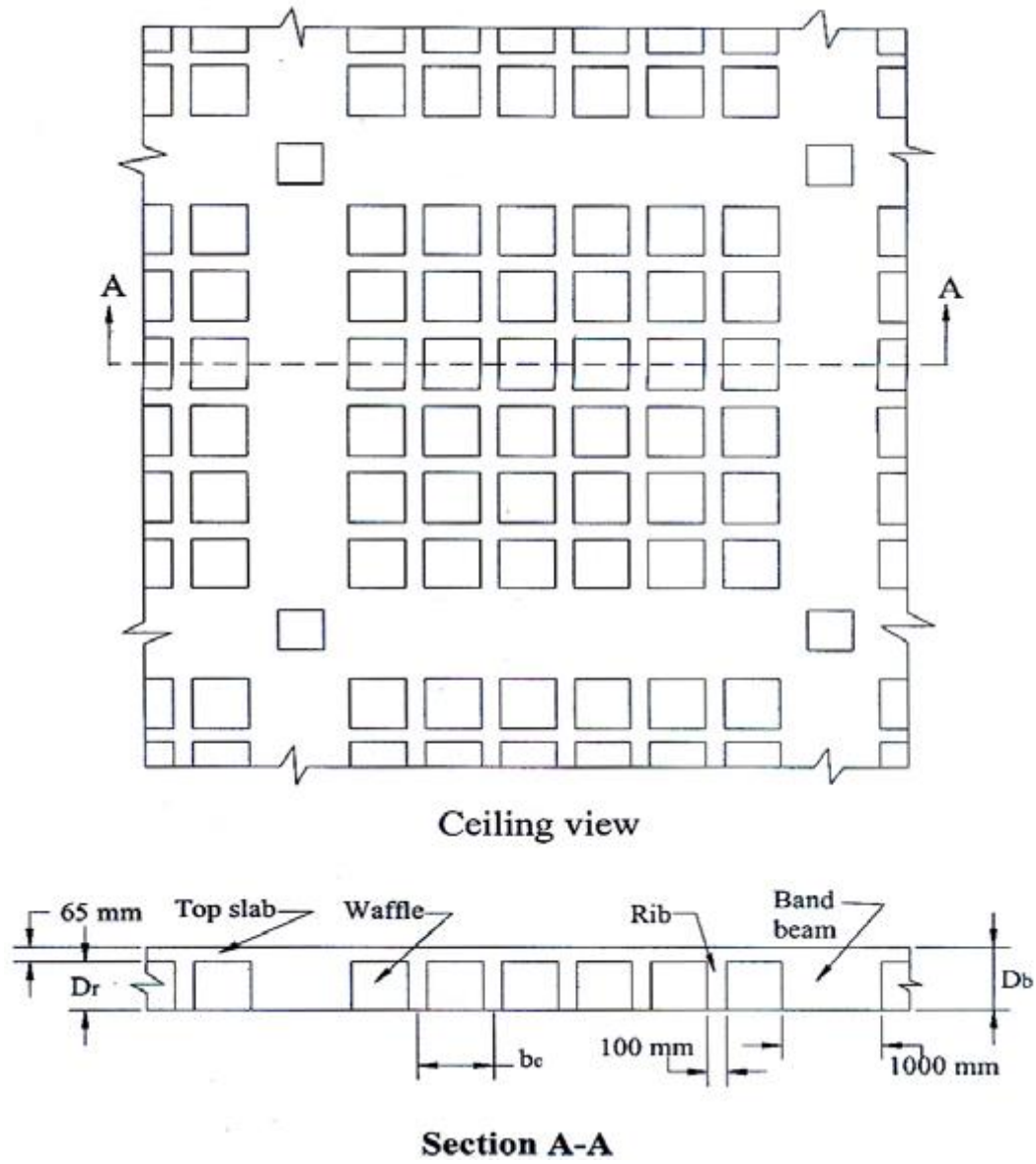


Figure 1.1: Detailed view of a waffle slab floor

According to Howard and Hansen (2002), waffle floors are used extensively in semiconductor factories as they provide high impedance mounts for manufacturing equipment that is extremely vibration sensitive. Also it has been used for cooling towers, storage tanks, communication shelters, barriers and retaining walls, artificial reefs, building walls, hybrid columns and beams (Waffle-Crete international, 2009).

Deflection or failure in slab can occur due to some factors like overloading, under-reinforcement and poor span-depth ratio. Under overloading, failure of a slab will occur when the reinforcement yields first in a region of high moments. According to Wang *et al* (2003), when concrete is under triaxial compressive loading, both its strength and ductility will have a significant increase as a result of resistance to the compressive force by the concrete materials (molecules). Initially, at service load, the response of a slab is elastic with maximum steel stress and deflection occurring at the centre of the slab. At this stage, it is possible that some hairline cracking will occur on the suffix where the flexural tensile capacity of the concrete has been exceeded at mid span. Increasing the load hastens the formation of these hairline cracks. Further increment of the load will increase the size of the cracks and induce yield of the reinforcement, initiating the formation of large cracks emanating from the point of maximum deflection (Kennedy and Goodchild, 2004). This portion acts like a plastic hinge. On increasing the load further, the hinging region rotates plastically and the moments due to additional loads are redistributed to adjacent sections, the concrete section at the position of a yield line is incapable of carrying any further load, causing them to collapse (Thompson and Haywood, 1986; Macgregor, 1997).

Yield line theory investigates failure mechanism at the ultimate limit state. It does not deal with serviceability issues such as deflection *per se*. Nonetheless, deflection

can be dealt with by simple formulae based on yield line moment (Kennedy and Goodchild, 2004). The basic assumption of the yield line theory is that a reinforced concrete slab, similar to a continuous beam or frame of a perfectly plastic material will develop yield line hinges under overload, but will not collapse until a mechanism is formed (Dunham,1964). The theory also permits the prediction of the ultimate load of a slab system by postulating a collapse mechanism which is compatible with the boundary conditions (Buyukozturk 2004). Yield-line analysis is seen as a useful technique to determine the collapse load of slabs (Johansen, 1963). The band in which yielding has occurred are referred to as yield lines which divide the slab into a series of elastic plates.

The use of yield line analysis requires knowledge of the plastic flexural capacity of the slab, and the results serve as an upper bound on the ultimate load of the system. The results of yield line analysis for a given slab are either correct or too high and are highly dependent on proper selection of a failure mechanism (Park and Gamble, 2000).

The Rankine Grashoff theory of equating deflections at the junctions of ribs is used for the analysis of ribbed or grid floors. The method considers the load in a slab and distribute to all the ribs along both the longitudinal and transverse directions.

## **1.2 Aim and Objectives**

The aim of this work was to develop a method of analysing precast waffle slabs of different sizes, under various support conditions.

The objectives of this study are to:

- i. Apply Yield Line and Rankine Grashoff Theories (YLRGT) to waffle slabs subjected to axial loading.
- ii. Determine the structural characteristics of waffle slabs.

iii. Develop a computer based method called YLRGT for the analysis and design of waffle slabs.

iv. Validation of the program developed.

### **1.3 Justification**

Structural failure in building was attributed to both technical and human errors by Carper (1998). He listed some causes of failure in civil engineering structures to include: programming deficiency, site selection and site development errors, design errors, construction errors, material deficiencies, and operational errors. Of all the causes mentioned, the programming and design errors were the main problems that this study has tried to look at and proffer appropriate solutions.

Yield line design leads to slabs that are quick and easy to handle. The resulting slabs are light and have very low amounts of reinforcement in very regular arrangements. Above all, yield line design generates economic concrete slabs, because it considers failures at the ultimate limit state (Kennedy and Goodchild, 2004). Since waffle slab is considered to be economical due to its reduced concrete volume, if compared with solid slabs, Yield line theory will be applied in order to investigate its structural response to loading.

In most of the works that have been carried out on waffle slabs, it was only the ribs that were analysed with the provision of adequate reinforcements, but the slab portion has often being left unanalysed. Most designs simply provided wire meshes for the slab portion. Mosley *et al* (1999) simply adopted 12% of the cross section area of the slab portion as wire mesh reinforcements. However, this method has been found to be adequate for slabs that are cast in-situ only, since they are not subjected to the same eccentric forces generated as a result of lifting and transportation of precast waffle slab



from the manufacturing plant to the site. In some factories where precast waffle slabs are manufactured using wire mesh in the slab portions, the precast slabs were subjected to both hogging and sagging moments which resulted into cracks and some times outright failures during loading and placement.

Considering the problems of precast with wire mesh reinforcements, it became necessary to subject the loading conditions to further investigations. This study therefore, developed a method (YLRGT) which is based on yield line and Rankine Grashoff theories of analysis to provide adequate reinforcements for the precast slab.

#### **1.4 Scope of the Study**

Six waffle slabs were modelled as W1, W2, W3, W4, W5 and W6, in this research work. Six solid slabs of the same shape and size as the models labeled, S1, S2, S3, S4, S5 and S6, were constructed to serve as control to the models. Both one way and two-way slabs with simple supports on four, three and two sides respectively were investigated. The above specimens were used to determine and compare deflection, crack width and the load bearing capacities of both waffle and solid slabs. The computer program developed (YLRGT) was used to determine the bending moments of both types of slab. The models were tested at the Department of Mechanical Engineering laboratory of The Polytechnic, Ibadan, Oyo state.

#### **1.5 Problems Encountered**

- (i.) The inability of the universal tensile machine to stop at every incremental load of 1 kN made the reading of the crack width difficult at this interval, hence the crack widths were measured at failure loads only.
- (ii.) The non-availability of crack meter or a crack detective microscope led to the use of a Vernier caliper in determining the crack widths of all the models.

## **CHAPTER TWO**

### **LITERATURE REVIEW**

#### **2.1 History of Reinforced Concrete**

Concrete is a material used in building construction, consisting of hard, chemically inert particulate substances, known as aggregates that is bonded by cement and water.

The Assyrians and Babylonians used clay as the bonding substance or cement. The Egyptians used lime and gypsum cement. In 1756, British engineer, John Smeaton made the first modern concrete (hydraulic cement) by adding pebbles as a coarse aggregate and mixing powdered brick into the cement. In 1824, English inventor, Joseph Aspdin invented Portland cement, which has remained the dominant cement used in concrete production. Joseph Aspdin created the first true artificial cement by burning ground limestone and clay together. The burning process changed the chemical properties of the materials and he created cement, stronger than what using plain crushed limestone would produce.

The other major part of concrete besides the cement is the aggregates which include sand, crushed stone, gravel, slag, ashes, burned shale, and burned clay. Fine aggregate (fine refers to the size of aggregate) is used in making concrete slabs and smooth surfaces. Coarse aggregate is used for massive structures or sections of cement.

Concrete that includes imbedded metal (usually steel) is called reinforced concrete or ferroconcrete. Reinforced concrete was invented in 1849 by Joseph Monier, who received a patent in 1867. He was a Parisian gardener who made garden pots and tubs of concrete reinforced with an iron mesh. Reinforced concrete combines the tensile or bendable strength of metal and the compressional strength of concrete to withstand heavy loads. Joseph Monier exhibited his invention at the Paris Exposition of 1867.

Besides his pots and tubs, He promoted reinforced concrete for use in railway ties, pipes, floors, arches and bridges (Bellis, 2009).

Since the strength of concrete in tension is very poor, this disadvantage of low tensile strength was overcome by the introduction of reinforcement. Hence; the name 'Reinforced Concrete' (Rajagopalan, 2005).

## **2.2 Analysis of Waffle Slabs**

The exact analysis and design of waffle slab is complex, therefore designers adopted simple procedure based on the use of certain coefficients to distribute the load in both directions, with the assumption that the moment on parallel ribs in one direction are equal (Abdel-Karim and Mahmood, 2006). This type of assumption is not so reliable because so many parameters are left unconsidered during analysis. According to Abdel-Karim and Mahmood (2006), the stiffnesses of the beams on which the ribs are supported are not taken into consideration in distributing the moments along the different parallel ribs in each direction, and this is a disadvantage to the results of such analysis.

In the analysis of waffle slabs, Oyenuga (2001) used the coefficient for two-way spanning solid slabs in the BS 8110 part 1(1997) code. The analysis considered the pot/hollow dimensions of the waffle slabs in which provisions were made for top and bottom reinforcements of the ribs only. Mosley *et al* (1999) designed a waffle slab for a panel in which the ribs were designed as a "T" section. Adequate reinforcements were provided and the deflection of the ribs was also determined. However, there was provision for minimum reinforcements in the slab portion.

Rahman *et al* (2010), used a 3-D strut-and tie model to analyse waffle slab, the thickness of the bottom tie, vertical ties, inclined strut and nodal zones at bottom were taken as the thickness of the waffle ribs. Thickness of the top strut and nodal zones at top was taken as the effective width of top slab and was less than or equal to the rib spacing. It was observed from the work that the ribs were well analysed using the strut-and tie model method, while the slab portion was not analysed.

## **2.3 Analytical Procedures**

Different analytical methods had been developed in the past in order to solve the problem of analysis of waffle slabs. According to Prasad *et al* (2005), waffle slabs were generally analysed using the finite element method, grid or grillage analysis and orthotropic plate theory.

### **2.3.1 Finite element method**

The Finite Element Method (FEM) is based on the division of the structures into small pieces (elements) whose behaviours are formulated to capture the local behaviour of the structure. Each element's definition is based on its material properties, geometry, location in the structure, and relationship with surrounding elements. These elements can be in the form of line elements, two dimensional elements and three-dimensional elements to represent the structure. The intersection between the elements are called nodal points in one dimensional problems, while in two and three dimensional problems, they are called nodal line and nodal planes respectively (Maher, 2007). At the nodes, degrees of freedom (which are usually in the form of the nodal displacements and/ or their derivatives, stresses, or combinations of these) are assigned.

Models which use displacements are called displacements models and some models use stresses defined at the nodal points as unknown. Models based on stresses are called force or equilibrium models, while those based on combination of both displacements and stresses are termed mixed models or hybrid models (Beckett, 1973). The mathematical assemblage of these elements into the complete structure allows for automated computation of the response of the entire structure. With finite element method, the entire floor can be analyzed at once.

Aalami and Kelly (2001), used FEM to analyze waffle slab, and two options were adopted. The first option was the modelling of the floor system with each waffle represented by its true geometry. Alternatively, the waffle stems were lumped together and positioned along the lines of support without changing the area, moment of inertial and section moduli of the structure. The result of the analysis was compared with two other methods, namely the Simple Frame Method (SFM), and Equivalent Frame Method (EFM). It was concluded that EFM was a refinement of the SFM, and that both methods were approximate. The degree of approximation depends on the extent to which a floor system deviates from a uniform, orthogonal support layout and constant slab thickness.

Although both the EFM and SFM gave safe solutions, they are not as reliable as the FEM. The study concluded that since the selection of load path is a prerequisite for the design of a concrete floor, using the FEM was able to prove that the selection of load path and analysis of the slab can be automated and give satisfactory results. This method needs more time and efforts in modelling than the grillage. The results obtained from the FEM depend on the mesh size, but by optimizing the mesh, the results of this method are considered more accurate than grillage. The FEM is a well-known tool for

the solution of complicated structural engineering problems, as it is capable of accommodating many complexities in the solution. (Maher, 2007).

Tiedman *et al* (1993) showed that FEM is a numerical method with powerful technique for solution of complicated structural engineering problems. It most predicted accurately the bridge behaviour under truck axle load. Quaqish (2005) presented the effect of skew angle on distribution of bending moments in bridge slabs using FEM, and the results were very reliable.

According to Bakht and Jaeger (1985), the finite element method has a number of advantages. These include the ability to model irregular shaped bodies composed of different materials, handle general loading and different kinds of boundary conditions.

### **2.3.2 Grillage analysis**

This method is commonly used in the analysis of bridge decks. In this method, the deck is represented by an equivalent grillage of beams. The orientation of the longitudinal members should always be parallel to the free edges while the orientation of transverse members can be either parallel to the supports or orthogonal to the longitudinal beams (Maher, 2007). The method can be used to consider boundary conditions in waffle slab analysis, in which the shear force and bending moments are determined (Lee *et al*, 2006). The grillage numerical method has gained increased popularity in the static and dynamic analysis of plate structures (Zeng *et al*, 2007).

West (1973) reported a study, where 53 models and full-sized bridges were compared. The work recommended the use of grillage analysis for slabs and pseudo-slabs bridge decks. Tan *et al*. (1998) reported that the accuracy, simplicity and speed of grillage analysis make it the most suitable model for bridge analysis. It was found that the results obtained from grillage analysis compared with experiments and more rigorous method is accurate enough for design purposes (Maher, 2007).

Zeng *et al* (2007) in their work on grillage analysis of skewed bridges were able to develop some governing equations of motion for transverse and torsional vibration of the bridge deck as follows:

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + \rho A \frac{\partial^2 w(x,t)}{\partial x^2} = f(x,t) \quad \dots\dots\dots 2.1$$

$$GJ \frac{\partial^2 \theta(x,t)}{\partial x^2} + m(x,t) = I_0 \frac{\partial^2 \theta(x,t)}{\partial t^2} \quad \dots\dots\dots 2.2$$

Where:

$w(x,t)$  and  $\theta(x,t)$  are the transverse and torsional displacements

$EI$  and  $GJ$  are the flexural and torsional rigidity

$\rho$  and  $A$  are the mass density and cross sectional area,  $I_0$  is the mass moment of inertial per unit length.

$f(x,t)$  and  $m(x,t)$  are the external transverse force (including bending moment) and torsional moment per unit length, applied at the nodes.

The effective flexural or torsional rigidity of a grillage element is equivalent to the corresponding rigidity of the strip of the plate,

$$\begin{aligned} EI &= bD, \\ D &= Eh^3 / 12(1-\nu^2), \\ GJ &= bD, \end{aligned}$$

Where:

$h$  is the thickness of the plate

$\nu$  is the poison ratio, and

$b$  is the width of a strip of the plate.

The deck was idealized as a grillage, with the girders and diaphragms coinciding with certain torsion beam members. Close spacing between grillage members provides accurate results, but increase computational efforts. The optimum assembly results form a compromise between accuracy, simplicity and efficiency.

According to Jaeger and Bakht (1982), the grillage analysis has become popular because of the following reasons:

- a. It can be used in cases where the bridges exhibit complicating features such as a heavy skew, edge stiffening and deep hunches over supports.
- b. The representation of a bridge as a grillage is ideally suited to carrying out the necessary calculations associated with analysis and design on digital computer.
- c. The grillage representation is conducive to giving the designer an idea of both the structural behaviour of the bridge and the manner in which bridge load is distributed and eventually taken to the supports.

In grillage design, the central intersection point load may be used as a worst-case loading condition, especially with a larger number of beams in each direction as long as local collapse is prevented. But a point load may often move around on the grid system. In such a case, the worst load point would not necessarily be at the central point. In this case, the worst load point is located between intersections (Ki-Sung *et al*, 2001).

Recently, Gordon and May (2004), showed that under certain conditions, the grillage analysis of slabs can give incorrect results, so that the use of FEM was preferred in their study. Grillage analysis was applied to 3 slabs: a square plate simply supported on four sides; a rectangular plate simply supported on two sides; and a skew plate simply supported on two sides. The results were compared with those obtained using a FEM software package (LUSAS), and where available, with theoretical



solutions. In certain cases the grillage results exhibit significant errors, and the situation was not improved by a local refinements. It was concluded that the use of finite element method (FEM) is to be preferred, and that the use of the grillage method should be avoided.

### 2.3.3 The Plate theory

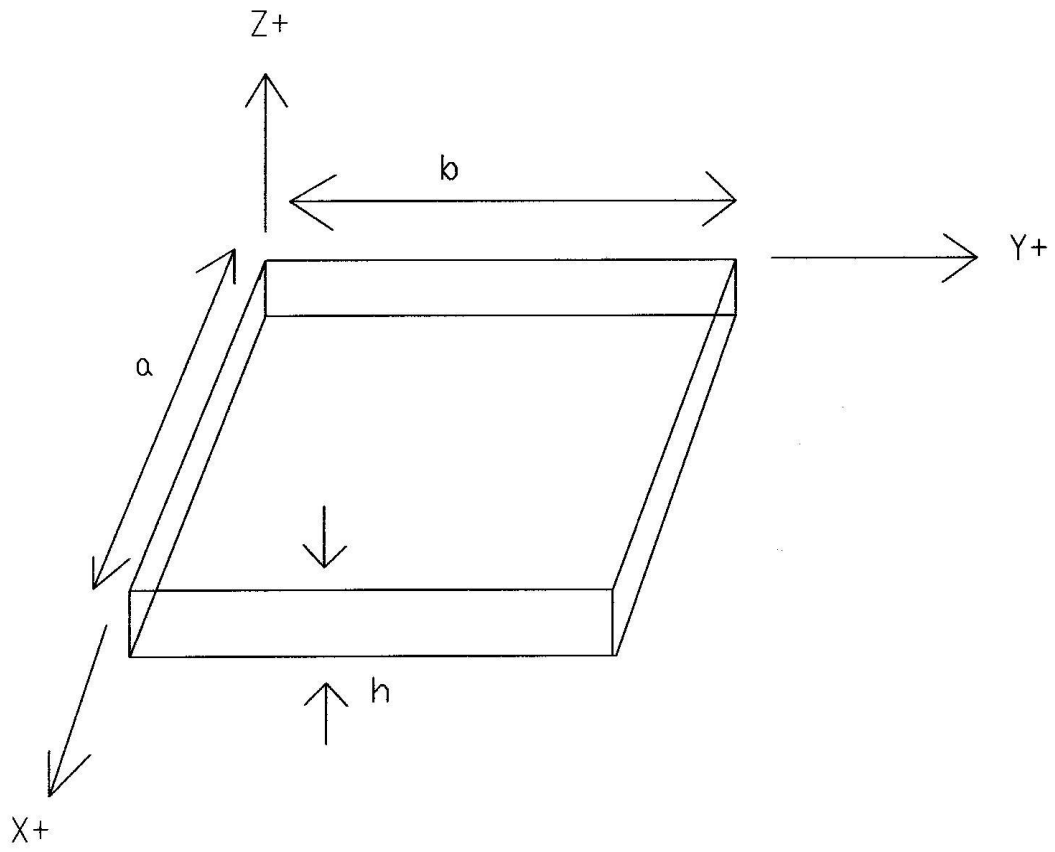
The plate theory is another method of analysing complex engineering problems especially thin walled plate structures. This method involves the use of solution in the form of simultaneous differential equations of the fourth order. Some assumptions are made in the use of the plate theory in analysing thin walled structures.

From Fig.2.1, the assumptions are:

- a. linear element of the plate extending through the plate thickness, normal to the mid surface, x-y plane, in the unstressed state, upon the application of load:
  - i. Undergoes at most a translation and a rotation with respect to the original coordinate system.
  - ii. Remains normal to the deformed middle surface.
- b. A plate resists lateral and in-plane loads by bending, transverse shear stresses, and in-plane action, not through block like compression or tension in the thickness direction. This assumption results from the fact that  $h/a \ll 1$  and  $h/b \ll 1$ .

From (ai), the following implied:

- c. A linear element through the thickness does not elongate or contract.
- d. The linear element remains straight upon load application.
- e. A normal stress in the direction normal to the mid surface is negligibly small and such can be discarded. (i.e.  $\partial_z = 0$ ). (Vinson, 1989).



**Figure 2.1: Plate Structure**

According to Murray (1986), flat plates which are stiffened to different degrees in orthogonal directions (waffle slabs) behave like orthotropic plates, the theory of which was developed by Gening in 1860 and Boussinesq in 1879.

For transverse load Y, the governing equation is:

$$D_x \frac{\partial^4 \omega}{\partial x^4} + 2H \frac{\partial^4 \omega}{\partial x^2 \partial z^2} + D_z \frac{\partial^4 \omega}{\partial z^4} = Y \quad \dots\dots\dots 2.3$$

It was developed by Huber in 1914, and known as the “Huber’s equation”

Where:

$D_x = (EI)_x / (1 - \nu_x \nu_z)$  = average flexural rigidity of the stiffened plate under bending moment,  $M_x$ .

$D_z = (EI)_z / (1 - \nu_x \nu_z)$  = average flexural rigidity of the stiffened plate under bending moment,  $M_z$ .

$$H = \frac{1}{2}(\nu_x D_z + \nu_z D_x) + 2(GI)_{xz}$$

$$2(GI)_{xz} = \frac{\partial^2 \omega}{\partial x \partial z} / M_{xz} = \text{average torsional rigidity} = G_{xz} t^3 / 12$$

$\nu_x \nu_z$  = Poisson’s ratio in the X- and Z- directions.

$E = \sqrt{(E_x E_z)}$  = modified Young’s modulus

$$G_{xz} \approx E / 2 \left( 1 + \sqrt{\nu_x \nu_z} \right) = \text{modified shear modulus.}$$

Huber applied these equations to the analysis of a reinforced concrete slab stiffened by orthogonal ribs (waffle slabs).

Jamal (1998), when studying the effects of shear reinforcements in rib stiffened (waffle) concrete slab, used the orthotropic plate theory to analyse the structures.

This theory assumes that the orthotropy of the structures may be replaced by the orthotropy of the constituent material. Although the actual structural behaviour of a stiffened slab cannot be entirely replaced by that of an equivalent orthotropic slab, previous theoretical and experimental investigations indicated good agreement (El-Sebakhly, 1979).

Abdul-Wahab and Khalil (2000) tested eight large-scale (1/4 scale) models of reinforced concrete waffle slabs with varied rib spacing and rib depth to failure. The results for the rigidities in the elastic-uncracked and elastic-cracked ranges were compared with predicted values obtained from three different methods which included the orthotropic plate theory. The theoretical analysis based on the conventional orthotropic plate theory gave satisfactory predictions but involved an elaborate procedure for determining the torsional rigidities.

## **2.4 Computer Programs**

There have been different computer programs that were developed by different researchers for the analysis of slabs of different shapes and configurations. It has been discovered that most of these programs, except very few, adopted the FEM of analysis of structures. Few of these programs are discussed here especially those that are applicable to the analysis of waffle slabs.

### **2.4.1 Fortran 77**

In their study of the determination of the optimum dimensions of waffle slabs for medium size floors, Prasad *et al* (2005) adopted the computer program for grid analysis, written in Fortran 77 (Formula Translation). Before adopting the said program, waffle slabs were considered as made of grid or grillage beams. The loads

were distributed between longitudinal beams by bending and twisting of transverse beams. The stiffness matrix is developed on the basis of writing joint equilibrium in terms of stiffness co-efficients and unknown joint displacements. Straight members of constant cross-section were considered. The deformations considered were two orthogonal rotations in the horizontal plane and a vertical deflection at each node. Nodal displacements in the horizontal plane and rotations along the vertical axis were not considered keeping in view that they did not significantly contribute to the structural behaviour and hence were ignored.

The computer analysis resulted in determining the moment, shear force and torsion for each of the elements and deflection and rotation about the two orthogonal axes at each of the nodes.

#### **2.4.2 SAP 2000**

Structural Analysis Program (SAP) is another computer based program that was used to analyze both steel and concrete structures. Abdel-Karim and Mahmood (2006) used SAP 2000 program to analyze two-way ribbed simply supported rectangular waffle slab models, supported on beams of different stiffnesses. The analysis was done to determine the moment and shear distribution in the ribs and beams in each direction, to study the effect of the panel aspect ratio and the beam stiffnesses on the distribution of moment and shear in each reaction.

The model was a grid system and the ribs were 0.5 m apart in each direction. The slab rested on four pin supports at the corners. The used load was  $2 \text{ kN/m}^2$ . The results of the analysis gave the shear force and moment values at each joint from which the load factors could be calculated using empirical equations such as:

$$M_a = C_a w l_a^2 / 8 \quad \dots\dots\dots 2.4$$

Where:

$C_a$  = *moment load factor in certain direction.*

$W$  = *uniform load per metre square.*

$l_a$  = *span length.*

From the results of analysis, it was discovered that the distribution of moments and shear in each direction depended on the panel aspect ratio and the perimeter beam stiffnesses. In actual fact, these factors depended on the relative stiffnesses of the perimeter beams to the slab or ribs stiffnesses and not the absolute stiffnesses of the beams.

The beam shear load factors in each direction were calculated by the shear values in the beams from the results of the analysis using SAP 2000. Part of the conclusion reached was that, the results of the research could be used to calculate the moments and shears in the perimeter beams and in the ribs in each direction for the given aspect ratio; panel dimension; slab thickness, and beams dimensions for a given slab load.

### **2.4.3 Adapt Floor Program**

This program also adopts FEM for the analysis of waffle slabs. It was found to be good for the analysis and design of post-tensioned waffle slabs, in which iron rod is replaced by cables that are under tension as in pre-stressed concrete design. The cables were placed between the waffle ribs and held under tension, while wire mesh was used in the slab area (Adapt Technical Note, 2006).

#### **2.4.4 SAFIR Program**

In his work on membrane action in fire exposed concrete floor system, Lim (2003) used this program to analyse slabs that are subjected to fire.

SAFIR is a non-linear finite element program which was developed at the University of Liege, Belgium, and is based on an earlier program, CEFICOSS. It consists of thermal and structural analysis components integrated into a single program. The thermal analysis component was used to determine the temperature distributions of the structural members which were used in the structural analysis. SAFIR's structural analysis capabilities include 2D and 3D analysis of steel, concrete and composite members and can account for geometrical and material non-linearity.

#### **2.4.5 RCC Program**

The Reinforced Concrete Council (RCC) also developed a computer program that adopted the Microsoft excel package to analyse and design different civil engineering structures based on BS 8110, and the results of this program have been found to be appropriate.

#### **2.4.6 ETABS**

ETABS is a program that can greatly enhance an engineer's analysis and design capabilities for structures. Part of that power lies in an array of options and features. The other part lies on the simplicity of its use.

The basic approach for using the program is very straightforward. The user establishes grid lines, places structural objects relative to the grid lines using points, lines and areas, and assigns loads and structural properties to those structural objects. (For example, a line object can be assigned section properties; a point object can be assigned

spring properties; an area object can be assigned slab or deck properties). Analysis and design are then performed based on the structural objects and their assignments. Results are generated in graphical or tabular form that can be forwarded to a printer or to a file for use in other programs. (ETABS User guide, 2005).

## **2.5 The Yield Line Theory**

Yield line theory is an ultimate load analysis. It establishes either the moments in an element (e.g. a load) at the point of failure or at which an element will fail. Yield lines not only signify the location of maximum principal moments (where yielding occurs), but also the location of zero shears (Gohnert, 2006). The shapes and locations of Yield zone are affected by the support structures and their rigidity (Geng *et al*, 2006). The theory gives a conservative estimate of strength (Ferguson, 1965). It may be applied to many slabs both with and without beams. It can deal with openings, holes, irregular shapes and with any support condition (Punmia *et al*, 2006; Chee *et al* 2008). Yield line design is a plastic method: it is different from 'normal' elastic method (Kennedy and Goodchild, 2004). The technique requires the postulation of a kinematically admissible yield-line or fracture pattern from which the corresponding collapse load is determined through the principle of virtual displacements (Ramsay and Johnson, 1997).

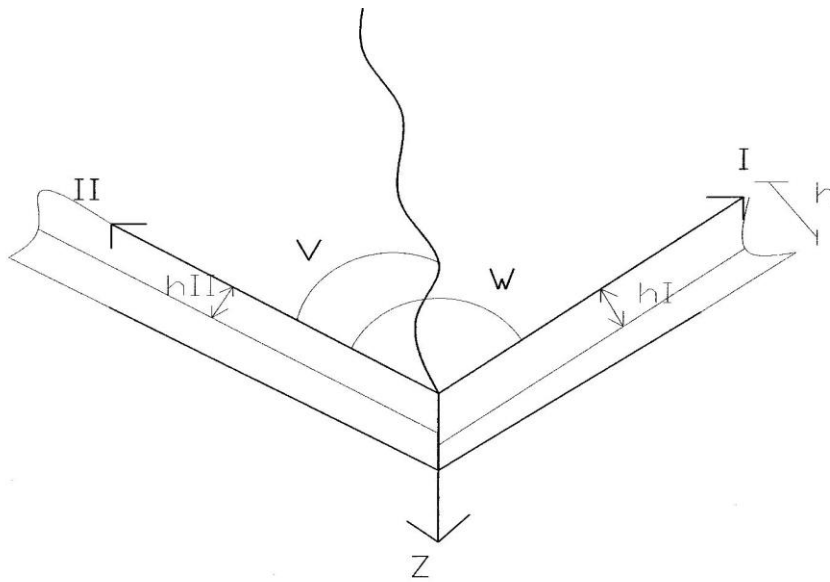
### **2.5.1 Energy dissipation**

In order to calculate the load-carrying capacity from an upper bound solution, the energy dissipation has to be known. In yield line theory, it is the energy dissipated that is used in the analysis of the slabs, because both the external energy and internal energy dissipated must be considered.



Gudmand-Høyer (2003) calculated dissipation in a yield line on the basis of Coulomb yield condition for concrete in order to verify K.W. Johansen's method. An effort was made to evaluate the error made using Johansen's proposal for orthotropic rectangular slabs and it was found that the method is sufficiently correct for practical purposes. Also, for deflected slabs that are believed to have a high load carrying capacity, it was assumed that the axis of rotation corresponds to the neutral axis of a slab part and dissipation was found from the moment capacities about these axes. The Johansen's proposal was also used to find the load carrying capacities in these cases. He compared his results with that of numerical calculations of the dissipation and generated some numerical equations for energy dissipation in slabs, based on contribution from concrete and the reinforcements. Some of the equations are shown below in conjunction with Figures 2.2, 2.3 and 2.4.

If the axes of rotation for two slab parts are not at the same depth measured from the slab surface, the relative displacement discontinuity is no longer perpendicular to the yield line. The angle between the displacement discontinuity and the yield line changes with the depth from the slab surface and this must be taken into account when calculating the dissipation (Figure 2.2).



**Figure 2.2: Yield line from the angle of a slab ( Source: Gudmand-Hoyer 2003)**

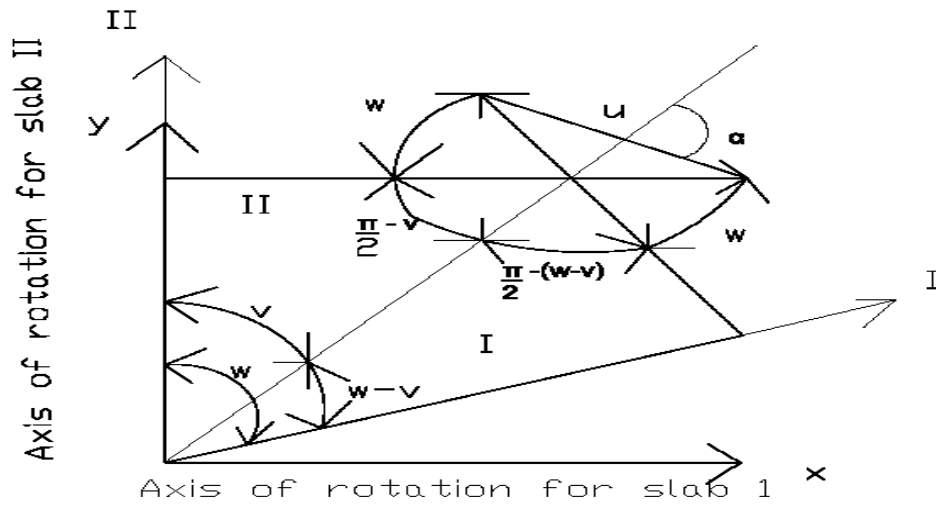


Figure 2.3: Displacement for slab parts ( Source: Gudmand-Hoyer 2003)

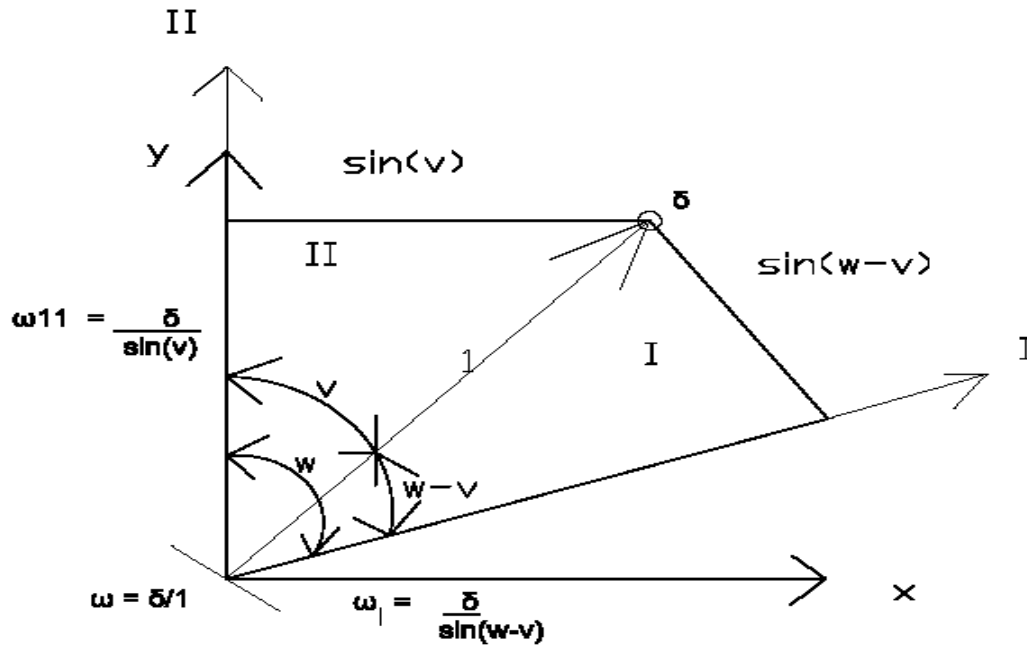


Figure 2.4: Geometry relation between rotations (Source: Gudmand-Hoyer 2003)

**2.5.2 The contribution from the concrete:**

The concrete contribution from the yield line was calculated from the dissipation formulas for plane stress assuming a modified Coulomb material. Setting the tensile strength of concrete to zero, the contribution to the dissipation (per unit length) from the concrete was calculated as:

$$W_c = \int_0^h \frac{1}{2} f_c u (1 - \sin(\alpha)) dx \quad \dots\dots\dots 2.5$$

$u$  being the relative displacement and  $\alpha$  the angle between the displacement and the yield line, Figure 2.3.

But  $u$  and  $\alpha$  depend on  $z$ , which is the depth from the top surface to the point considered.  $u_I$  and  $u_{II}$  are the displacements of slab part I and II, respectively,  $u$  was calculated as:

$$u = \sqrt{u_I^2 + u_{II}^2 + 2u_I u_{II} \cos(w)} \quad \dots\dots\dots 2.6$$

The angle between the displacement and the yield line  $\alpha$  varies with respect to  $u_I$  and  $u_{II}$  depending on whether they are positive or negative.

The relationship between the two rotations about  $I$  and  $II$  were found from the geometrical conditions demanding the same displacement at a point of yield line.

From Figure 2.4, the rotations may be calculated as:

$$\omega_I = \frac{\omega}{\sin(w-v)}$$

..... 2.7

$$\omega_{II} = \frac{\omega}{\sin(v)}$$

Here  $\omega$  is the rotation of slab part line about an axis along the yield line.

In the calculation of the displacement, it was assumed that the rotation is small and the displacement may therefore be calculated as the product of rotation and the height.

The displacements  $u_I$ ,  $u_{II}$  and  $u$  was calculated as:

$$u_I = h \frac{\omega}{\sin(w-v)} \left( \frac{h_I}{h} - \frac{z}{h} \right) \Leftrightarrow$$

..... 2.8

$$\frac{u_I}{h\omega} = \frac{1}{\sin(w-v)} \left( \frac{h_I}{h} - \frac{z}{h} \right)$$

$$u_{II} = h \frac{\omega}{\sin(v)} \left( \frac{h_{II}}{h} - \frac{z}{h} \right) \Leftrightarrow$$

..... 2.9

$$\frac{u_{II}}{h\omega} = \frac{1}{\sin(v)} \left( \frac{h_{II}}{h} - \frac{z}{h} \right)$$

Inserting (2.8) and (2.9) into (2.6) leads to:

$$u = \sqrt{\left(h \frac{\omega}{\sin(w-v)} \left(\frac{h_I}{h} - \frac{z}{h}\right)\right)^2 + \left(h \frac{\omega}{\sin(v)} \left(\frac{h_{II}}{h} - \frac{z}{h}\right)\right)^2} \Leftrightarrow$$

$$+ 2 \left(h \frac{\omega}{\sin(w-v)} \left(\frac{h_I}{h} - \frac{z}{h}\right)\right) \left(h \frac{\omega}{\sin(v)} \left(\frac{h_{II}}{h} - \frac{z}{h}\right)\right) \cos(w)$$

.....2.10

$$\frac{u}{h\omega} = \sqrt{\left(\frac{\left(\frac{h_I}{h} - \frac{z}{h}\right)}{\sin(w-v)}\right)^2 + \left(\frac{\left(\frac{h_{II}}{h} - \frac{z}{h}\right)}{\sin(v)}\right)^2 + \frac{2\left(\frac{h_I}{h} - \frac{z}{h}\right)\left(\frac{h_{II}}{h} - \frac{z}{h}\right)\cos(w)}{\sin(w-v)\sin(v)}}$$

The angle  $\alpha$  varies and if the situation in which:

( $u_I > 0$  &  $u_{II} > 0$ ): is considered, then  $\alpha$  can be calculated as:

$$\alpha = -\left(\frac{\pi}{2} + v - \text{Arc cos}\left(\frac{u^2 + u_{II}^2 - u_I^2}{2u_{II}u}\right)\right)$$

.....2.11

It is seen that the contribution to the dissipation from the concrete is a function of both the position of the axis of rotation  $h_I$ ,  $h_{II}$ , and  $h$ , the rotation  $\omega$  and the compressive strength,  $f_c$ . The dissipation may be calculated in a dimensionless form as:

$$\frac{W_c}{h^2 \omega f_c} = \int_0^1 \frac{1}{2} \frac{u}{h} (1 - \sin(\alpha)) d \frac{z}{h}$$

..... 2.12

### 2.5.3 Contribution from reinforcement

Gudmand-Høyer went further to determine the contribution from the reinforcement, based on Figure 2.5. If the reinforcement is placed in a direction perpendicular to the axis of rotation at a distance from the slab surface as shown in Figure 2.5, the contribution from the reinforcement to the dissipation per unit length becomes:

$$W_s = \omega_I \cos(w - v) (A_{s,I} |h - h_{cl} - h_I| + A_{s,I} |h_I - h_{cl}|) + \omega_2 \cos(v) (A_{s,II} |h - h_{cII} - h_{II}| + A_{s,II} |h_{II} - h_{cII}|) \quad \dots\dots\dots 2.13$$

Where  $\omega_I$  and  $\omega_{II}$  are the rotations about axis I and II, respectively. These are determined in Equation 2.6 and the expression may be written as:

$$\frac{W_s}{\omega h^2 f_c} = \frac{1}{\tan(w - v)} \left( \Phi_{0,I} \left| 1 - \frac{h_{cl}}{h} - \frac{h_I}{h} \right| + \Phi_{0,I} \left| \frac{h_I}{h} - \frac{h_{cl}}{h} \right| \right) + \frac{1}{\tan(v)} \left( \Phi_{0,II} \left| 1 - \frac{h_{cII}}{h} - \frac{h_{II}}{h} \right| + \Phi_{0,II} \left| \frac{h_{II}}{h} - \frac{h_{cII}}{h} \right| \right) \quad \dots\dots\dots 2.14$$

If the corner is right-angled the dissipation becomes:

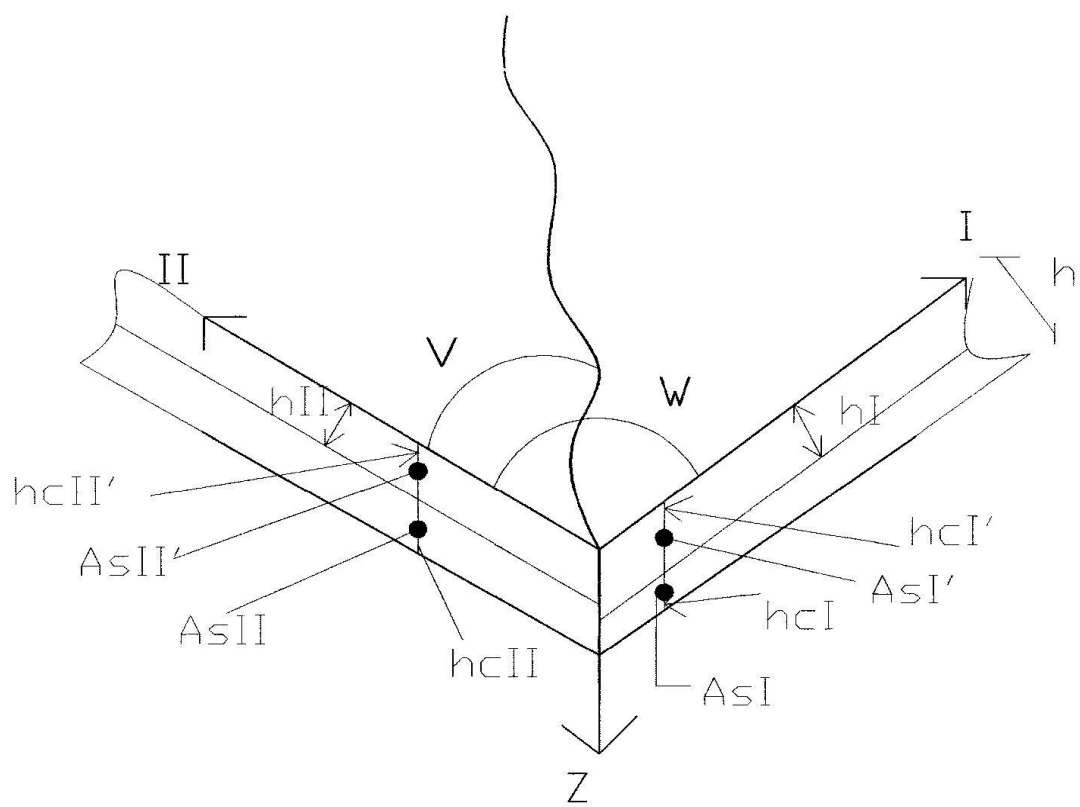
$$\frac{W_s}{\omega h^2 f_c} = \tan(v) \left( \Phi_{0,I} \left| 1 - \frac{h_{cl}}{h} - \frac{h_I}{h} \right| + \Phi_{0,I} \left| \frac{h_I}{h} - \frac{h_{cl}}{h} \right| \right) + \frac{1}{\tan(v)} \left( \Phi_{0,II} \left| 1 - \frac{h_{cl}}{h} - \frac{h_{II}}{h} \right| + \Phi_{0,II} \left| \frac{h_{II}}{h} - \frac{h_{cII}}{h} \right| \right) \quad \dots\dots\dots 2.15$$

Finally, the results from the theoretical equations derived were compared with those obtained from laboratory test specimens, where some slabs were loaded to failure.



It was discovered that the theoretical load – carrying capacity was too high. This phenomenon was ascribed to the fact that concrete does not behave entirely according to plastic theory and an effectiveness factor was proposed to make the theory reliable.

Beside the load-carrying capacity, it was discovered that the deflection was wrong. Not only wrong when it came to numerical value, but also when it came to the relationship between axial force and deflection. However, the comparison with test results showed that the theory developed may be used if the deflection at failure is known and a proper effectiveness factor is introduced. But if deflection at failure is not known, a conservative simplified method, which will lead to a large underestimation for low axial forces was proposed, the method was therefore recommended as rough estimate.



**Figure 2.5: Reinforcement arrangement**

## 2.6 Isotropic and Orthotropic Slabs

The arrangement of reinforcements in slabs has given rise to both isotropic and orthotropic slabs. An isotropic slab is one with the same amount of bottom reinforcement both ways. This is because the yield moments on both ways are equal, and by assuming equal effective depth, this type of slab is easy to deal with when using the yield line analysis and design method. Orthotropic slabs have different amounts of reinforcements in the two directions; in this case, the yield moments in both axes are mutually perpendicular to each other (Nilson, 1997; Buyukozturk, 2004).

According to Kennedy and Goodchild (2004), the analysis of such orthotropic slabs can be done using the affine transformation. In these, the stronger direction is assumed to have the moment capacity,  $M$ , and in the weaker direction the capacity of the slab is assumed to be  $\mu M$ . The value of  $\mu$  is usually based on the relative amounts of reinforcement the designer wishes to use in the two directions. The use of  $\mu$  will help to transform the orthotropic slabs to isotropic slabs and can be treated as such, applying all the usual formulae and methods.

## 2.7 Definition of Upper Bound and Lower Bound Theorem

- i. The lower bound of the true collapse load is that external load for which a distribution of moments can be found satisfying the requirements of equilibrium and boundary conditions so that the moments at any location do not exceed the yield moment.
- ii. The upper bound of the true collapse load is that external load for which the internal work done by the slab for a small increment of displacement is equal to the

external work done by that external load for the same amount of small increment of displacement.

Thus, the collapse load satisfying the lower bound theorem is always lower than or equal to the true collapse load. On the other hand, the collapse load satisfying the upper bound theorem is always higher than or equal to the true collapse load. The yield line analysis is an upper bound method in which the predicted failure load of a slab for given moment of resistance (capacity) may be higher than the true value. Thus, the solution of the upper bound method (yield line analysis) may result into unsafe design if the lowest mechanism could not be chosen. However, it has been observed that the prediction of the most probable true mechanism in slab is not difficult. Thus, the solution is safe and adequate in most of the cases (Kharagpur, 2009).

## **2.8 Difference in Load Distribution in Waffle Slabs and Solid Slabs**

Load distribution in waffle slabs and solid slabs are different due to the orientation of these slabs. Waffle slabs have hollows in between the slabs, while solid slabs have no hollow in between. The arrangement of reinforcements in both slabs is different as a result of the hollow in the waffle slabs. Due to the arrangements of reinforcement and the span ratio of solid slab, there is possibility for one-way slab, however, waffle slabs do not have a one-way slab because of the reinforcement orientation and the presence of ribs within the slab. In a typical one-way slab, the floor comprises beams spanning in one direction between columns and a slab spanning between them. The system is designated as one-way slab because all loads in the slab are transferred primarily in one direction (to the beams), since the transverse slab span is infinite.

For a typical two-way slab, the floor comprises beams spanning both directions between columns, and a slab framing between the beams. The system is designated two-way because, in a situation where the aspect ratio of the slab has a value near unity, the slab transfers load to the beams in two directions. This type of floor system is generally quite efficient.

### **2.8.1 Load path designation in waffle slabs**

In waffle slabs, loads are transferred from the slabs to the ribs and to the perimeter beams. Abdel-Karim and Mahmood (2006), when investigating the effect of beam stiffnesses on the load distribution in waffle slabs, was able to discover that the distribution of moments along the parallel ribs in the short and long directions depends, on the panel aspect ratio, the relative stiffness of the slab ribs and on the beams on which these ribs are supported.

From the model analysis of slabs, using SAP2000, the following observations were made. It was noticed that, as the beam stiffness was increased, the shear load factor in the short direction increased for a specified panel aspect ratio. Also, as the panel aspect ratio was increased, the beam shear load factor increased in the short direction and decreased in the long direction. It was also observed that moments varied in each direction along the panel width. It was seen that edge strip ribs had larger moments than the middle strip ribs for beams with small cross section (moment of inertia: stiffness) and the middle strip ribs had larger moments than the edge strip ribs for larger beams cross section (moment of inertia: stiffness). Hence, the ribs gave different bending moments and shear forces in each direction as a result of varying stiffnesses. The orientation and the amount of the reinforcement provided depended on

the bending moments (the function of the reinforcements is to resist the bending moment).

### **2.8.2 Load path designation in solid slabs**

Reinforced concrete is very unique in its behaviour, and this has made it popular as construction material. In solid slabs; at flexural failure, concrete slabs develop hinge lines. A hinge line causes much of the reinforcement passing through it to resist the moment along its length, contributing to the safety of the slab.

Once a slab has cracked, the reinforcement determines the manner in which the applied load is resisted; it is the orientation and the amount of reinforcement that govern the path that the load takes to the supports (Aalami, 2005).

Prior to the calculation of the design moments and shears, the first thing that must be considered is to anticipate the load path, which sets the orientation and position of the reinforcement. Sivagamasundari and Kumara, 2008, opined that the major work of the longitudinal reinforcement is to provide flexural strength for the concrete slab. For example, in a solid two-way slab, the function of the distribution bar is to distribute the load from the slab to the bottom or main bar, while the bottom bar will distribute the load to the supports at the edges of the slab, both the distribution and main bars are designed for in this type of slab. The amount of bending moment in each direction will depend on the ratio of the two spans and the condition of restraint at each support (Mosley and Bungey, 1990), while in one-way slab it is only the main bars that are designed, although appropriate provision is made for distribution bar in this type of slab. Top (torsion) reinforcement is provided at the supports or edges of slabs to prevent cracks as concrete is known to be weak in tension (BS 8110, 1997).

## CHAPTER THREE

### METHODOLOGY

#### 3.1 Preamble

Mathematical equations were imputed into the developed computer program and used to analyse developed models, while basic experimentation was carried out to determine, the deflection, crack width and the ultimate failure of the various types of slab models.

#### 3.2 Mathematical Formulation for the Experimental Studies

The bending moment formula based on the yield line theory was derived as given below. These formulae have been taken from the solutions given by Johansen in 1943 and 1968. The solution below is for a one-way simply supported slab on two sides, using the work method of analysis (Figure 3.1)

##### 3.2.1 Analysis of Slabs adopting Yield Line Theory

Assuming:

*Internal work done, E = External work done, D*

$$\sum(N \times \delta) = \sum(m \times l \times \theta)$$

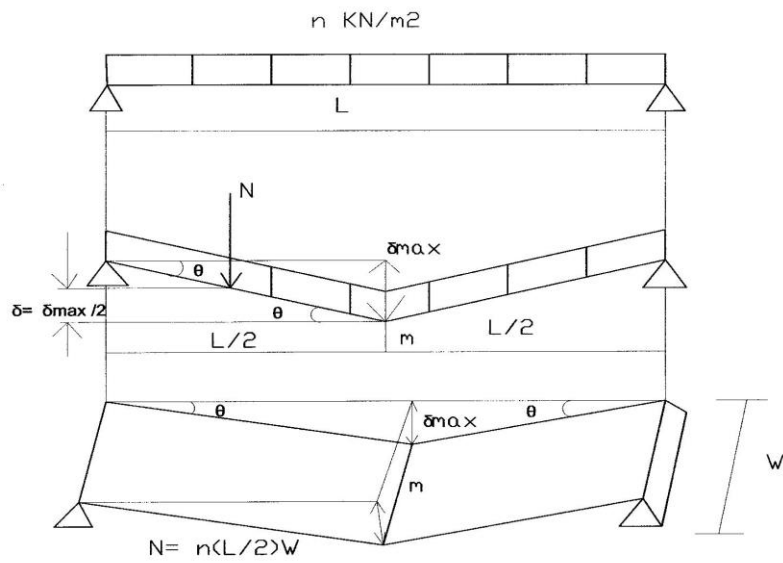
$$2 \times n \times \frac{L}{2} \times w \times \frac{\delta_{\max}}{2} = 2 \times m \times l \times \theta$$

Here, the length of the projected yield line, 'l', onto the axis of rotation is w.

Also  $\theta$ , equate to  $\tan \theta = \delta_{\max} / (L/2)$ .

Therefore:

$$\frac{2nLw}{2} \times \frac{\delta_{\max}}{2} = 2mw \times \frac{\delta_{\max}}{L/2}$$



**Figure 3.1: Simply supported one-way slab**



$$\therefore \frac{2nL}{4} = \frac{4m}{L}$$

Rearranging

$$m = nL^2/8 \quad \dots\dots\dots 3.1$$

For two way slabs, the equation can be developed using Fig 3.2, as follows,

(Kong and Evans, 1987):

Assuming the slab measures b and L.

$$E = D$$

$$\sum(N \times \delta) = \sum(m \times L \times \theta)$$

E = external energy.

$$E = N \left[ 2 \times \frac{1}{3} b \alpha L + \frac{1}{2} b (1 - 2\alpha) L \right]$$

$$= \frac{NbL}{6} (3 - 2\alpha) \quad \dots\dots\dots 3.2$$

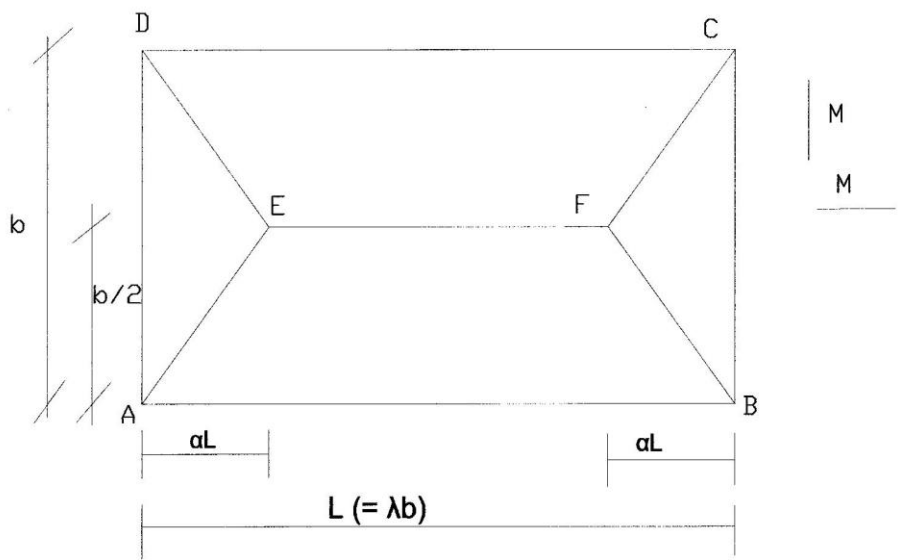
D= internal energy.

Energy dissipation for yield line AE

$$= m \left[ \frac{\alpha L}{b/2} + \frac{b/2}{\alpha L} \right]$$

Energy dissipation for yield line EF

$$= m \left[ 2 \times \frac{(1-2\alpha)L}{b/2} \right]$$



**Figure 3.2: Yield lines on two-way simply supported slab**

Therefore, total energy dissipation for yield lines AE, DE, BF, CF, and EF

$$\begin{aligned}
 &= 4m \left[ \frac{\alpha L}{b/2} + \frac{b/2}{\alpha L} \right] + 2m \frac{(1-2\alpha)L}{b/2} \\
 &= \frac{2m(1+2\lambda^2\alpha)}{\lambda\alpha} \quad (\text{Where } \lambda = l/b) \dots\dots\dots 3.3
 \end{aligned}$$

Equating (3.2) and (3.3)

$$D = E$$

$$\begin{aligned}
 \frac{2m(1+2\lambda^2\alpha)}{\lambda\alpha} &= \frac{NbL}{6} (3-2\alpha) \\
 m &= \frac{NL^2}{12} \cdot \frac{(3-2\alpha)\alpha}{1+2\lambda^2\alpha} \dots\dots\dots 3.4
 \end{aligned}$$

Equations 3.1 and 3.4 have been simplified by Kennedy and Goodchild (2004), to accommodate different boundary conditions expected in a slab. Equation 3.1 was simplified to 3.5, while 3.4 became 3.6. Furthermore, equation 3.6 has been modified in this work to equation 3.7, in order to take care of the conservative nature of yield line theory in solid slabs, (Akinyele 2011). Equations 3.5 and 3.7 were therefore adopted in the computer program in Tables 3.1, 3.2 and 3.3.

$$m = \frac{nL^2}{2(\sqrt{1+i_1} + \sqrt{1+i_2})^2} \dots\dots\dots 3.5$$

$$m = \frac{n \times a_r \times b_r}{8 \left( 1 + \frac{b_r}{a_r} + \frac{a_r}{b_r} \right)} \dots\dots\dots 3.6$$

$$m = \frac{n \times a_r \times b_r}{8 \left( 1 + \frac{b_r}{a_r} + \frac{a_r}{b_r} \right)} (k + v) \dots\dots\dots 3.7$$

Where:

N = load acting within a particular region (kN)

$\delta$  = vertical displacement of the load N on each region expressed as a fraction of  
Unity (m)

m = ultimate design moment of resistant for the slab (kNm)

L = length of slab (m)

$\theta$  = rotation of the region about its axis of rotation (m/m)

n = the ultimate uniformly distributed load (kN/m<sup>2</sup>)

$a_r$  = reduced short span dimension

$b_r$  = reduced long span dimension

k = is a constant = 1

v = is a variable =  $0 \leq 0.5$

i = the fixity ratio at supports, e.g  $i_1, i_2$ .

If :  $i_1 = i_2 = 1$ , that support is a continuous support

And  $i_1 = i_2 = 0$  that support is a simple support.

**Table 3.1: Template for the computer analysis and design of simply supported waffle slab adopting the yield line theory.**

| References  | Calculation/output                           |
|---|--|
| 1 ANALYSIS AND DESIGN OF WAFFLE<br>SLABS USING THE YIELD LINE<br>THEORY |  |
| 2 CODE BS. 8110 PARTS 1 1997.   |  |
| 3 Designed by: AKINYELE J.O   |  |
| 4 Simply supported on all Four Sides                                    |  |
| 5 Slab Long Span (ly) (m)   | 0.00   |
| 6 Slab Short Span (lx) (m)  | 0.00   |
| 7 Rib Width (mm)  | 0.00   |
| 8 Distance between ribs on Long Span (m)                                | 0.00   |
| 9 Distance between ribs on Short Span (m)                               | 0.00   |
| 10 Depth of Slab portion (mm)   | 0.00   |
| 11 Depth of ribs (mm)   | 0.00   |
| 12 Total depth of Slab (mm)   | =SUM(H10,H11)                                |
| 13 Effective depth of slab (mm)   | =H10-H29                                     |
| 14 Slab dead load (kN/m <sup>2</sup> )                                  | =H10*24*10 <sup>-3</sup> *1.4                |
| 15 Imposed load on Slab (kN/m <sup>2</sup> )                            | 0.00   |
| 16 Finishes and partition loads   | 0.00   |
| 17 Total load on slab (kN/m <sup>2</sup> )                              | = SUM(H14:H15:H16)                           |
| 18 For discontinuous supports, fixity                                   | 0.00   |
| 19 For continuous supports on any side, fixity                          | 1  |
| 20 Fixity ratio for short span left. I <sub>1</sub>                     | 0.00   |
| 21 Fixity ratio for long span bottom. I <sub>2</sub>                    | 0.00   |
| 22 Fixity ratio for short span right. I <sub>3</sub>                    | 0.00   |
| 23 Fixity ratio for long span top. I <sub>4</sub>                       | 0.00   |
| 24 Reduced side a <sub>r</sub>  | =2*H6/(SQRT(1+H21)+SQRT(1+H23))              |
| 25 Reduced side b <sub>r</sub>  | =2*H5/(SQRT(1+H20)+SQRT(1+H22))              |
| 26 Bending moment in slab (kNm)   | =1.0*H17*H24*H25/(8*(1+(H25/H24)+(H24/H25))) |
| 27  |  |
| 28 DESIGN   |  |
| 29 Cover to reinforcement (mm)  | 0.00   |
| 30 Concrete characteristic strength (N/mm <sup>2</sup> )                | 0.00   |
| 31 Steel characteristic strength (N/mm <sup>2</sup> )                   | 0.00   |
| 32 Maximum lever arm factor   | 0.95   |

---

|    |  |                         |
|----|--|-------------------------|
| 33 | Maximum K-value                                    | 0.156                   |
| 34 | Moment M   | =H26*1.0E06             |
| 35 | K-value  | =H34/(H30*1.0E03*H13^2) |
| 36 | Lever arm factor                                   | =0.5+SQRT(0.25-H35/0.9) |
| 37 | Used Lever arm                                     | =MIN(H32,H36)           |
| 38 | Area of Steel (mm <sup>2</sup> )                   | =H34/(0.95*H31*H37*H13) |
| 39 | PROVIDE=   |                         |
| 40 | ANALYSIS & DESIGN OF RIBS                          |                         |
| 41 | Adopting Rankine Grashoff theory                   |                         |
| 42 | Number of ribs on long span                        | 0.00                    |
| 43 | Number of ribs on short span                       | 0.00                    |
| 44 | Weight of slab (kN/m <sup>2</sup> )                | =H17                    |
| 45 | Total load on slab (kN)                            | =H44*H5*H6              |
| 46 | Weight of ribs kN/m                                | =(H7*H11)*1.0E-06*24    |
| 47 | Total weight of ribs on long span (kN)             | =H42*H46*H5             |
| 48 | Total weight of ribs on short span (kN)            | =H43*H46*H6             |
| 49 | Total weight of Finishes (kN)                      | =H5*H6*1.0              |
| 50 | Total Live load (kN)                               | =H5*H6*1.5              |
| 51 | Total load on grid floor (kN)                      | =H45+H47+H48+H49+H50    |
| 52 | Q= load per area (W2)                              | =H51/(H5*H6)            |
| 53 | Denominator for Qx and Qy                          | =SUM(H5^4,H6^4)         |
| 54 | Load sheared on short span Qx (kN/m <sup>2</sup> ) | =H52*(H5^4/H53)         |
| 55 | Load sheared on long span Qy (kN/m <sup>2</sup> )  | =H52*(H6^4/H53)         |
| 56 | Moment on short span Mx (kNm)                      | =(H54*H9*H6^2)/8        |
| 57 | Moment on Long Span My (kNm)                       | =(H55*H8*H5^2)/8        |
| 58 | Shear force on short span Vx (kN)                  | =(H54*H8*H6)/2          |
| 59 | Shear force on long span Vy (kN)                   | =(H55*H9*H5)/2          |
| 60 | DESIGN   |                         |
| 61 | Steel characteristic strength (N/mm <sup>2</sup> ) | 0.00                    |
| 62 | Short span design moment (kNm)                     | =H56*1.0E06             |
| 63 | Cover to ribs                                      | 0.00                    |
| 64 | Effective depth of ribs                            | =H11-H63                |
| 65 | K-value  | =H62/(H30*H7*H64^2)     |
| 66 | Lever arm factor                                   | =0.5+SQRT(0.25-H65/0.9) |
| 67 | Used lever arm                                     | =MIN(H66,H32)           |
| 68 | Area of steel (mm <sup>2</sup> )                   | =H62/(0.95*H61*H67*H64) |
| 69 | PROVIDE  |                         |

---

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|    |                                  |                         |
|----|----------------------------------|-------------------------|
| 70 | Long span design moment (kNm)    | =H57*1.0E06             |
| 71 | K-value                          | =H70/(H30*H7*H64^2)     |
| 72 | Lever arm factors                | =0.5+SQRT(0.25-H71/0.9) |
| 73 | Used laver arm                   | =MIN(H72,H32)           |
| 74 | Area of steel (mm <sup>2</sup> ) | H70/(0.95*H61*H73*H64)  |
| 75 | PROVIDE                          |                         |
| 76 | END DESIGN                       |                         |

---

**Table 3.2: Template for the computer analysis of simply supported waffle slab adopting the yield line theory (3 sides simply supported)**

| References   | Calculation / output                            |
|--|---|
| 1 ANALYSIS AND DESIGN OF WAFFLE SLAB USING THE YIELD LINE THEORY |   |
| 2 CODE B.S. 8110 PART 1 1997.                                    |   |
| 3 Designed by: Akinyele J.O                                      |   |
| 4 Supported on 3-sides   |   |
| 5 Slab long Span (ly) (m)  | 0.00  |
| 6 Slab short Span (lx) (m)                                       | 0.00  |
| 7 Depth of slab part (mm)  | 0.00  |
| 8 Distance between ribs on long span (m)                         | 0.00  |
| 9 Distance between ribs on short span (m)                        | 0.00  |
| 10 Rib width (mm)  | 0.00  |
| 11 Depth of ribs (mm)  | 0.00  |
| 12 Total depth of slab (mm)                                      | =SUM(H7,H11)                                    |
| 13 Cover to reinforcements (mm)                                  | 0.00  |
| 14 Effective depth of slab (mm)                                  | =H7-H13   |
| 15 Slab Dead load (kN/m <sup>2</sup> )                           | =H7*24*10 <sup>-3</sup> *1.4                    |
| 16 Slab Imposed load (kN/m <sup>2</sup> )                        | 0.00  |
| 17 Finishes and Partition loads                                  | 0.00  |
| 18 Total load on Slab (kN/m <sup>2</sup> )                       | =SUM(H15:H16:H17)                               |
| 19 CASE 1 = h <sub>1</sub> + h <sub>2</sub> < b                  |   |
| 20 Fixity for 1 <sup>st</sup> short Span I <sub>1</sub>          | 0.00  |
| 21 Fixity for supported long span I <sub>2</sub>                 | 0.00  |
| 22 Fixity for 2 <sup>nd</sup> short span I <sub>3</sub>          | 0.00  |
| 23 Reduced span b <sub>r</sub>                                   | =2*H5/(SQRT(1+H20)+SQRT(1+H22))                 |
| 24 K' = redistribution factor                                    | =2*H6/(3*H23)                                   |
| 25 .h = factor to determine yield length                         | =H6/H24+SQRT(H24^2+(H21+H23+H24/2*H6)+1)        |
| 26 Bending moments in slab (kN.m)                                | =1.0*(H18*H6*H23)/8*(((H21*H23)/4*H6)+(H6/H25)) |
| 27 CASE 2 = h <sub>1</sub> + h <sub>2</sub> = b                  |   |
| 28 .b' = dimensions  | =2*H6/SQRT(1+H21)                               |
| 29 Bending moments in slab (kN.m)                                | =1.0*(H18*H23*H28)/8*(1+(H28/H23)+(H23/H28))    |
| 30 Design  |   |



---

|    |  |                         |
|----|--|-------------------------|
| 31 | Concrete characteristic strength                   | 0.00                    |
| 32 | Steel characteristic strength                      | 0.00                    |
| 33 | Maximum lever arm factor                           | 0.95                    |
| 34 | Maximum K-value                                    | 0.156                   |
| 35 | Used bending moment                                | =MAX(H26,H29)           |
| 36 | Moments M  | H35*1.0E06              |
| 37 | K-value  | =H36/(H31*1.0E03*H14^2) |
| 38 | Lever arm factor                                   | =0.5+SQRT(0.25-H37/0.9) |
| 39 | Used laver arm                                     | =MIN(H33,H38)           |
| 40 | Area of steel (mm <sup>2</sup> )                   | =H36/(0.95*H32*H39*H14) |
| 41 | Provide  |                         |
| 42 | Analysis and design of ribs                        |                         |
| 43 | ADOPTING RANKINE GRASHOFF THEORY                   |                         |
| 44 | No of ribs on long span (ly)                       | 0.00                    |
| 45 | No of ribs on short span (lx)                      | 0.00                    |
| 46 | Weight of slab (kN/m <sup>2</sup> )                | =H18                    |
| 47 | Total load on slab (kN)                            | =H46*H5*H6              |
| 48 | Weight of ribs (kN/m)                              | =(H10*H11)*1.0E-06*24   |
| 49 | Total weight of ribs on long span(kN)              | =H44*H48*H5             |
| 50 | Total weight of ribs on short span(kN)             | =H45*H48*H6             |
| 51 | Total weight of finishes (kN)                      | =H5*H6*1                |
| 52 | Total Live Load (kN)                               | =H5*H6*1.5              |
| 53 | Total load on grid floor (kN)                      | =H47+H49+H50+H51+H52    |
| 54 | .q = load per area (m <sup>2</sup> )               | =H53/(H5*H6)            |
| 55 | Denominator for qx and qy                          | =SUM(H5^4,H6^4)         |
| 56 | Load sheared on short span qx(kN/m <sup>2</sup> )  | =H54*(H5^4/H55)         |
| 57 | Load sheared on long span qy (kN/m <sup>2</sup> )  | =H54*(H6^4/H55)         |
| 58 | Moments on short span Mx (kNm)                     | =(H56*H9*H6^2)/8        |
| 59 | Moments on long span My (kNm)                      | =(H57*H8*H5^2)/8        |
| 60 | Shear force on short span Vx (kN)                  | =(H56*H8*H6)/ 2         |
| 61 | Shear force on long span Vy (kN)                   | =(H57*H9*H5)/ 2         |
| 62 | Design   |                         |
| 63 | Steel characteristic strength (N/mm <sup>2</sup> ) | 0.00                    |
| 64 | Short span design moment (kNm)                     | =H58*1.0E06             |
| 65 | Cover to ribs                                      | 0.00                    |
| 66 | Effective depth of ribs                            | =H11-H65                |

---

---

|    |                                  |                          |
|----|----------------------------------|--------------------------|
| 67 | K-value                          | =H64/(H31*H10*H66^2)     |
| 68 | Lever arm factor                 | =0.5+SQRT(0.25-H67/ 0.9) |
| 69 | Used lever arm                   | =MIN(H33,H68)            |
| 70 | Area of steel (mm <sup>2</sup> ) | =H64/(0.95*H63*H66*H69)  |
| 71 | Provide                          |                          |
| 72 | Long span design moments (kNm)   | =H59*1.0E06              |
| 73 | k-value                          | =H72/(H31*H10*H66^2)     |
| 74 | Lever arm factor                 | =0.5+SQRT(0.25-H73/ 0.9) |
| 75 | Used lever arm                   | =MIN(H74,H33)            |
| 76 | Area of steel (mm <sup>2</sup> ) | =H72/(0.95*H63*H75*H66)  |
| 77 | Provide                          |                          |
| 78 | END DESIGN                       |                          |

---

**Table 3.3: Template for computer analysis of waffle slabs, supported on two short sides, adopting the yield line theory.**

|    | References  | Calculation / output  |
|----|---|---|
| 1  | ANALYSIS AND DESIGN OF WAFFLE SLAB<br>USING THE YIELD LINE THEORY |   |
| 2  | CODE B.S. 8110 PART 1 1997.                                       |   |
| 3  | Designed by: Akinyele J.O   |   |
| 4  | Simply supported on two sides                                     |   |
| 5  | Slab long Span (ly) (m)   | 0.00  |
| 6  | Slab short Span (lx) (m)  | 0.00  |
| 7  | Depth of slab part (mm)   | 0.00  |
| 8  | Distance between ribs on long span (m)                            | 0.00  |
| 9  | Distance between ribs on short span (m)                           | 0.00  |
| 10 | Rib width (mm)  | 0.00  |
| 11 | Depth of ribs (mm)  | 0.00  |
| 12 | Total depth of slab (mm)  | =SUM(H7,H11)  |
| 13 | Cover to reinforcements (mm)                                      | 0.00  |
| 14 | Effective depth of slab (mm)                                      | =H12-H13  |
| 15 | Slab Dead load (kN/m <sup>2</sup> )                               | =H7*24*10 <sup>-3</sup> *1.4  |
| 16 | Slab Imposed load (kN/m <sup>2</sup> )                            | 0.00  |
| 17 | Finishes and Partition loads                                      | 0.00  |
| 18 | Total load on Slab (kN/m <sup>2</sup> )                           | =SUM(H15:H16:H17)   |
| 19 | Fixity for 1 <sup>st</sup> support I <sub>1</sub>                 | 0.00  |
| 20 | Fixity for 2 <sup>nd</sup> support I <sub>2</sub>                 | 0.00  |
| 21 | Bending moments in slab (kN.m)                                    | =H17*H5 <sup>2</sup> /(2*(((SQRT(1+ H19)) + (SQRT(1+H20)))) <sup>2</sup> )) |
| 22 | Design of slab  |   |
| 23 | Concrete characteristic strength                                  | 0.00  |
| 24 | Steel characteristic strength                                     | 0.00  |
| 25 | Maximum lever arm factor  | 0.95  |
| 26 | Maximum K-value   | 0.156   |
| 27 | Moments M   | =H21*1.0E06   |
| 28 | K-value   | =H27/(H23*1.0E03*H14 <sup>2</sup> )   |
| 29 | Lever arm factor  | =0.5+SQRT(0.25-H28/ 0.9)  |
| 30 | Used lever arm  | =MIN(H25,H29)   |
| 31 | Area of steel (mm <sup>2</sup> )                                  | =H27/(0.95*H24*H30*H14)   |
| 32 | Provide   |   |

---

|    |  |                         |
|----|--|-------------------------|
| 33 | ANALYSIS & DESIGN OF RIBS                          |                         |
| 34 | Adopting Rankine Grashoff theory                   |                         |
| 35 | Number of ribs on long span                        | 0.00                    |
| 36 | Number of ribs on short span                       | 0.00                    |
| 37 | Weight of slab (kN/m <sup>2</sup> )                | =H18                    |
| 38 | Total load on slab (kN)                            | =H37*H5*H6              |
| 39 | Weight of ribs kN/m                                | =(H10*H11)*0.000001*24  |
| 40 | Total weight of ribs on long span (kN)             | =H35*H39*H5             |
| 41 | Total weight of ribs on short span (kN)            | =H36*H39*H6             |
| 42 | Total weight of Finishes (kN)                      | =H5*H6*1                |
| 43 | Total Live load (kN)                               | =H5*H6*1.6              |
| 44 | Total load on grid floor (kN)                      | =H38+H40+H41+H43+H42    |
| 45 | Q= load per area (m <sup>2</sup> )                 | =H44/(H5*H6)            |
| 46 | Denominator for Qx and Qy                          | =SUM(H5^4,H6^4)         |
| 47 | Load sheared on short span Qx (kN/m <sup>2</sup> ) | =H45*(H5^4/H46)         |
| 48 | Load sheared on long span Qy (kN/m <sup>2</sup> )  | =H45*(H6^4/H46)         |
| 49 | Moment on short span Mx (kNm)                      | =(H47*H9*H6^2)/8        |
| 50 | Moment on Long Span My (kNm)                       | =(H48*H8*H5^2)/8        |
| 51 | Shear force on short span Vx (kN)                  | =(H47*H8*H6)/2          |
| 52 | Shear force on long span Vy (kN)                   | =(H48*H9*H5)/2          |
| 53 | DESIGN   |                         |
| 54 | Steel characteristic strength (N/mm <sup>2</sup> ) | 0.0                     |
| 55 | Short span design moment (kNm)                     | =H49*1.0E06             |
| 56 | Cover to ribs                                      | 0.00                    |
| 57 | Effective depth of ribs                            | =H11-H56                |
| 58 | K-value  | =H55/(H23*H10*H57^2)    |
| 59 | Lever arm factor                                   | =0.5+SQRT(0.25-H58/0.9) |
| 60 | Used lever arm                                     | =MIN(H59,H25)           |
| 61 | Area of steel (mm <sup>2</sup> )                   | =H55/(0.95*H54*H60*57)  |
| 62 | PROVIDE  |                         |
| 63 | Long span design moment (kNm)                      | =H50*1.0E06             |
| 64 | K-value  | =H63/(H23*H10*H57^2)    |
| 65 | Lever arm factors                                  | =0.5+SQRT(0.25-H64/0.9) |
| 66 | Used laver arm                                     | =MIN(H65,25)            |
| 67 | Area of steel (mm <sup>2</sup> )                   | =H63/(0.95*H54*H66*H57) |
| 68 | PROVIDE  |                         |
| 69 | END DESIGN   |                         |

---

### 3.2.2 Analysis of ribs adopting the Rankine Grashoff Formulae

The Rankine Grashoff formulae were adopted for the analysis of the rib portion of the waffle slab, the formulae are very simple and they considered both the bending moments and shear forces at the ribs which are designed as flanged sections. This formulae were developed by Grashoff. (Krishna Raju, 1988).

The parameters for the equation are as follows;

- $a_1$  and  $b_1$  = are the spacing of the ribs on the short and long span respectively.
- $q$  = total load per unit area
- $q_1$  and  $q_2$  = the load shared on the short and long span respectively
- $a$  = Shorter dimension of grid
- $b$  = Longer dimension of grid
- $M_x$  and  $M_y$  are moments on the short - and long- span ribs respectively
- $Q_x$  and  $Q_y$  are the shear forces on the short and long span ribs respectively.

Grashoff equations were derived as follows:

$$q_1 = q \left( \frac{b^4}{a^4 + b^4} \right) \text{ and } q_2 = q \left( \frac{a^4}{a^4 + b^4} \right) \dots\dots\dots 3.8$$

The bending moment were given as shown below:

$$M_x = \left( \frac{q_1 b_1 a^2}{8} \right) \text{ and } M_y = \left( \frac{q_2 a_1 b^2}{8} \right) \dots\dots\dots 3.9$$

The shear force equation was given as shown below:

$$Q_x = \left( \frac{q_1 b_1 a}{2} \right) \text{ and } Q_y = \left( \frac{q_2 a_1 b}{2} \right) \dots\dots\dots 3.10$$

### **3.3 Computer Program for the Analysis and Design of Waffle Slab.**

The computer program YLRGT for the analysis and design of waffle slab was developed adopting the Microsoft excel package. The complete templates for the program are shown in Tables 3.1, 3.2 and 3.3.

### **3.4 Materials**

#### **3.4.1 Concrete**

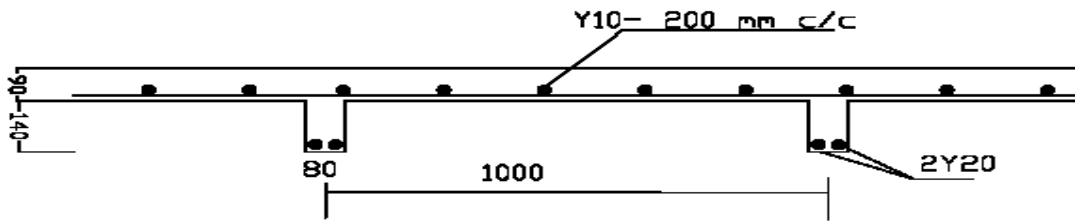
The ordinary Portland cement was used in the concrete, mixed with fine aggregate (sand) and coarse aggregate (granite), the maximum size of the granite was limited to 9.5 mm (3/8 in). This is because of the small models used and narrow dimensions between the sides of the form work.

The combined aggregate were prepared based on the recommendation of BS 1881 Part 116; natural clean water without contamination was used in the concrete mix. Water cement ratio of 0.5 was selected to achieve the required 28-day concrete strength of 20 N/mm<sup>2</sup>, as was used in the case study; concrete samples were taken for the cube test (BS 1881, 1971).

#### **3.4.2 Reinforcement**

The steel reinforcements used in the case study were, 2Y20 mm for main ribs, 2Y12 mm for minor ribs, and Y10-200 mm c/c distribution steel in the slab for panels P1, P5, and for P3, (Figure 3.3), 2Y16mm was adopted for the main ribs and 2Y12 mm for minor ribs. When ratio 1:4 was adopted for the model, the main reinforcement became 1Y8 mm for the major ribs and 1R6 mm steel for the minor ribs, while plain 2.5 mm /D98 wire mesh was used in the slab as distribution steel.

For panels P2, P4 and P6, in the prototype, 2Y12 mm was used in the major and minor ribs respectively. And Y8-200 mm c/c distribution steel was used in the slab.



**Figure 3.3: Reinforcement arrangement for slab and ribs in waffle slab**

Adopting ratio 1:4, the major and minor ribs became 1R6 mm steel as main steel, while 2.5 mm/D98 wire mesh was also adopted as distribution steel in our models. The wires were cut to sizes and cleaned from rust using metal brush, in such a way to keep the bond forces that will be developed between the interface of concrete and steel. These bond forces are essential to prevent slip from occurring at the interfaces (Nilson and Winter, 1991).

### **3.5 Description of the Specimens**

A total of sixty model samples were used for the experiment. Thirty samples each were used for the waffle and solid slabs respectively. These samples were subjected to axial loads, five samples per panel. Six types of panels were used for each of the waffle slabs and solid slabs respectively. Table 3.4 shows the geometric properties of waffle slab prototypes, while Tables 3.5 and 3.6 show the scaled down geometric description of the panel properties of the models for both the waffle and solid slabs respectively.

Different support conditions were adopted for all the slabs. Slabs W1, W3, S1 and S3 were supported on the two short sides, slabs W5, W6, S5 and S6 were all supported on three sides, i.e. two short and one long sides, while slabs W2, W4, S2 and S4 were supported on all the four sides. The reason for the different support conditions is to apply eccentric loading in the slabs.



**Table 3.4: Panel properties for a waffle slab prototype.**

| <b>Panel No</b> | <b>Length mm</b> | <b>Width mm</b> | <b>Slab thickness mm</b> | <b>Rib width (mm)</b> | <b>Rib spacing long span (mm)</b> | <b>Rib spacing Short span (mm)</b> | <b>Slab reinforcement (spacing)</b> | <b>Longitudinal Rib reinforcement</b> | <b>Transverse Ribs reinforcement</b> |
|-----------------|------------------|-----------------|--------------------------|-----------------------|-----------------------------------|------------------------------------|-------------------------------------|---------------------------------------|--------------------------------------|
| P1              | 5415             | 1720            | 90*                      | 80                    | 1040                              | 460                                | Y10/200 mm                          | 2Y20                                  | 2Y12                                 |
| P2              | 3600             | 1200            | 50**                     | 80                    | 800                               | 680                                | Y8/200 mm                           | 2Y10                                  | 1Y12                                 |
| P3              | 4340             | 1720            | 90*                      | 80                    | 1000                              | 460                                | Y10/200 mm                          | 1Y16                                  | 1Y12                                 |
| P4              | 1635             | 1475            | 50**                     | 80                    | 760                               | 1370                               | Y8/200 mm                           | 2Y10                                  | 1Y12                                 |
| P5              | 5250             | 1200            | 90*                      | 80                    | 1280                              | 560                                | Y10/200 mm                          | 2Y20                                  | 2Y12                                 |
| P6              | 3280             | 1440            | 50**                     | 80                    | 1040                              | 680                                | Y8/200 mm                           | 2Y10                                  | 1Y12                                 |

\*Total Waffle Slab Thickness was 230 mm.

\*\*Total waffle Slab Thickness was 200 mm

**Table 3.5. Panel properties of the waffle models. (Scale ¼)**

| <b>Panel No</b> | <b>Length mm</b> | <b>Width mm</b> | <b>Slab thickness (mm)</b> | <b>Rib width (mm)</b> | <b>Rib spacing Long span (mm)</b> | <b>Rib spacing short span (mm)</b> | <b>Slab reinforcement (spacing)</b> | <b>Longitudinal Rib reinforcement</b> | <b>Transverse Ribs reinforcement</b> |
|-----------------|------------------|-----------------|----------------------------|-----------------------|-----------------------------------|------------------------------------|-------------------------------------|---------------------------------------|--------------------------------------|
| W1              | 1353             | 430             | 22.5*                      | 20                    | 260                               | 135                                | 2.5/D98                             | 1Y8                                   | 1R6                                  |
| W2              | 900              | 300             | 12.5**                     | 20                    | 200                               | 170                                | 2.5/D98                             | 1R6                                   | 1R6                                  |
| W3              | 1085             | 430             | 22.5*                      | 20                    | 250                               | 135                                | 2.5/D98                             | 1R6                                   | 1R6                                  |
| W4              | 407              | 364             | 12.5**                     | 20                    | 190                               | 344                                | 2.5/D98                             | 1R6                                   | 1R6                                  |
| W5              | 1312             | 300             | 22.5*                      | 20                    | 320                               | 140                                | 2.5/D98                             | 1Y8                                   | 1R6                                  |
| W6              | 860              | 360             | 12.5**                     | 20                    | 260                               | 170                                | 2.5/D98                             | 1R6                                   | 1R6                                  |

\*Total Waffle slab Thickness was 58 mm.

\*\*Total Waffle slab Thickness was 50 mm.

**Table 3.6. Panel properties of the solid slab models.**

| <b>Panel No</b> | <b>Length (mm)</b> | <b>Width (mm)</b> | <b>Slab thickness (mm)</b> | <b>Slab Reinforcement (spacing)</b> |
|-----------------|--------------------|-------------------|----------------------------|-------------------------------------|
| S1              | 1353               | 430               | 40                         | 6R-75c/c                            |
| S2              | 900                | 300               | 40                         | 6R-100c/c                           |
| S3              | 1085               | 430               | 40                         | 6R-100c/c                           |
| S4              | 407                | 364               | 40                         | 6R-100c/c                           |
| S5              | 1312               | 300               | 40                         | 6R-75c/c                            |
| S6              | 860                | 360               | 40                         | 6R-100c/c                           |

### 3.5.1 Modelling of specimens

The areas of reinforcements for the models were obtained from equation 3.11, and the equation is derived below.

Adopting dimensional analysis, (Rajput, 1998: Bankole, 2007).

Using stresses relationship between the prototype and the model,

For Prototype.

$$\text{Stress } \sigma = \frac{\text{Force}}{\text{Area}} = M_p \cdot L_p^{-1} \cdot T_p^{-2}$$

And for the Model.

$$\text{Stress } \sigma = \frac{\text{Force}}{\text{Area}} = M_m \cdot L_m^{-1} \cdot T_m^{-2}$$

Where

M = Mass

L = length

T = Time

Subscript  $p$  and  $m$  are symbol for the prototype and model respectively.

At scale 1 : 4

$$4. M_m \cdot L_m^{-1} \cdot T_m^{-2} = 1. M_p \cdot L_p^{-1} \cdot T_p^{-2} \dots\dots\dots i$$

Substituting Mass as a function of reinforcement unit weight in dimensions symbol,

$$\text{Unit weight} = \text{weight} / \text{Volume} = \delta$$

$$\delta = M \cdot L^{-2} \cdot T^{-2}$$

$$M = \delta \cdot L^2 \cdot T^2 \dots\dots\dots ii$$

Substituting for M in equation i,

$$4. (\delta \cdot L^2 \cdot T^2)_m \cdot L_m^{-1} \cdot T_m^{-2} = 1. (\delta \cdot L^2 \cdot T^2)_p \cdot L_p^{-1} \cdot T_p^{-2} \dots\dots\dots \text{iii}$$

From equation iii:

$$4. \delta_m L_m = 1. \delta_p L_p$$

$$L_m = \frac{\delta_p \cdot L_p \cdot 1}{\delta_m \cdot 4} \dots\dots\dots 3.11$$

But  $L_p =$  Diameter of bars in the prototype

$L_m =$  Diameter of bars in the model

$\delta_p =$  Characteristic strength of steel used in the prototype

$\delta_m =$  Characteristic strength of steel used in the model

For example, the area of the steel reinforcement for the major rib in panel P1 of the prototype is as follows

$$2Y20 = 628 \text{ mm}^2$$

But for the model W1 using equation 3.11 above,

$$L_m = \frac{410 \cdot 20 \cdot 1}{410 \cdot 4} = 5 \text{ mm}$$

The bar size is 5 mm diameter (area=19.64 mm<sup>2</sup>), and since 2Y20 was required from the prototype, hence, 2 No.5 mm diameter bar will be obtained (area= 39.27 mm<sup>2</sup>). Since there is no 5 mm diameter bar available in the market, then 1Y8 bar (area= 50 mm<sup>2</sup>) was adopted for the main rib of the model W1.

For the solid slab models, the reinforcement was obtained as follows:

The reinforcement in the prototype was calculated to be Y12- 275 mm c/c.

For panel S1,

L= 5415 mm, total length.

$$\text{Hence, } \frac{5415}{275} = 19$$

19 bars were used,

$$\text{Total area of bars per length} = 113 \times 19 = 2147 \text{ mm}^2$$

$$\text{At scale } 1 : 4, \text{ area required} = 2147 / 4 = 537 \text{ mm}^2$$

$$\text{Area of } 5 \text{ mm bar} = 19.64 \text{ mm}^2, \text{ (since Y12 will give 5 mm bar from scale 1:4)}$$

$$\text{No of bar required} = 537 / 19.64 = 27$$

Since there is no 5mm available in the market,

Adopt 6R- 75 mm c/c. ( $377 \text{ mm}^2$ ).

Hence, Length of model = 1354mm

$$\therefore \frac{1354}{75} = 18.05 \quad \text{Say } 19$$

$$19 \times 28.30 = 538 \text{ mm}^2 \quad \text{area of bars required is } \quad \text{O.K}$$

### 3.6 Casting of Specimens

All specimens were cast in forms made of plywood. The voids between the ribs were made by gluing Polystyrene blocks to the plywood form by following a marked pattern on the plywood. The reinforcement was then placed between the Polystyrene blocks in both directions. The reinforcement in the longer direction was placed first on small concrete blocks (biscuit) to keep it 5 mm from the bottom of the forms. This provided for the minimum cover required for the steel. The steel in the shorter direction was then placed on the top of the reinforcement in the longer direction. Both layers of

reinforcement were tied together with thin binding wire for stability during casting of the concrete. The wire mesh was placed and fixed properly.

The concrete was then cast, tamped and vibrated to prevent honey comb. After that, the top surface of the concrete was given a smooth final finish with the use of hand trowel. All specimens were moist cured for 28 days which helped the concrete to stabilize its own properties, like compressive strength and modulus of elasticity. To determine the compressive strength of the concrete, 150 x 150 mm concrete cubes were cast with the specimen. These cubes were submerged in water for curing and crushed at 7, 21 and 28 days (BS 1881, 1971).

### **3.7 Instrumentation**

The deflections of the concrete specimens for the bending test were measured by means of a digital dial gauge with sensitivity of 0.01 mm. The dial gauges were placed at the top of the slab and held in place by the use of steel frame clamped on to the test machine. The crack widths were measured using a vernier caliper. Plate 1 showed both the dial gauge and the vernier caliper.

### **3.8 Experimental Set-up and Procedure**

The slabs were loaded using Universal tensile testing machine. The slabs were placed on welded steel frame to give it a good support, and placed in between the loading machine. The machine loader applies the loads to the slabs from the top.

Deflections of the slabs were measured at load increment of 1 kN, and these were done during the loading of the concrete specimen. Crack widths were measured at failure load. The average mean value of the five results for each of the models samples was used.

The bending moments of each slab were obtained using the yield line theory formulae (equations 3.5 and 3.7.).

Plates 1 and 2 showed the precast waffle and solid slabs well stacked waiting transportation to the laboratory. Plate 3 showed the universal tensile testing machine while Plate 4 showed the test arrangements in the machine.

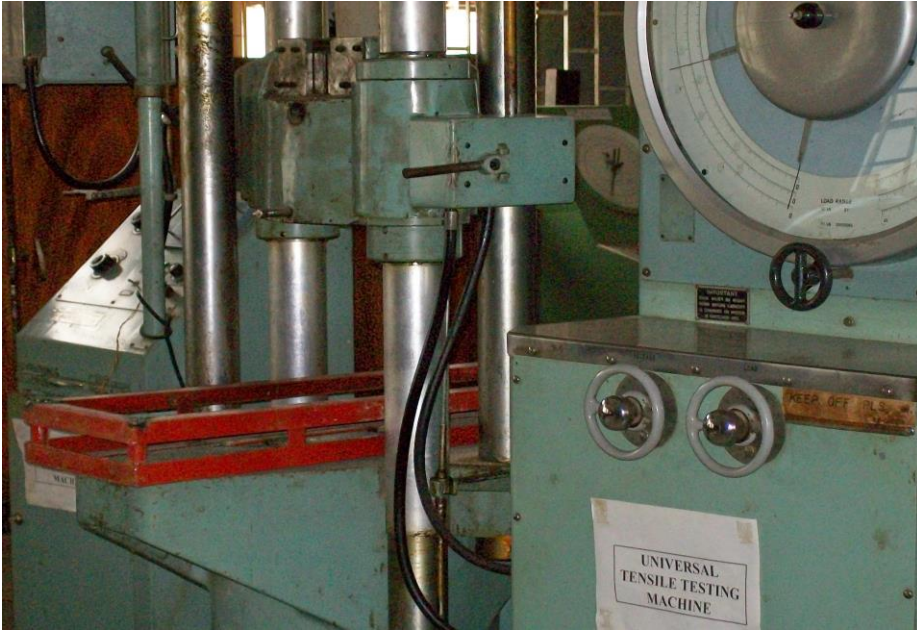




**Plate 1: Stacked slabs.**



**Plate 2: Some samples of waffle slabs**



**Plate 3: The Universal Tensile Testing Machine, with welded steel support for slabs.**



**Plate 4: Test arrangement for a slab sample**

### **3.9 The Analysis of the Prototype**

The Prototype was analysed by the Grillage analysis method which is mainly used in the analysis of bridges.

In grillage analysis, the longitudinal members are arranged to represent the main beams with the transverse members representing the deck slab and diaphragm beams. All loads are proportioned to the grillage members (longitudinal members only) and grillage joints before the moments, shear and torsion are calculated. The loads are applied to each longitudinal member as uniformly distributed loads (Childs, 2008). Hence the transverse members are neglected in this analytical method. This method may not be the most reliable for the analysis of waffle slabs because the transverse members in the waffle slab also carry some proportion of loads which should not be neglected. This might have led to some of the discrepancies observed in the laboratory results.

This study adopted the Rankine Grashoff theory which accommodated loading in both the longitudinal and transverse members of the waffle slab.

### **3.10 Analysis of Data**

With the experimental set up above, coupled with the equations, some data were generated at the end of the work. The results obtained from both the waffle slabs and the solid slabs were subjected to Descriptive statistics and analysis of variance (ANOVA). Significant means were separated using least significant difference (LSD).

## CHAPTER FOUR

### RESULTS AND DISCUSSION

#### 4.1 Structural Response of Specimens to Loads

Waffle slabs and solid slabs were all simply supported, deflection was observed from the dial gauge immediately loading started. The cracks emerged from the middle of the slabs where the load was being applied and extended toward the slab edges, the crack width continued to expand until the failure load was reached. The cracks at the surface of the slab were very small compared to the ones formed at the slab bottom. This is as a result of the weakness of concrete in tension as the bottom parts of the slabs are under tension.

Deflection and the extent of cracking of a reinforced concrete slab are highly dependent on its support conditions, nonlinear and inelastic properties of concrete and the surrounding structure (Gilbert and Guo, 2005). The initial load at which deflection was observed, failure load and final deflection are showed in Table 4.1, for example, W1 started deflecting at 2 kN and the initial deflection was 0.10 mm, the final deflection was 1.19 mm at 18 kN failure load, while the load at which the first crack was observed was 9 kN and the final crack width was 0.60 mm, at a failure load of 18 kN. Table 4.2, showed the initial and final loads for the crack width of other slabs. The final crack patterns on the slab surface are showed in Plates 5 – 12. Each crack pattern for solid slabs depends on the support condition and loading orientation, whereas the crack pattern in waffle slabs did not show define pattern, Plate 13 showed the crushing of a waffle slab ribs.

**Table 4.1: Deflection of slabs under load**

| <b>Specimens</b> | <b>Initial Load<br/>(kN)</b> | <b>Failure load<br/>(kN)</b> | <b>Initial deflection<br/>(mm)</b> | <b>Deflection<br/>at failure load<br/>(mm)</b> |
|------------------|------------------------------|------------------------------|------------------------------------|--|
| W1               | 2.00                         | 18.00                        | 0.10                               | 1.19   |
| W2               | 1.00                         | 9.00                         | 0.34                               | 3.64   |
| W3               | 4.00                         | 12.00                        | 1.11                               | 3.90   |
| W4               | 1.00                         | 6.00                         | 1.45                               | 6.60   |
| W5               | 1.00                         | 8.00                         | 1.44                               | 8.17   |
| W6               | 2.00                         | 12.00                        | 0.43                               | 3.28   |
| S1               | 2.00                         | 12.00                        | 0.73                               | 3.56   |
| S2               | 1.00                         | 7.00                         | 1.10                               | 9.28   |
| S3               | 2.00                         | 14.00                        | 0.71                               | 7.44   |
| S4               | 1.00                         | 6.00                         | 1.19                               | 6.44   |
| S5               | 1.00                         | 6.00                         | 2.14                               | 12.18  |
| S6               | 2.00                         | 10.00                        | 0.14                               | 3.89   |

**Table 4.2: Crack properties of slabs under load**

| <b>Specimens</b> | <b>Initial crack load<br/>(kN)</b> | <b>Final crack load<br/>(kN)</b> | <b>Crack width<br/>at failure<br/>(mm)</b> | <b>Crack patterns</b> |
|------------------|------------------------------------|----------------------------------|--|-----------------------|
| W1               | 9.00                               | 18.00                            | 0.60                                       | Plate 5               |
| W2               | 6.00                               | 9.00                             | 0.35                                       | Plate 6               |
| W3               | 6.00                               | 12.00                            | 0.34                                       | -                     |
| W4               | 3.00                               | 6.00                             | 0.25                                       | Plate 7               |
| W5               | 4.00                               | 8.00                             | 0.75                                       | Plate10               |
| W6               | 7.00                               | 12.00                            | 0.62                                       | -                     |
| S1               | 5.00                               | 12.00                            | 1.20                                       | -                     |
| S2               | 2.00                               | 7.00                             | 0.75                                       | Plate 8               |
| S3               | 6.00                               | 14.00                            | 0.90                                       | -                     |
| S4               | 2.00                               | 6.00                             | 0.70                                       | Plate 9               |
| S5               | 3.00                               | 6.00                             | 1.10                                       | Plate 11              |
| S6               | 4.00                               | 10.00                            | 1.30                                       | Plate 12              |



#### **4.1.1 General observation of structural response of specimens to load**

Generally, it was observed that the crack widths on all the waffle slabs were small compared to the corresponding solid slabs. The crack patterns in the solid slabs were more definite than those of waffle slabs.

This was due to the presence of ribs at the bottom and was also responsible for the small crack width and irregular crack pattern observed on the toping of the waffle slabs. The ribs were in the tension zone and were subjected to direct tensile stresses, unlike the solid slab where the bottom part was exposed directly to the tensile force.

In the waffle slabs, the ribs failed first before the slab portion started showing any sign of distress. The different crack patterns formed were as a result of the type of support (boundary) conditions that each slab was exposed to. Appendices 39 to 50 show details of the loads, deflections and crack widths of the slabs.



**Plate 5: Crack pattern for waffle slab W1 supported on 2 short sides**



**Plate 6: Crack pattern for waffle slab W2 Supported on all 4 sides**



**Plate 7: Crack pattern for waffle slab W4 Supported on all 4 sides**



**Plate 8: Crack pattern for Solid Slab S2 supported on all 4 sides**





**Plate 9: Crack pattern for solid slab S4 supported on all 4 sides**



**Plate 10: Crack pattern for waffle slab W5 Supported on 1 long side and 2 short sides**

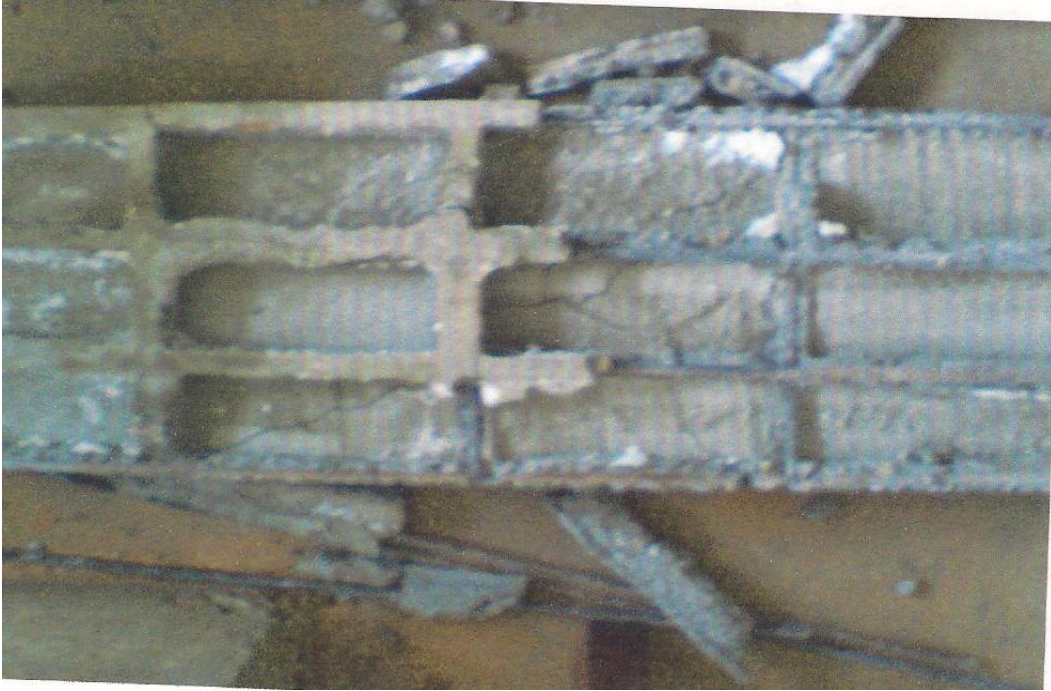


**Plate 11: Crack pattern for Solid Slab S5 supported on 1 long side and 2 short sides**





**Plate 12: Crack pattern for Solid Slab S6 supported on 1 long side and 2 short sides**



**Plate 13: Crack pattern of a failed waffle slab bottom**

## 4.2 Bending Moments of Specimens

The theoretical bending moments of the panels were compared with the experimental bending moments. The results for that of waffle and solid slabs are presented in Tables 4.3 and 4.4 respectively. The theoretical collapse load was obtained using the design load of  $2.0 \text{ kN/m}^2$ , Live load (for Hostels) and Dead load based on the depth of each slab, finishes and partition. The factors of safety 1.4 for dead load and 1.6 for live load were applied.

The experimental collapse load was obtained from the results of the laboratory work. The bending moments were obtained using the yield line theory for all the different supports conditions such as: simply supported on two, three, and four sides respectively.

From the results in Tables 4.3 and 4.4, it was observed that the experimental bending moments of the various panels were higher than the theoretical values (both for waffle and solid slabs). This is in contrast with the findings of Gudmand-Høyer in his work on yield line theory for concrete slabs subjected to axial forces, but concluded that the findings could be a rough estimate and an effectiveness factor could be introduced to make the findings reliable (Gudmand-Høyer 2003). Although it is expected that the experimental results be greater than the theoretical, because the final reinforcement provided is expected to be higher than the designed reinforcement area due entirely to the safety of the structure. But the difference between the theoretical results and the real results (experimental) should not be too high, since the economy of the structures is also very important. The differences may be as a result of the following factors:

- The design package used in the (Prototypes) experiment was different from the one used in the theoretical analysis.

**Table 4.3: Comparison of theoretical and experimental bending moments  
(Waffle slab)**

| Panels | Theoretical              |  | Experimental          |                    |
|--------|--------------------------|--|-----------------------|--------------------|
|        | Bending Moments<br>(kNm) |  | Bending Moments (kNm) | Deflection<br>(mm) |
| W1     | 2.157                    |  | 5.526                 | 1.19               |
| W2     | 0.084                    |  | 1.122                 | 3.64               |
| W3     | 1.653                    |  | 2.880                 | 3.90               |
| W4     | 0.066                    |  | 0.590                 | 6.60               |
| W5     | 0.436                    |  | 0.947                 | 8.17               |
| W6     | 0.297                    |  | 1.276                 | 3.29               |

**Table 4.4: Comparison of theoretical and experimental bending moments  
(Solid slab)**

| Panels | Theoretical           | Experimental          |                 |
|--------|-----------------------|-----------------------|-----------------|
|        | Bending Moments (kNm) | Bending Moments (kNm) | Deflection (mm) |
| S1     | 2.257                 | 3.684                 | 3.56            |
| S2     | 0.0924                | 0.872                 | 9.28            |
| S3     | 1.452                 | 3.360                 | 7.44            |
| S4     | 0.072                 | 0.590                 | 6.44            |
| S5     | 0.459                 | 0.710                 | 12.18           |
| S6     | 0.324                 | 1.062                 | 3.89            |

- The theoretical results are based on ideal situations that are assumed to be perfect, while the practical results are based on the situation on ground which may not be as perfect as expected due to various unforeseen conditions.
- The reinforcements provided to resist bending moments of reinforced concrete were sometimes over estimated. This was evident in all the reinforcements provided in the ribs of the prototypes. For example, panel P1 which had the largest dimensions of 5415 mm x 1720 mm, had reinforcement of 2Y20 (628 mm<sup>2</sup>) in the long ribs and 2Y12 (226 mm<sup>2</sup>) was provided in the short ribs. In the model, this was scaled down to ratio (1/4), and dimension 1353 mm x 430 mm was obtained. Using dimensional analysis, the reinforcements for the long ribs were 1Y8 (50 mm<sup>2</sup>), and 1R6 (28 mm<sup>2</sup>) for the short ribs (Tables 3.4 and 3.5). In the analysis of the ribs adopting the Rankine Grashoff theory (Appendix 1), the area of reinforcements for both long and short ribs were 3.320 mm<sup>2</sup> and 1.136 mm<sup>2</sup> respectively. This implies that if a reinforcement of 1R2.5 (5 mm<sup>2</sup>) was used for both ribs, the bending moment would be supported without causing any structural problem. The use of 1R2.5 in the model suggested that 1Y10 (79mm<sup>2</sup>) steel could be used in both ribs for the prototype P1. In order to reduce service stress in the steel, 2Y10 could also be used. The reinforcements used in the ribs of the prototype were very high and this was not economical. The reason for the high number of reinforcements was because the grillage analytical method that was used in the analysis of the prototype allowed excess load in the longitudinal ribs, the method did not distribute any load to the transverse ribs (nominal reinforcements were provided for the transverse ribs). This might have led to the

over reinforcement in the main ribs, thus affecting the bending moment of the structure. The grillage method is good for the analysis of bridge decks, not for precast waffle slab (Akinyele and Alade, 2011).

- The discrepancies in the results might also be due to the fact that concrete does not behave entirely as an elastic material (Mindes and Young, 1981; Neville, 1990). This is because, concrete was assumed to be elastic during the design stage. The analysis and design of other slabs can be seen in Appendices 2-24.

#### **4.2.1 Comparison of bending moments of waffle slabs and solid slabs**

The results for waffle W1 and solid S1 showed that the bending moments of the waffle slab were higher than that of the solid slabs by 33 %. Both slabs were supported on two short sides. For W2 and S2, the waffle was 22 % greater than that of solid slab in strength; both slabs were supported on four sides. Waffle slab W3 was 16.7% lower than solid slab S3, although they were supported on two sides. Slabs W4 and S4 gave the same results. However, these were supported on four sides.

The bending moments of W5 was 24.96 % higher than that of S5 while W6 was 16.67% greater than that of S6. These slabs were supported on three sides. Apart from the slabs that gave the same results (Tables 4.3 and 4.4); it was generally observed that waffle slabs had higher bending moments than solid slabs. This supports the advantages of waffle slabs over solid slabs in terms of ability to support heavy loads over a long span without increasing the depth.

This advantage was as a result of the presence of ribs in waffle slabs which act as flanged beams within the slab system. The bulk of the tensile load was carried by the ribs that spanned in both ways. This helped in the equal distribution of the load over the entire

span to the supports. The contribution of reinforcement to this advantage can not be ruled out also, because the presence of reinforcement in the slab portion and the ribs of waffle slabs gave it advantage over solid slabs which had reinforcement in it tensile zone only.

#### **4.2.2 Effects of slab sizes and ribs on the bending moments of waffle slabs**

The effects of the sizes of slabs on their bending moments were investigated. The sizes of waffle slabs that were set up under the same support conditions were compared with each other. The collapse loads (ultimate load) were used to determine the bending moments and the following observations were made.

##### **4.2.2.1 Comparison of Bending moments of W1 and W3**

Waffle slabs W1 and W3 were both simply supported on two sides, along the small spans while the large spans were free. From Table 3.5; it was observed that W1 has a larger dimension than W3. The collapse load of W1 was 18 kN, the bending moment obtained was 5.526 kNm, while for W3, the collapse load was 12 kN and bending moment was 2.88 kNm, W1 gave a 9 kN shear force at the supports while the shear force for W3 was 6 kN at the supports.

The above results showed that W1 has a higher bending moments than W3 despite it long span (Theoretically, slabs with long span are expected to fail earlier than those with short span for the same depth). The higher collapse load and moment may be attributed to the number of ribs in slab W1 which had four equally spaced ribs on the long axis while W3 had only three that were also equally spaced. They both had two ribs each on their short sides.



#### **4.2.2.2 Evaluation of the bending moments of W2 and W4**

Waffle slab W2 had larger dimensions than W4 (Table 3.5). The collapse load of W2 was 9 kN, and the bending moments was 1.122 kNm. The slab W4 had a collapse load of 6 kN, and its bending moments was 0.590 kNm. W4 was smaller in size; it had the lower collapse load and bending moments. This can be attributed to the single rib in its soffit, unlike W2 that had 3 short ribs and two long ribs in its soffit (plate 13), and both slabs were supported on all four sides.

#### **4.2.2.3 Assessment of the bending moments of W5 and W6**

The two waffle slabs W5 and W6 were supported on three sides, leaving one of the long span free. W5 had an ultimate load of 8 kN, and a bending moment of 0.947 kNm, while W6 had an ultimate load of 12 kN, and a bending moment of 1.276 kNm. It can be observed that although W5 had the higher dimension (Table 3.5), both the ultimate load and bending moment were lower than those of W6. While W5 had three ribs on the long side compared to the two of W6, they both had one rib each on the shorter side.

The reason for the low bending moment of W5 can be attributed to the type of support and the length of the long span. The length of W5 was about 1.5 times that of W6, this created a kind of in-balance in the slab. This also had effect on the load carrying ability of the slab since loads that were distributed to the unsupported long span area caused the early failure of the slab. The other area of the slab that were supported could not carry the excess load from the unsupported long span because the three ribs on the long span were responsible for the transfer of the load from the middle of the slab to the edge ribs. Since the edge ribs were to convey the loads to the supports, the edge rib on the unsupported side was under greater tensile stress as it had no supports under it. The three

mid ribs that transferred the load from the middle of the slab acted like a point load on this unsupported but long edge rib. The reinforcements on the edge rib could not withstand the excess load on its rib; therefore it yielded earlier, leading to the eventual failure of the slab system. Although W6 also reacted like W5 in terms of load distribution, it did not yield earlier than W5 because the edge rib (long span) was shorter than that of W6.

#### **4.2.3 Effects of slab size on the bending moments of solid slabs**

The sizes of solid slabs were also investigated to determine their contribution to the bending moment of solid slabs. Slabs with the same support conditions were compared with each other and the results discussed below:

##### **4.2.3.1 Relationship between the bending moments of S1 and S3**

The two solid slab samples were simply supported on the two short spans and the long spans were free. S1 was larger than S3, and both slabs were one way slab. It was observed that the collapse load for S1 was 12 kN, while the bending moment was 3.684 kNm; S3 had 14 kN for its collapse load, while bending moment of 3.36 kNm was generated, slab S1 had a shear force of 6 kN while S3 had 7 kN at the support respectively (Appendices 19 and 21).

From the above results, it was observed that both slabs had different collapse loads, but the bending moment generated in S1 was higher than S3 when the analytical method of a uniform load over part of a simply supported slab was used. This difference was because the span/ depth ratio of S1 was higher than that of S3, so that an appropriate reinforcement could be provided to resist the excess bending moment generated as a result of the longer span in the slab. If the result is compared to the corresponding waffle

slabs W1 and W3, the absence of ribs that would have provided additional rigidity to the solid slab also contributed to the lower collapse load in S1.

#### **4.2.3.2 Difference in the bending moments between S2 and S4**

The collapse load for S2 was 7 kN, and its bending moment was 0.872 kNm. Solid slab S4 gave a collapse load of 6 kN, and bending moment of 0.590 kNm. Both slabs were supported on four sides. The bending moment of S4 was 1.4 times smaller when compared to that of S2. The above results had shown that the size of a slab had a way of contributing to the amount of bending moment generated in the structure.

#### **4.2.3.3 Comparison of the bending moments of S5 and S6**

The results in slabs S5 and S6 showed that S5 had a collapse load of 6 kN and a bending moment of 0.710 kNm. The result of S6 gave a collapse load of 10 kN, while its bending moment was 1.062 kNm.

Both slabs were supported on three sides while one large side was free. They both had the same depth, and were both one way slabs. This condition gave rise to the lower collapse load of the longer slab S5. The in-balance was a result of one long side not supported, causing the tensile or bottom parts of the slabs to be over stressed, especially the non-supported slab edge. It can be recalled that all loads were transferred from the middle of the slabs where the load was being applied to the edges. Since there were no ribs on this edge as it was in waffle slabs, the bottom reinforcement in this part of the slab yielded easily, leading to the eventual failure of the entire slab system. It was the same failure mode that happened in slab S6, but the amount of failure was lower when compared with S5.

### 4.3 Assessment of the Rankine Grashoff and BS 8110 Slab Coefficients Factors

The BS 8110 slab coefficients factor was compared with the Rankine Grashoff theory in the analysis of waffle slab ribs. This factor was used to analyse a waffle slab with dimension 6000 mm by 5000 mm. Ribs spacing was 400 mm while the total depth of waffle slab was 200 mm. The topping was 60 mm depth and rib depth was 140 mm. This example was obtained from Mosley *et al* (1999) (Appendices 25 and 26).

The appropriate coefficient from BS 8110 (for slab analysis) was used for the ribs analysis, while minimum reinforcement was obtained by adopting  $0.12bh$  (sectional area) for the topping or slab portion of the waffle slabs. But in this work, the ribs were analysed using the Rankine Grashoff theory of ribs analysis, while the topping was analyzed using the Yield line theory.

The section used was a waffle slab for an internal panel of a floor system. The characteristic material strengths for concrete and steel were  $f_{cu} = 30 \text{ N/mm}^2$  and  $f_y = 460 \text{ N/mm}^2$  respectively. The design ultimate load was  $12.4 \text{ kN/m}^2$ . The moment coefficients were taken from Table 3.14 of BS 8110 with  $l_y/l_x = 1.2$ . The results are shown in Table 4.5. From the results obtained, it was observed that the BS 8110 gave very low values for the moments generated in the ribs for each direction compared to the analysis using the Rankine theory. The moments obtained were 1.3 times greater than BS 8110 coefficients along the long span, and 1.4 times greater along the short span. The required area of steel from the Rankine theory was higher than the BS 8110 for both spans. The difference was between 1.8 – 2.0 times more than that of BS 8110 slab coefficients.

**Table 4.5: Summary results for BS 8110 and Rankine theory**

| <b>Analytical method</b> | <b>Moment in long span ribs (kNm)</b> | <b>Moment in short span ribs (kNm)</b> | <b>A<sub>s</sub> required long span (mm<sup>2</sup>)</b> | <b>A<sub>s</sub> required short span (mm<sup>2</sup>)</b> | <b>Shear force (kN)</b> | <b>Top Flange A<sub>s</sub> required (mm<sup>2</sup>)</b> |
|--------------------------|---------------------------------------|--|--|---|-------------------------|---|
| BS 8110                  |                                       |  |  |   |                         |   |
| Coefficients             | 2.98                                  | 3.97                                   | 45   | 60  | 5.20                    | 72  |
| Rankine                  |                                       |  |  |   |                         |   |
| Theory                   | 3.13                                  | 4.50                                   | 80   | 122   | 3.60                    | 462   |

Although Mosley *et al* (1999) provided 2T10 bars in each rib at the bottom ( $A_s = 157 \text{ mm}^2$ ) which was adequate to cater for the required steel area and also adequate for the results from Rankine analysis. But the low analytical results from the use of BS 8110 coefficients may lead to under reinforcements of the ribs and hence allow for service stress in the steel provided. For the topping or slab portion, the use of  $0.12bh$  gave the required area of steel to be  $72 \text{ mm}^2$  which was the minimum required, and a wire mesh D98 ( $A_s = 98 \text{ mm}^2$ ) was provided. If this is compared to the yield line analysis results of  $462 \text{ mm}^2$  with the provision of Y8@100 mm c/c ( $503 \text{ mm}^2$ ), then the yield line results was adequate for the type of eccentric loading that the slabs in the prototypes were subjected to during loading and transportation to the site.

As discussed in chapter one, the reason for cracks and sometimes outright failures in the initial waffle slab panels was due to the use of wire mesh that can not withstand the eccentric load from cranes and transportation to site and eventual placements of the slabs to their positions on the site. The results obtained for the minimum reinforcements might be adequate for casting in-situ waffle slabs that were not exposed to eccentric forces, but not for precast waffle slabs that would be transported from the factory to the site and subjected to different types of loading conditions.

The reason for the cracks and failure in the precast slab of this research prototype was because the topping of the slab was subjected to both sagging and hogging moments during lifting. Since this portion of the slab was not designed to resist tensile moments (sagging / hogging), the forces generated as a result of lifting produced the tensile force that caused the slab topping to fail.

#### **4.4 Comparison of YLRGT and ETABS**

The computer program ETABS (developed by computers and structures Inc.) was also compared with the Yield line and Rankine Grashoff theory (YLRGT) in order to test the reliability of the proposed computer program. The result is showed in Table 4.6.

##### **4.4.1 Outcome of W1 and W3**

It was observed that the new program gave a higher moment of 2.157 kNm in the theoretical and 5.526 kNm in the experimental results of waffle W1 while ETABS results were 1.925 kNm and 4.729 kNm for theory and experiment respectively.

Also the new program gave the results for W3 as 1.653 kNm and 2.880 kNm for theory and experiment respectively whereas ETABS gave 1.613 kNm and 3.338 kNm for both theoretical and experimental result respectively for the same panel W3.

Both W1 and W3 were simply supported on the two short sides. The results showed that ETABS was conservative in the analysis of a simply supported waffle slabs on two short sides, although the difference in the results of both analysis was about 10% which is still reliable for any analysis.

##### **4.4.2 Results of W2 and W4**

The yield line result for W2 was 0.084 kNm for theory and 1.122 kNm for the experiment, while ETABS result was 0.502 kNm for theory and 0.581 kNm for experiment. Also W4 gave 0.066 kNm for theory and 0.590 kNm for experiment in yield line, whereas ETABS gave the theoretical result to be 0.472 kNm and experimental to be 0.733 kNm. Both slabs were simply supported on all four sides; yield line gave conservative results in the theory for both slab and the experimental result for W4, while ETAB gave conservative result in the experimental results of W2.

**Table 4.6: Comparison of the bending moment of ETABS and YLRGT**

| Panels | ETABS             |                    | YLRGT             |        |        |                    |        |        |
|--------|-------------------|--------------------|-------------------|--------|--------|--------------------|--------|--------|
|        | Theoretical (kNm) | Experimental (kNm) | Theoretical (kNm) |        |        | Experimental (kNm) |        |        |
|        |                   |                    | SP                | L. Rib | S. Rib | SP                 | L. Rib | S. Rib |
| W1     | 1.925             | 4.729              | 2.157             | 0.0074 | 0.0378 | 5.526              | 0.9183 | 0.0345 |
| W2     | 0.502             | 0.581              | 0.084             | 0.0031 | 0.0240 | 1.122              | 0.0365 | 0.2792 |
| W3     | 1.613             | 3.338              | 1.653             | 0.0112 | 0.0370 | 2.880              | 0.6199 | 0.0272 |
| W4     | 0.427             | 0.733              | 0.066             | 0.0200 | 0.0213 | 0.590              | 0.1609 | 0.1710 |
| W5     | 0.568             | 0.851              | 0.436             | 0.0023 | 0.1898 | 0.947              | 0.0044 | 0.0370 |
| W6     | 0.530             | 0.686              | 0.297             | 0.0102 | 0.0255 | 1.276              | 0.0362 | 0.0904 |

SP = Slab portion of waffle

L. Rib = Long rib of waffle

S. Rib = Short rib of waffle



#### **4.4.3 Comparison of W5 and W6**

The results for W5 were 0.436 kNm theoretical and 0.947 kNm experimental for yield line, while ETABS was 0.568 kNm theoretical and 0.851 kNm experimental. Waffle slab W6 gave theoretical result to be 0.297kNm and experimental result of 1.276 kNm for the yield line theory, while ETABS results were 0.530 kNm and 0.686 kNm for both theoretical and experimental results respectively. The experimental results showed that ETABS was conservative for both type of waffle slabs, whereas yield line was conservative in the theoretical results. The differences in all the values obtained varied and the variation had to do with the support conditions, size of slabs and the magnitude of load that caused the bending moments in each slab type. Both slabs were supported on three sides.

#### **4.4.4 General observations of YLRGT and ETABS**

Generally, it was observed that both analytical methods showed variation in the analysis of waffle slabs. The differences in their moments generated were very minimal and it all depended on the size of the slabs and support conditions. ETABS gave conservative results in slabs simply supported on two short sides, YLRGT gave conservative results in slabs supported on four sides, while the results in slabs supported on three sides varies between the two methods. These observations have showed that YLRGT method for the analysis of waffle slabs was reliable if compared with existing methods of analysis. Its advantage over ETABS is the ability to analyse the slab portion and ribs part of a waffle slab separately as shown in Table 4.6.

In the design of concrete elements, ETABS and other existing programs have a constant figure of 460 N/mm<sup>2</sup> for high yield steel and 250 N/mm<sup>2</sup> for low yield steel as recommended by BS 8110. Also, in most of these packages, the characteristic strength of

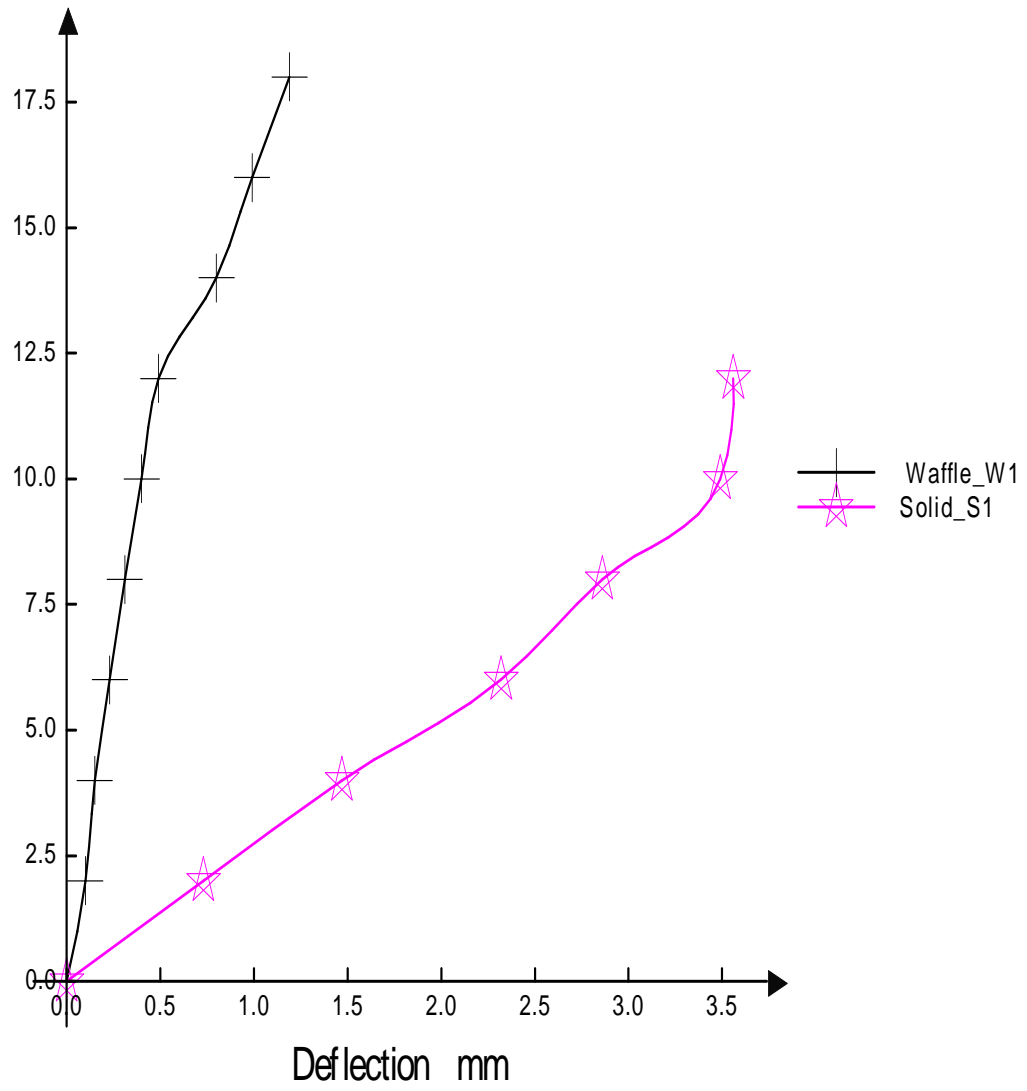
concrete had a minimum value of  $30 \text{ N/mm}^2$  because they were developed to meet the environmental challenges of specific countries. Most of these programs are not suitable for the peculiar conditions that are prevalent in Nigeria.

In a research, Salau and Farayola (2006) carried out chemical and mechanical tests on some samples of locally available steel reinforcing bars in Nigeria, it was discovered that the chemical contents of all the steel did not conform to international standards. This non-conformity led to the poor performance of all the locally available steel bars under the tensile strength test. It was recommended that the characteristics yield strength of between  $280 \text{ N/mm}^2$  and  $300 \text{ N/mm}^2$  should be adopted during the design stage. Consequently, YLRGT has introduced some flexibility to all these parameters so that the designer can use any value desired, which must be within the recommendation of BS 8110.

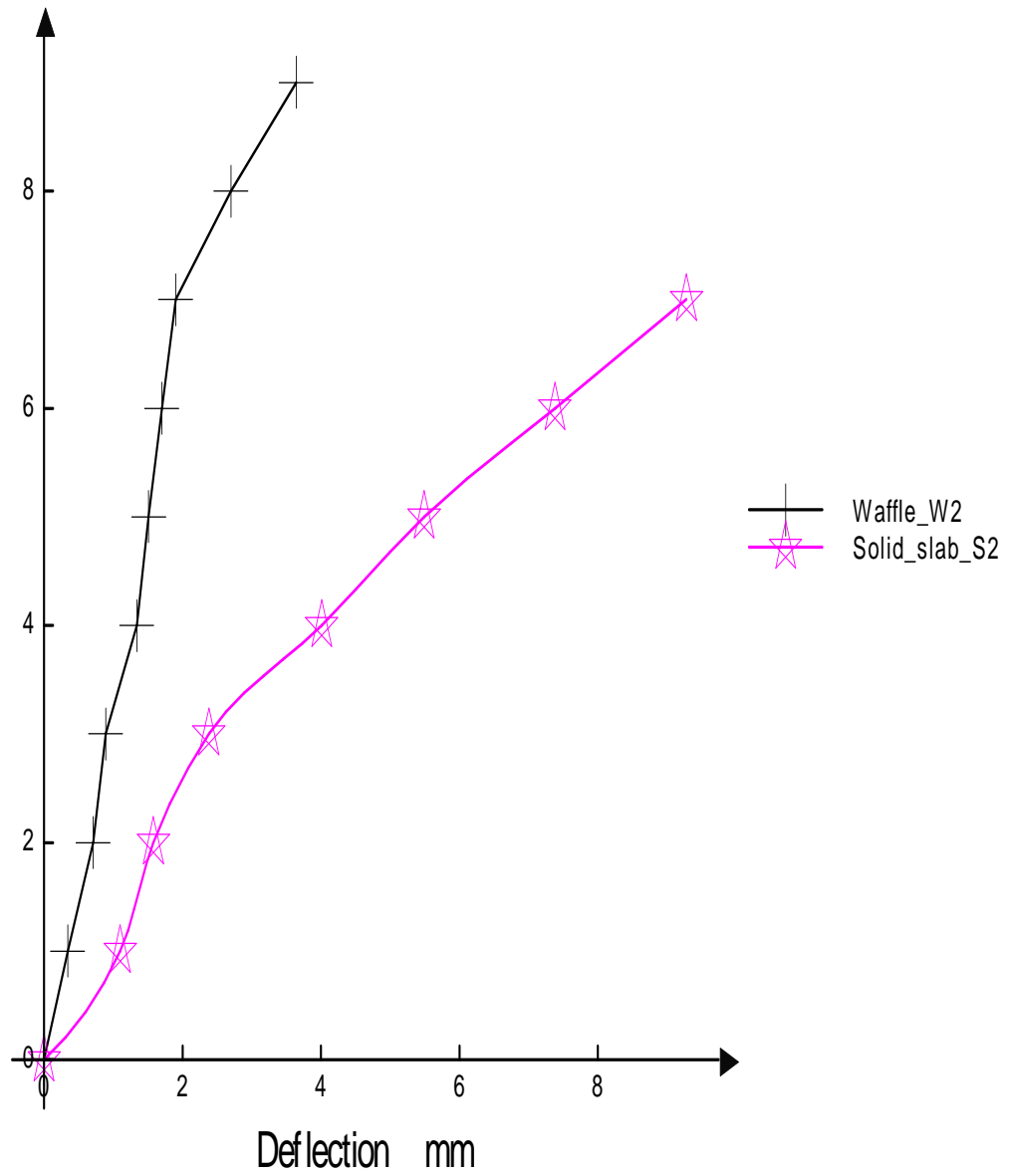
This will therefore enhance the acceptability of the proposed method in countries like Nigeria where most of the steel that are used as reinforcement in a good number of sites have yield strength that are lower than the BS 8110 recommended values. Meanwhile, the common concrete characteristic strength that is used in Nigeria is between  $20 \text{ N/mm}^2$  and  $30 \text{ N/mm}^2$ . Hence, this program is friendly to the Nigerian environment.

#### **4.5 Deflections in Specimens**

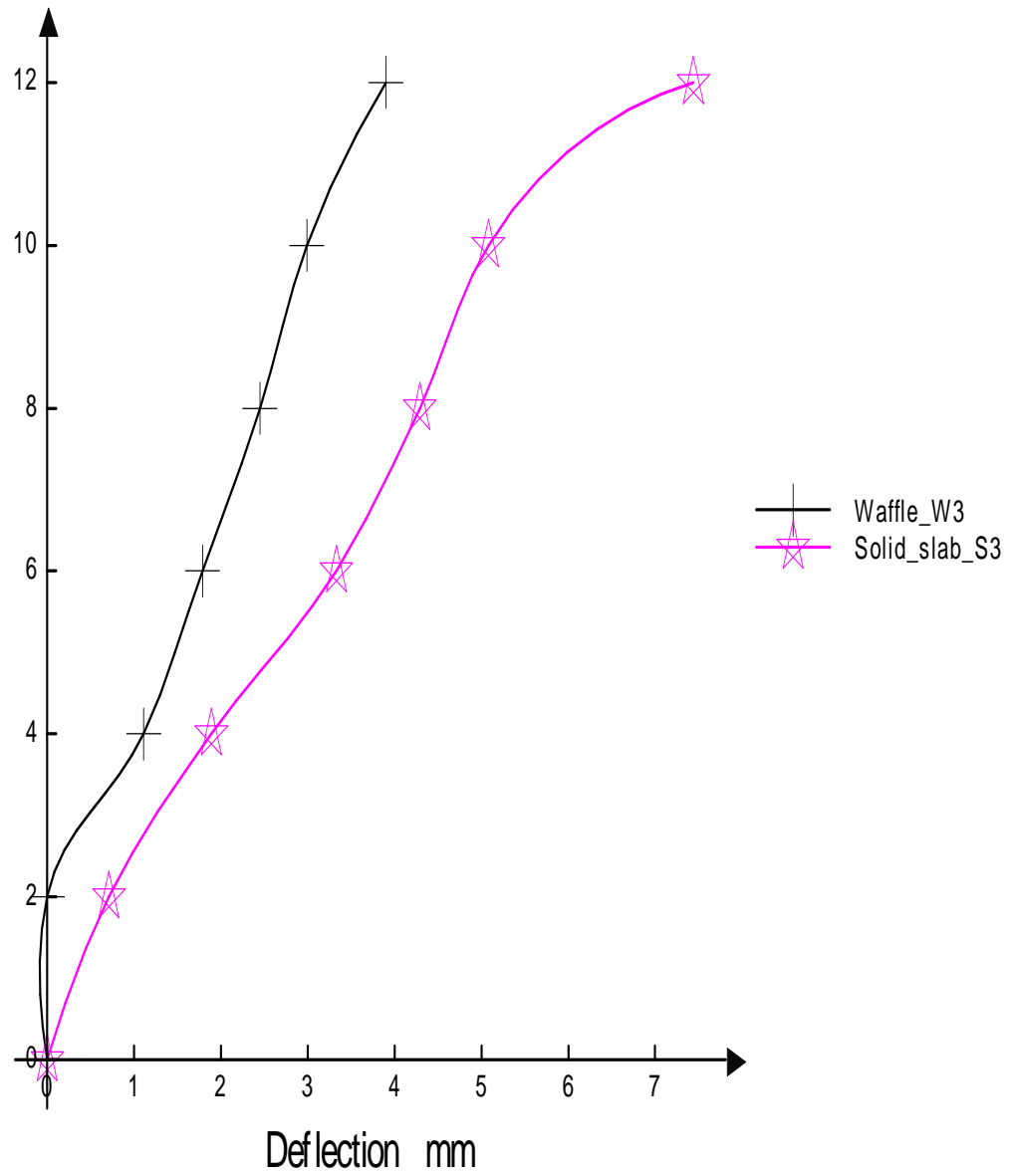
The deflection of each slab was obtained, the variation in the results of waffle slabs and the corresponding solid slabs were compared with each other and the graph of load against deflection was plotted (Figures 4.1 to 4.6).



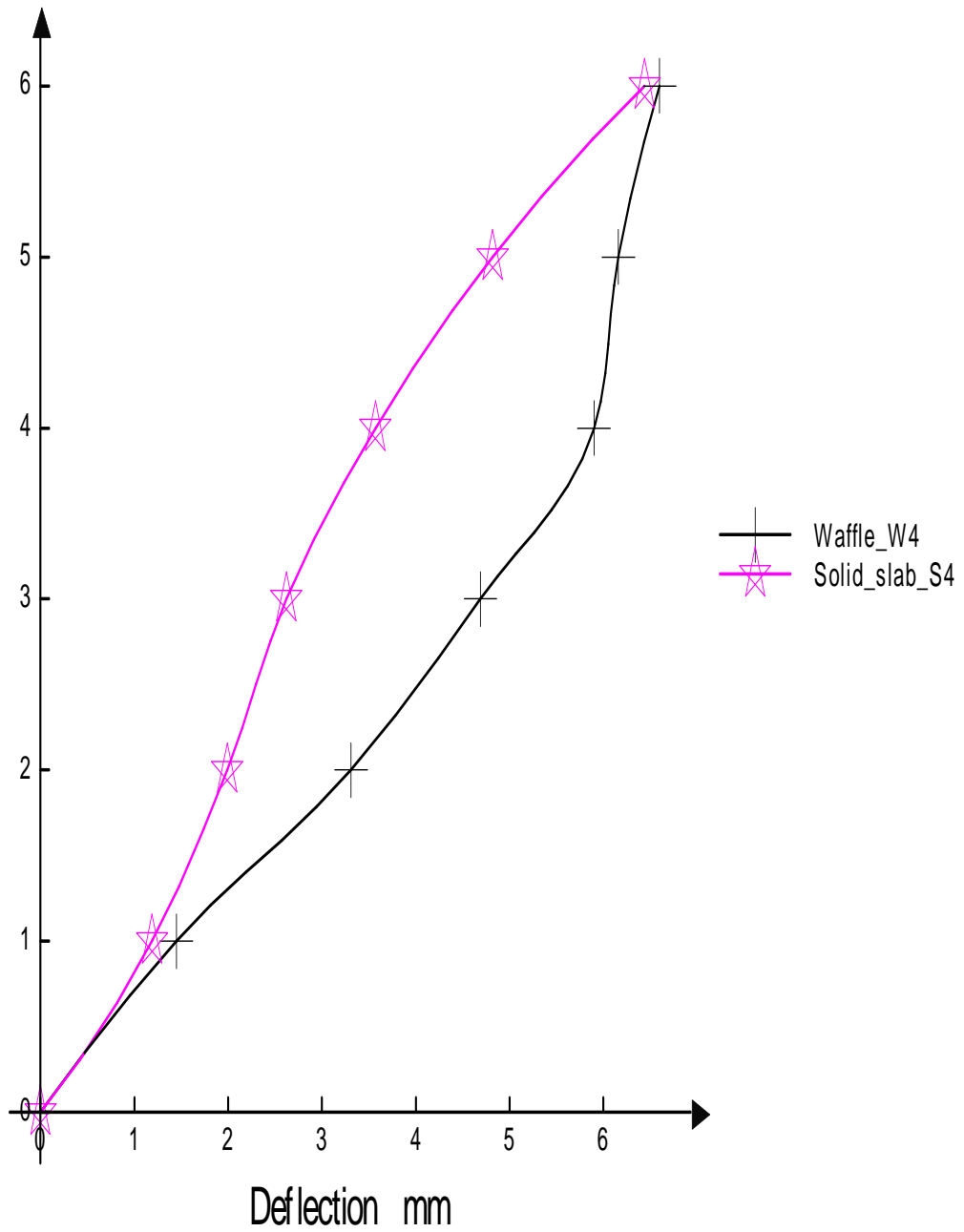
**Figure 4.1:** Variation in the deflection of W1 and S1



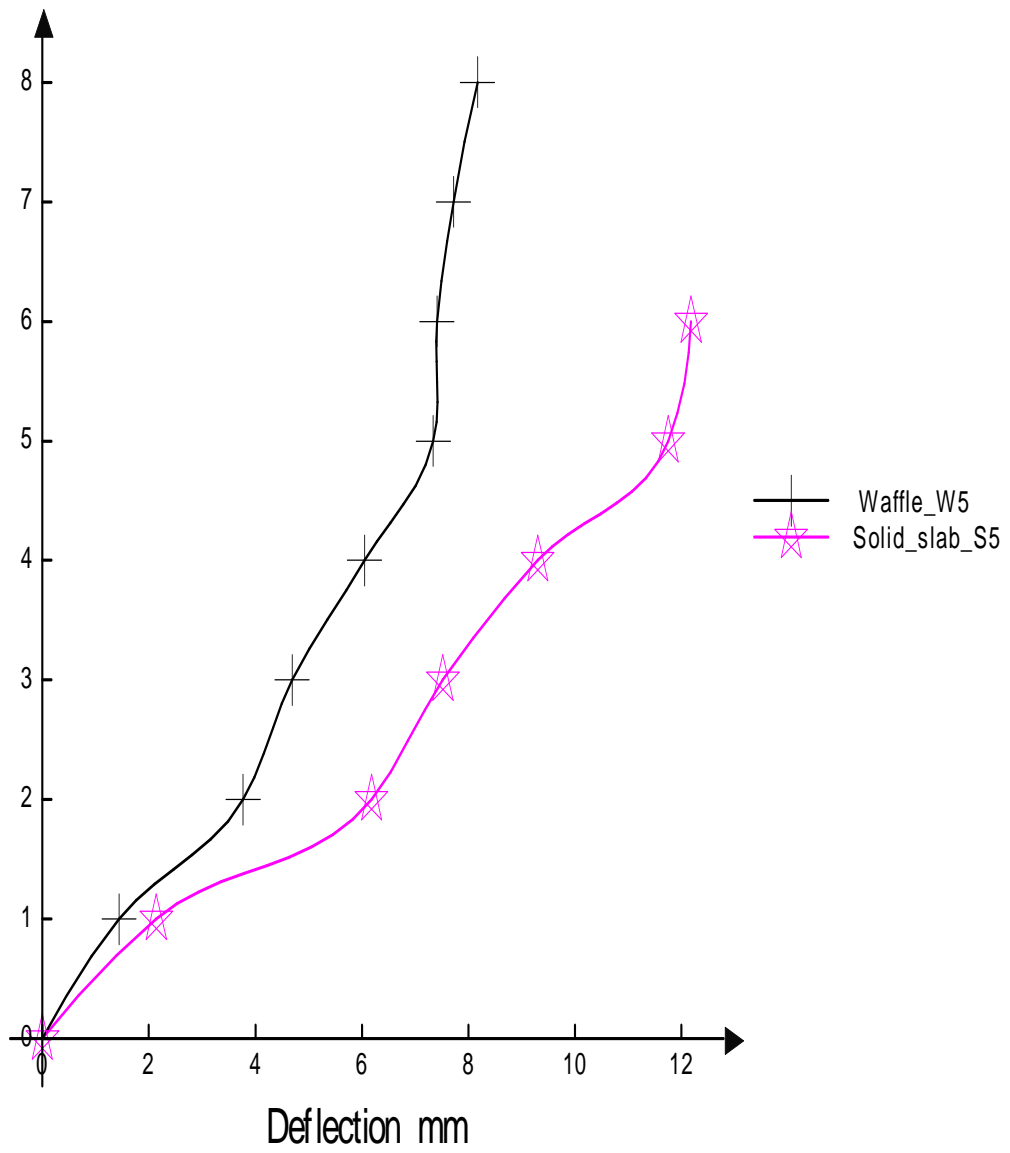
**Figure 4.2:** Variation in the deflection of W2 and S2



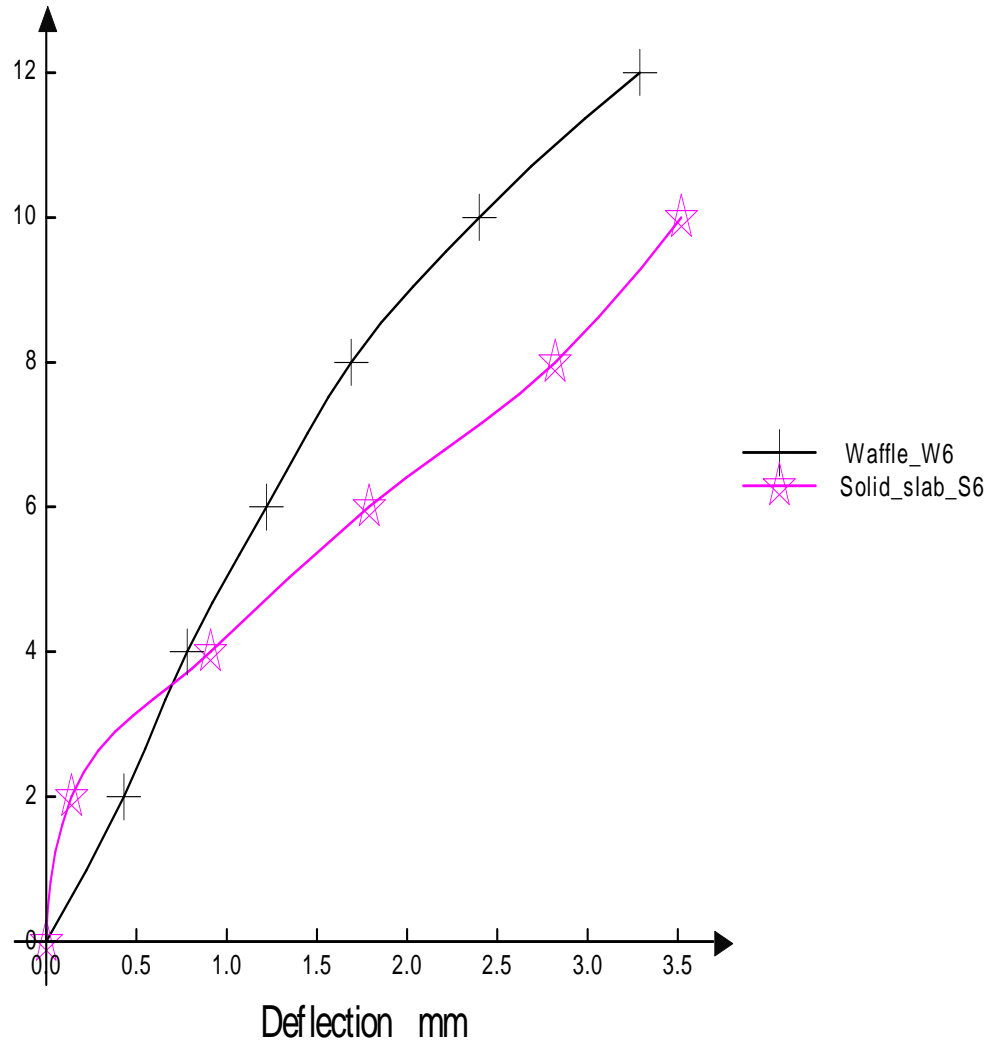
**Figure 4.3:** Variation in the deflection of W3 and S3



**Figure 4.4:** Variation in the deflection of W4 and S4



**Figure 4.5:** Variation in the deflection of W5 and S5



**Figure 4.6:** Variation in the deflection of W6 and S6



It was observed from Figure 4.1, that solid slab S1 deflected more than waffle slab W1. The deflection of S1 continued gradually forming a curve until there was a sudden change in direction at 10 kN. This continued on a straight line until failure at 12 kN. The waffle W1 curve was gradual; in fact it maintained a parallel deflection below 1mm for a long time until it got to 12kN where the elastic limit was exceeded. The same condition applied to S1, when it got to 10 kN, the elastic limit was exceeded. The rate at which slab S1 deflected showed that its ability to fail rapidly under overload was higher compared with W1 and it actually failed earlier. This confirmed that it can not withstand as much load as the corresponding waffle slab. Both slabs were simply supported on two sides.

Figure 4.2, showed the graph of waffle slab W2 and solid slab S2, it was observed that there was gradual deflection in W2. The curve formed between 1 kN and 7 kN was gradual until a sudden change at 7 kN which was the elastic limit. S2 also showed a gradual curve from the beginning until it got to 3 kN where it maintained a straight line to 7 kN which was the failure point. The elastic limit was exceeded at the 3 kN. The amount of deflection in S2 was greater than that of W2, hence it showed signs of failure earlier than W2. Both slabs were supported on four sides.

The graph of W3 and S3 (Figure 4.3) showed that S3 failed at 14 kN, and the amount of deflection in S3 is greater than that of W3. Both graphs showed gradual deflection from the beginning but as soon as W3 got to 12 kN it collapsed. The elastic limit for S3 was exceeded at 10 kN. Both slabs were simply supported on two sides.

The graph of W4 and S4 showed some differences when compared with the previous graphs Figure 4.4. In this case, W4 showed a slightly higher amount of

deflection over S4. The curve at W4 was constant until it got to 4 kN where it maintained a straight line to 6 kN. The elastic limit was exceeded after the 4 kN load point. S4 was able to maintain its curve to the 2 kN before it changes its direction. Both slabs failed at the 6 kN, they were supported on four sides.

In the W5 and S5 graph Figure 4.5, the slab S5 showed a gradual rise to 1 kN and maintain a curve to 2 kN, and change the direction of the curve up to 4 kN. The elastic limit was exceeded after the 4 kN. The amount of deflection in this slab was higher than that of its W5 counterpart. The W5 waffle slabs also showed a gradual rise from the beginning to 2 kN, it then sustained its curve between the 2 kN and 5 kN, after which a straight line was formed to the 8 kN failure point. The elastic limit was exceeded after the 5 kN load. Both slabs were supported on three sides during the test.

Figure 4.6 showed that, Slab W6 maintained a good curve from the beginning to the 12 kN load point. The amount of deflection in this slab is lower than that of S6. The shape of S6 showed that it maintained a curve from the beginning to 4 kN and the elastic limit was exceeded at this point and it eventually failed at 10 kN. Both slabs were supported on three sides.

All slabs with higher amount of displacements have lower slab stiffness when compared with those with lower amount of displacements. Waffle W1 had higher slab stiffness than solid S2; waffle W2 had better stiffness than solid S2, while waffle W3 showed good slab stiffness over solid S3. However, waffle W4 had a lower stiffness when compared with solid slab S4. Waffle W5 had better slab stiffness than solid S5 while waffle W6 also showed good slab stiffness over solid S6. All the slabs maintained their stiffnesses within the elastic limit stage, but lost it when ever the elastic limit was

exceeded, if the loads were removed before the loss of stiffnesses, there is tendency for the slab to return to its initial stage, but after losing its stiffnesses, the slab will never recover its pre-loading stage because the elastic limit was exceeded.

In general, it was observed that all the waffle slab specimens, with the exception of waffle W4, had better slab stiffness over corresponding solid slab. This must be as a result of the presence of ribs that were under tension in the various waffle slabs, and also the slab thickness.

If the corresponding solid slab was to have the same amount of slab stiffness like the waffle slab, the depth of such solid slab and the reinforcement provided would have to be increased. This solution was not the best and it was not economical. The nature of all the curves (both for waffle and solid slabs) had shown that concrete is ductile, and it can be classified as an elastoplastic material. Such material behaves in an elastic manner until the elastic limit is reached after which they behave plastically (Megson, 2002).

#### **4.6 Mean Deflection in Waffle Slabs and Solid Slabs**

Mean deflections in waffle and solid slabs were compared using analysis of variance (ANOVA) (Miller and Freund, 1987). Mean deflection for waffle slab W1 and solid slab S1 were significantly different at 5% probability level (Appendix 51). Least significant difference was used to separate the means (Table 4.7). The results showed that S1 had the higher mean value of 2.405 mm.

The results of W2 and S2 (Appendix 52) showed that the mean deflection for both slabs were significantly different at 5% probability level. Least significant difference was also used to separate the means (Table 4.8). However, S2 had the higher mean of 4.46 mm.

**Table 4.7: Least significant difference table for S1 and W1**

| <b>Parameters</b> | <b>Deflection Means</b> | <b>LSD</b> | <b>P 0.05</b> |
|-------------------|-------------------------|------------|---------------|
| S1                | 2.405a                  | 1.885      | 0.955         |
| W1                | 0.520b                  |            |               |

Means with the same letter under each column are not different at P=0.05.

Means with different letters under each column are different at P=0.05

**Table 4.8: Least significant difference table for S2 and W2**

| <b>Parameters</b> | <b>Deflection Means</b> | <b>LSD</b> | <b>P= 0.05</b> |
|-------------------|-------------------------|------------|----------------|
| S2                | 4.46a                   | 2.970      | 2.520          |
| W2                | 1.49b                   |            |                |

Means with the same letter under each column are not different at P=0.05.

Means with different letters under each column are different at P=0.05

The W3 and S3 analysis (Appendix 53) showed that there was no significant difference in the deflections of both slab at the 5% probability level, since the calculated variance ratio (F) 3.637 was less than the tabulated (F) 4.96, S3 gave a mean value of 4.15 mm, which was higher than that of W3 (2.04 mm).

In the analysis of W4 and S4 (Appendix 54), the results also showed that there was no significant difference in the mean deflection for both slabs at the 5% probability level. The calculated variance ratio (F) 0.591 is less than the tabulated (F) 4.60. W4 had the higher mean value of 5.68 mm compared to the 4.65 mm of S4.

From Appendix 55, the results also showed that there was no significant difference in the average mean values of both slab W5 and S5, as the calculated (F) 2.076 is less than the tabulated (F) 4.75. Solid slab S5 showed a higher mean value of 8.18 mm, compared with 5.82 mm of W5.

Waffle slab W6 and solid slab S6 also did not reflect any significant difference in their mean values. This is because the calculated (F) 0.000 is far below the tabulated (F) 4.96. The mean value of both slab was 1.91 mm, this same mean value contributed to the 0.00 variance ratio obtained (F). (Appendix 56)

Apart from slabs W1, S1, W2 and S2 that showed significant difference in their mean deflections with standard deviations of  $\pm 1.131$ ,  $\pm 0.388$ ,  $\pm 3.080$ ,  $\pm 1.025$  respectively, all other slab samples showed that there was no significant difference in their mean value at  $p < 0.05$  level. It can be generally concluded that despite the ability of solid slabs to fail earlier than waffle slabs, due to its low stiffness which resulted in higher deflection value, it can still maintain its stiffness up to  $p < 0.05$  level with its corresponding waffle slabs in

the case of solid slabs S1 and S2. The remaining results showed that there was minimal difference between the mean deflection of solid and waffle slabs.

#### **4.7 Correlation between the Deflections in Slabs of Similar Support Condition**

The Pearson's correlation analysis method was used to determine the correlation of the deflections of slabs that are subjected to the same boundary conditions but of different sizes. From Appendix 57, the result of the analysis shows that the correlation coefficient in the deflection of both slab W1 and W3 is 0.984 and it is significant at 0.01 levels.

For slabs W2 and W4, the correlation coefficient is 0.988, and it is significant at 0.01 levels. See Appendix 58. Waffle slabs W5 and W6 showed a correlation coefficient of 0.919 and it is also significant at 0.01 levels (Appendix 59). The results for S1 and S3 also gave a correlation coefficient of 0.983, and it is significant at 0.01 levels as well. See Appendix 60. S2 and S4 gave a coefficient of 0.997 and were significant at the 0.01 levels (Appendix 61). From Appendix 62, solid slabs S5 and S6 gave a coefficient of 0.974 values, and it is also significant at 0.01 levels.

It can be generally concluded that the results obtained in the comparison showed that the deflections were within the same range, indicating a good relationship between the displacements values and positive correlation results. All results were significant at the 0.01 levels. This can imply that slabs of the same boundary conditions have corresponding displacement, notwithstanding the sizes of the slabs.

#### 4.8 Flexural Cracking

Members subjected to bending generally exhibit a series of distributed flexural cracks, even at service load. These cracks are harmless and can not cause any damage to the structure unless the widths become excessive (Mosley *et al*, 1999). Excessively wide cracks can be unsightly and spoil the appearance of an exposed concrete surface. They can allow the ingress of moisture and accelerate corrosion of the reinforcement and durability failure. In exceptional cases; they can reduce the contribution of the concrete to the shear strength of a member (Gilbert, 2001).

The crack width at service for both the waffle and solid slabs were estimated, Table 4.9, the example in Appendix 63 is the estimated crack width for solid slab S1 at service. The results showed that all the solid slabs have lower crack widths at service, when compared with corresponding waffle slabs. This is because the estimated crack width in waffle at service was based on the slab portions only, the ribs were not considered. The depth of the slab portion was small if compared with that of the solid slab, also the size of reinforcement used (wire mesh) in the slab portion was small if compared to the steel reinforcement used in the solid slabs and these were the parameters used to determine the estimated crack width. In the experiment, the crack widths at failure in waffle slabs were smaller than those of solid slabs. The presence of ribs in the waffle reduced the effect of load on the slab portion, since the ribs were directly exposed to tensile forces instead of the slab portion. The solid slabs were however exposed to the direct tensile forces which resulted in the flexural cracks that were formed.



**Table 4.9: Crack width at failure and service**

| <b>Specimens</b> | <b>Crack width at failure load (mm)</b> | <b>Estimated crack width at service (mm)</b> |
|------------------|---|--|
| W1               | 0.60                                    | 0.425  |
| W2               | 0.35                                    | 0.065  |
| W3               | 0.34                                    | 0.232  |
| W4               | 0.25                                    | 0.063  |
| W5               | 0.35                                    | 0.054  |
| W6               | 0.62                                    | 0.283  |
| S1               | 1.20                                    | 0.035  |
| S2               | 0.75                                    | 0.002  |
| S3               | 0.90                                    | 0.049  |
| S4               | 0.70                                    | 0.003  |
| S5               | 1.10                                    | 0.006  |
| S6               | 1.30                                    | 0.009  |

#### **4.8.1 Comparison of mean crack width between waffle slab and solid slab**

Crack width mean at failure load in waffle and solid slabs were compared using analysis of variance. The crack width mean for waffle slab and solid slab were significantly different at 5% probability level (Appendix 64). Least significant difference was used to separate the means, and the results showed that the differences in means were very minimal (Table 4.10). The results showed that solid slab had the higher mean value of 0.99 mm, compared to that of waffle slab which is 0.48 mm. These results confirmed that the crack widths formed in solid slabs were wider than those of waffle slabs in the experiment.

Consequently, the presence of excess crack width in solid slabs samples must have contributed to their earlier failure when compared with corresponding waffle slabs.

**Table 4.10: Least significant difference table of crack width for Waffle and Solid slabs**

| <b>Parameters</b> | <b>Crack width Means</b> | <b>LSD</b> | <b>P= 0.05</b> |
|-------------------|--------------------------|------------|----------------|
| Solid slab        | 0.99a                    | 0.507      | 0.907          |
| Waffle slab       | 0.48a                    |            |                |

Means with the same letter under each column are not different at P=0.05.

Means with different letters under each column are different at P=0.05

## CHAPTER FIVE

### CONCLUSIONS AND RECOMMENDATIONS

#### 5.1 Conclusions

The structural characteristics of waffle slabs have been determined through the investigation of the slabs and its response to applied loads. The bending moments of the slabs were determined using YLRGT.

- i. The response of waffle slabs to load afforded their advantages over solid slabs because the crack width formed as a result of loading to failure is generally small when compared to corresponding solid slabs. The crack patterns formed in solid slabs follow the effects of the types of support conditions which the slabs were exposed to, unlike waffle slabs where the crack patterns were not precise and did not follow a particular pattern. This was due to the presence of ribs in the waffle slabs.
- ii. The bending moments of waffle slabs are generally higher than that of solid slabs in all the tests carried out except in two cases where the collapse load of both slabs were equal. This implied that the stiffness of waffle slab is higher than that of solid slabs in its response to failure loads.
- iii. The grillage analytical method that was used in the analysis of the ribs in the prototype was defective because it considered moments in the longitudinal ribs only while neglecting the transverse ribs, although nominal reinforcements were provided for the transverse ribs, this eventually led to over reinforcement of the ribs. The Rankine Grashoff method adopted in the analysis of the model was able to overcome this defect in the prototype by providing appropriate reinforcement for both transverse and longitudinal ribs.

iv. Statistical analysis revealed that solid slabs generally have higher mean deflections than waffle slabs. However, solid slab S1 and S2 are significantly different in mean deflections at ( $p=0.05$ ) over the corresponding waffle slabs W1 and W2 respectively. The significant difference between the other solid slabs and corresponding waffle slabs are less than 1%. This implied that solid slabs deflect more than waffle slabs under failure loads.

v. The graph of load against deflection also showed the gradual deflection of both waffle and solid slabs, but solid slabs generally deflect more than waffle slabs.

vi. The Pearson's correlation analysis method revealed that slabs that are subjected to the same boundary conditions showed positive correlation results in their deflections and are all significant at 0.01 levels.

vii. The crack width formed in solid slab is significantly different from that of waffle slabs at the ( $p=0.05$ ) probability level, although the difference is very minimal. This implied that the average crack width in solid slab is wider than that of waffle slab at failure load. This must have contributed to the low flexural strength of solid slabs if compared with waffle slabs.

viii. YLRGT also gave more dependable results when it was compared with the BS 8110 slab coefficients method that some engineers have adopted in the analysis of the waffle slab ribs.

ix. The study has showed that the reason for the failures observed in the prototype was as a result of approximate analytical procedures adopted in the design of the waffle slabs and the provision of inadequate reinforcements at the slab portions while the ribs were over reinforced.

x. YLRGT developed in this study for solving the problem was validated in the case study; by comparing the results of YLRGT analytical method with the results obtained from ETABS which is based on the finite element method (FEM). The results obtained showed good relationship between the two methods. However, the advantage of YLRGT over ETABS is the analysis of the slab portion separately using the Yield line theory and the rib portion by adopting the Rankine Grashoff theory in waffle slabs, the program can also be effectively used for the analysis of solid slabs.

xi. The introduction of YLRGT has facilitated the improved analysis of pre-cast waffle slabs. The use of YLRGT will enhance the structural integrity of pre-cast waffle slab as revealed when compared with the finite element method. The program is safe and it is very easy to use.

## **5.2 Contribution to Knowledge**

- i. This research has provided an improved method of analysing and designing separately, the slab and rib of waffle slabs. Adequate reinforcements can therefore be put in the appropriate sections to ensure structural integrity and reliability.
- ii. The new method of analysis (YLRGT) can also be used in solving the problems of solid slabs.

## **5.4 Recommendations for Further Research**

The following suggestions are recommended for further research as extension of this investigation:

- (i.) Equipment that can give incremental results for every crack width measured is recommended.

(ii) The use of crack measuring meter or crack detecting microscope instead of Vernier caliper is suggested to give correct values of all measured crack widths.

(iii). During the conception and design of civil engineering structures, efforts should be made by the design engineer to adopt effective and reliable analytical methods in order to avoid the type of problems encountered in this study.

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## **APPENDICES**

## Appendix 1: Theoretical result of waffle slab W1.

### ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY

CODE B.S.8110 PART1 1997

Designed by Akinyele J.O

simply supported on two sides

|   |         |
|---|---------|
| Slab Long span (ly) (m)                 | 1.353   |
| Slab Short span (lx) (m)                | 0.43    |
| Depth of Slab part (mm)                 | 27      |
| Distance between ribs on Long Span (m)  | 0.26    |
| Distance between ribs on Short Span (m) | 0.135   |
| Rib width (mm)                          | 20      |
| Depth of ribs (mm)                      | 33      |
| Total depth of slab (mm)                | 60      |
| Cover to reinforcement (mm)             | 5       |
| Effective depth of slab (mm)            | 22      |
| Slab dead load (kN/m <sup>2</sup> )     | 0.9072  |
| Slab imposed load (kN/m <sup>2</sup> )  | 3.2     |
| Finishes and Partitions                 | 5.32    |
| Total load on slab (kN/m <sup>2</sup> ) | 9.4272  |
| I <sub>1</sub> fixity at 1st support    | 0       |
| I <sub>2</sub> fixity at 2nd support    | 0       |
| Bending moment in Slab (KNm)            | 2.15719 |

### DESIGN OF SLAB

|   |         |
|---|---------|
| Concrete characteristic strength (N/mm <sup>2</sup> ) | 20      |
| Steel characteristic strength (N/mm <sup>2</sup> )    | 250     |
| Maximum Lever arm factor                              | 0.95    |
| Maximum K- value                                      | 0.156   |
| Moment M  | 2157190 |
| K-value   | 0.22285 |
| Lever arm Factor                                      | 0.54887 |
| Used Lever arm  | 0.54887 |
| Area of Steel (mm <sup>2</sup> )                      | 752.192 |

[B785 structural mesh@100mmc/c bottom](#)

Provide [\(785mm<sup>2</sup>\)](#)

### ANALYSIS AND DESIGN OF RIBS

Adopting the Rankine Grashoff Theory.

|   |         |
|---|---------|
| Number of Ribs on Long Span (ly)                      | 4       |
| Number of Ribs on Short Span (lx)                     | 2       |
| Weight of Slabs (kN/m <sup>2</sup> )                  | 9.4272  |
| Total Load on Slab (KN)                               | 5.48465 |
| Weight of Ribs (kN/m)                                 | 0.01584 |
| Total weight of Ribs on long span (ly), (kN)          | 0.10715 |
| Total weight of Ribs on Short span (lx), (kN)         | 0.01362 |
| Total weight of Finishes (kN)                         | 0.58179 |
| Total Live Load (kN)                                  | 0.93086 |
| Total Load on grid floor (kN)                         | 7.11808 |
| q=Load per Area (m <sup>2</sup> )                     | 12.2348 |
| Denominator for qx and qy                             | 3.38531 |
| Load sheared on short span (qx), (KN/m <sup>2</sup> ) | 12.1112 |
| Load sheared on Long span (qy), (kN/m <sup>2</sup> )  | 0.12355 |



|  |   |
|--|---|
| Moment on short span $M_x$ (kN.m)                  | 0.03778                                     |
| Moment on long span $M_y$ (kN.m)                   | 0.00735                                     |
| Shear force on short span, $V_x$ , (kN)            | 0.67701                                     |
| Shear force on long span, $V_y$ , (kN)             | 0.01128                                     |
| DESIGN   |   |
| Steel characteristic strength (N/mm <sup>2</sup> ) | 250   |
| Short span design moment (Nmm)                     | 37789.3                                     |
| Cover to ribs (mm)                                 | 5   |
| Effective depth of ribs                            | 28  |
| K.value  | 0.12050                                     |
| Lever arm Factor                                   | 0.84074                                     |
| Used Lever arm                                     | 0.84074                                     |
| Area of Steel (mm <sup>2</sup> )                   | 3.32020                                     |
| Provide  | <a href="#">1R6@Bottom-28mm<sup>2</sup></a> |
| Long span design moment (Nmm)                      | 7351.06                                     |
| K.value  | 0.02344                                     |
| Lever arm Factor                                   | 0.97323                                     |
| Used Lever arm                                     | 0.97323                                     |
| Area of Steel (mm <sup>2</sup> )                   | 1.13582                                     |
| Provide  | <a href="#">1R6@Bottom-28mm<sup>2</sup></a> |
| END DESIGN   |   |

## Appendix 2: Theoretical results of waffle slab W2.

ANALYSIS AND DESIGN OF WAFFLE SLABS USING THE YIELD LINE THEORY.

CODE B.S. 8110: PART 1: 1997.

Designed By : Engr. Akinyele J.O

Simply supported on all four sides.

|   |       |
|---|-------|
| Slab Length(Long span) ( $l_y$ ) (m)    | 0.9   |
| Slab Width (Short span) ( $l_x$ ) (m)   | 0.3   |
| Rib width (mm)                          | 20    |
| Distance between ribs on Long span (m)  | 0.2   |
| Distance between ribs on Short span (m) | 0.17  |
| Depth of slab portion (mm)              | 15    |
| Depth of ribs (mm)                      | 35    |
| Total depth of slab (mm)                | 50    |
| Effective depth of slab (mm)            | 10    |
| Slab dead load $\text{kN/m}^2$          | 0.504 |
| Imposed load on slab $\text{kN/m}^2$    | 5.32  |
| Finishes and Partition load             | 3.2   |
| Total load on slab ( $\text{kN/m}^2$ )  | 9.024 |

### ANALYSIS OF SLABS.

|  |         |
|--|---------|
| Fixity ratio for continuous supports on any side |         |
| Fixity ratio short span left side $I_1$          | 0       |
| Fixity ratio long span bottom $I_2$              | 0       |
| Fixity ratio short span right side $I_3$         | 0       |
| Fixity ratio long span top $I_4$                 | 0       |
| Reduced sides $a_r'$                             | 0.3     |
| Reduced sides $b_r'$                             | 0.9     |
| Bending moments in slab (KNm)                    | 0.08434 |

### DESIGN

|  |   |
|--|---|
| Cover to reinforcement. (mm)                       | 5   |
| Concrete characteristic stress ( $\text{N/mm}^2$ ) | 30  |
| Steel Characteristic Stress ( $\text{N/mm}^2$ )    | 250   |
| Maximum lever arm factor                           | 0.95  |
| Maximum K- Value                                   | 0.156   |
| Moment M   | 84339.69  |
| K- value   | 0.028113  |
| Lever arm factor                                   | 0.967721  |
| Used Lever arm                                     | 0.95  |
| Area of steel ( $\text{mm}^2$ )                    | 37.38047  |
| Provide  | <a href="#">R2.5@100mmc/c(49mm<sup>2</sup>)</a> |

### Appendix 3: Theoretical results of waffle slab W3.

#### ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY

CODE B.S.8110 PART1 1997

Designed by Akinyele J.O

SUPPORTED ON TWO SIDE (simply supported)

|   |         |
|---|---------|
| Slab Long span (ly) (m)                 | 1.085   |
| Slab Short span (lx) (m)                | 0.43    |
| Depth of Slab part (mm)                 | 25      |
| Distance between ribs on Long Span (m)  | 0.26    |
| Distance between ribs on Short Span (m) | 0.135   |
| Rib width (mm)                          | 20      |
| Depth of ribs (mm)                      | 33      |
| Total depth of slab (mm)                | 58      |
| Cover to reinforcement (mm)             | 5       |
| Effective depth of slab (mm)            | 20      |
| Slab dead load (KN/m <sup>2</sup> )     | 0.84    |
| Slab imposed load (KN/m <sup>2</sup> )  | 5.32    |
| Finishes and Partitions                 | 3.2     |
| Total load on slab (KN/m <sup>2</sup> ) | 9.36    |
| I <sub>1</sub> fixity at 1st support    | 0       |
| I <sub>2</sub> fixity at 2nd support    | 0       |
| Bending moment in Slab (KNm)            | 1.65282 |

#### DESIGN OF SLAB

|   |   |
|---|---|
| Concrete characteristic stress (N/mm <sup>2</sup> ) | 20  |
| Steel characteristic stress (N/mm <sup>2</sup> )    | 250   |
| Maximum Lever arm factor                            | 0.95  |
| Maximum K- value                                    | 0.156   |
|   | 165282  |
| Moment M  | 4   |
| K-value   | 0.20660   |
| Lever arm Factor                                    | 0.64297   |
| Used Lever arm                                      | 0.64297   |
| Area of Steel (mm <sup>2</sup> )                    | 541.178   |
| Provide   | <a href="#">R6@50mmc/c bottom (566mm<sup>2</sup>)</a> |

#### Appendix 4: Theoretical results of waffle slab W4.

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.

CODE B.S. 8110: PART 1: 1997.

Designed By : Engr. Akinyele J.O

Simple supported on all four sides.

|   |       |
|---|-------|
| Slab Length(Long span) ( $l_y$ ) (m)    | 0.407 |
| Slab Width (Short span) ( $l_x$ ) (m)   | 0.364 |
| Rib width (mm)                          | 20    |
| Distance between ribs on Long span (m)  | 0.2   |
| Distance between ribs on Short span (m) | 0.17  |
| Depth of slab portion (mm)              | 15    |
| Depth of ribs (mm)                      | 35    |
| Total depth of slab (mm)                | 50    |
| Effective depth of slab (mm)            | 10    |
| Slab dead load $\text{kN/m}^2$          | 0.504 |
| Imposed load on slab $\text{kN/m}^2$    | 5.32  |
| Finishes and Partition load             | 3.2   |
| Total load on slab ( $\text{kN/m}^2$ )  | 9.024 |

#### ANALYSIS OF SLABS.

Fixity ratio for continuous supports on any side

|  |          |
|--|----------|
| Fixity ratio short span left side $I_1$  | 0        |
| Fixity ratio long span bottom $I_2$      | 0        |
| Fixity ratio short span right side $I_3$ | 0        |
| Fixity ratio long span top $i_4$         | 0        |
| Reduced sides $a_r'$                     | 0.364    |
| Reduced sides $b_r'$                     | 0.407    |
| Bending moments in slab (KNm)            | 0.066567 |

#### DESIGN

|  |   |
|--|---|
| Cover to reinforcement. (mm)                       | 5   |
| Concrete characteristic stress ( $\text{N/mm}^2$ ) | 20  |
| Steel Characteristic Stress ( $\text{N/mm}^2$ )    | 250   |
| Maximum lever arm factor                           | 0.95  |
| Maximum K- Value                                   | 0.156   |
| Moment M   | 66567.44  |
| K- value   | 0.033284  |
| Lever arm factor                                   | 0.961539  |
| Used Lever arm                                     | 0.95  |
| Area of steel ( $\text{mm}^2$ )                    | 29.50357  |
| Provide  | <a href="#">R2.5@100mmc/c(49mm<sup>2</sup>)</a> |

## Appendix 5: Theoretical results of waffle slab W5.

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY

CODE: B.S. 8110: Part 1

1997

Designed By : Akinyele

J.O

Simply supported on 3 sides

|  |          |
|--|----------|
| Slab Long Span ( $l_y$ ) (m)                       | 1.312    |
| Slab Short Span ( $l_x$ ) (m)                      | 0.3      |
| Depth of Slab part (mm)                            | 25       |
| Distance between ribs on long span (m)             | 0.32     |
| Distance between ribs on short span (m)            | 0.14     |
| Rib width (mm)                                     | 20       |
| Rib depth (mm)                                     | 33       |
| Total depth of slab (mm)                           | 58       |
| Cover to reinforcement (mm)                        | 5        |
| Effective depth of slab (mm)                       | 20       |
| Slab dead load ( $\text{kN/m}^2$ )                 | 0.84     |
| Slab imposed load ( $\text{kN/m}^2$ )              | 5.32     |
| Finishes and Partition load                        | 3.2      |
| Total load on Slab ( $\text{kN/m}^2$ )             | 9.36     |
| For case A, $= h_1+h_3 < b$                        |          |
| $l_1$ = fixity1 for left side of short span        | 0        |
| $l_2$ = fixity2 for supported side of long span    | 0        |
| $l_3$ = fixity3 for right side of short span       | 0        |
| Reduced span $b_r$ (m)                             | 1.312    |
| $K'$ = redistribution factor                       | 0.152439 |
| $h$ = factor to determine yield length             | 0.236725 |
| 1st Bending moment in slab (kNm)                   | 0.436058 |
| For case B, $= h_1+h_3 = b$                        |          |
| $b'$ = dimensions                                  | 0.6      |
| 2nd. Bending moment in slab (kNm)                  | 0.303302 |
| DESIGN   |          |
| Concrete characteristic stress ( $\text{N/mm}^2$ ) | 20       |
| Steel Characteristic Stress ( $\text{N/mm}^2$ )    | 250      |
| Maximum Lever arm factor                           | 0.95     |
| Maximum K- value                                   | 0.156    |
| Used Bending moment (kNm)                          | 0.436058 |
| Moment $M$   | 436058.3 |
| K-value  | 0.054507 |
| Lever arm factor                                   | 0.935243 |
| Used Lever arm                                     | 0.935243 |
| Area of Steel ( $\text{mm}^2$ )                    | 98.15818 |

Provide [R5@B196Struct.mesh100mmc/c](#)  
[bottom\(196mm<sup>2</sup>\)](#)

## Appendix 6: Theoretical result of waffle slab W6.

### ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY

CODE: B.S. 8110: Part 1

1997

Designed By : Akinyele

J.O

Simply supported on 3 sides

|  |  |
|--|--|
| Slab Long Span ( $l_y$ ) (m)                       | 0.86                                       |
| Slab Short Span ( $l_x$ ) (m)                      | 0.36                                       |
| Depth of Slab part (mm)                            | 15   |
| Distance between ribs on long span (m)             | 0.32                                       |
| Distance between ribs on short span (m)            | 0.14                                       |
| Rib width (mm)                                     | 20   |
| Rib depth (mm)                                     | 35   |
| Total depth of slab (mm)                           | 50   |
| Cover to reinforcement (mm)                        | 5  |
| Effective depth of slab (mm)                       | 10   |
| Slab dead load ( $\text{kN/m}^2$ )                 | 0.504                                      |
| Slab imposed load ( $\text{kN/m}^2$ )              | 5.32                                       |
| Finishes and Partition load                        | 3.2  |
| Total load on Slab ( $\text{kN/m}^2$ )             | 9.024                                      |
| For case A, $= h_1 + h_3 < b$                      |  |
| $l_1 =$ fixity1 for left side of short span        | 0  |
| $l_2 =$ fixity2 for supported side of long span    | 0  |
| $l_3 =$ fixity3 for right side of short span       | 0  |
| Reduced span $b_r$ (m)                             | 0.86                                       |
| $K' =$ redistribution factor                       | 0.27907                                    |
| $H =$ factor to determine yield length             | 0.255007                                   |
| 1st Bending moment in slab (kNm)                   | 0.296852                                   |
| For case B, $= h_1 + h_3 = b$                      |  |
| $b' =$ dimensions                                  | 0.72                                       |
| 2nd. Bending moment in slab (kNm)                  | 0.276466                                   |
| DESIGN   |  |
| Concrete characteristic stress ( $\text{N/mm}^2$ ) | 20   |
| Steel Characteristic Stress ( $\text{N/mm}^2$ )    | 250  |
| Maximum Lever arm factor                           | 0.95                                       |
| Maximum K- value                                   | 0.156                                      |
| Used Bending moment (kNm)                          | 0.296852                                   |
| Moment $M$   | 296852.4                                   |
| K-value  | 0.148426                                   |
| Lever arm factor                                   | 0.791688                                   |
| Used Lever arm                                     | 0.791688                                   |
| Area of Steel ( $\text{mm}^2$ )                    | 157.8784                                   |
| Provide  | <a href="#">R5/B196Struct.mesh100mmc/c</a> |

[bottom\(196mm<sup>2</sup>\)](#)

## Appendix 7: Theoretical result of solid slab S1.

### ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY

CODE B.S.8110 PART1 1997

Designed by Akinyele J.O

simply supported on two short sides

|   |         |
|---|---------|
| Slab Long span (ly) (m)                 | 1.353   |
| Slab Short span (lx) (m)                | 0.43    |
| Depth of Slab part (mm)                 | 40      |
| Distance between ribs on Long Span (m)  | 0       |
| Distance between ribs on Short Span (m) | 0       |
| Rib width (mm)                          | 0       |
| Depth of ribs (mm)                      | 0       |
| Total depth of slab (mm)                | 40      |
| Cover to reinforcement (mm)             | 5       |
| Effective depth of slab (mm)            | 35      |
| Slab dead load (kN/m <sup>2</sup> )     | 1.344   |
| Slab imposed load (kN/m <sup>2</sup> )  | 3.2     |
| Finishes and Partitions                 | 5.32    |
| Total load on slab (kN/m <sup>2</sup> ) | 9.864   |
| I <sub>1</sub> fixity at 1st support    | 0       |
| I <sub>2</sub> fixity at 2nd support    | 0       |
| Bending moment in Slab (kNm)            | 2.25714 |

### DESIGN OF SLAB

|   |   |
|---|---|
| Concrete characteristic strength (N/mm <sup>2</sup> ) | 20  |
| Steel characteristic strength (N/mm <sup>2</sup> )    | 250   |
| Maximum Lever arm factor                              | 0.95  |
| Maximum K- value                                      | 0.156   |
| Moment M  | 225714  |
| K-value   | 0.09212   |
| Lever arm Factor                                      | 0.88423   |
| Used Lever arm  | 0.88423   |
| Area of Steel (mm <sup>2</sup> )                      | 307.086   |
| Provide   | <a href="#">R6@75mmc/c bottom (377mm<sup>2</sup>)</a> |

## Appendix 8: Theoretical result of solid slab S2.

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.

CODE B.S. 8110: PART 1: 1997.

Designed By : Engr. Akinyele J.O

Simply supported on all four sides.

|   |       |
|---|-------|
| Slab Length(Long span) ( $l_y$ ) (m)    | 0.9   |
| Slab Width (Short span) ( $l_x$ ) (m)   | 0.3   |
| Rib width (mm)                          | 0     |
| Distance between ribs on Long span (m)  | 0     |
| Distance between ribs on Short span (m) | 0     |
| Depth of slab portion (mm)              | 40    |
| Depth of ribs (mm)                      | 0     |
| Total depth of slab (mm)                | 40    |
| Effective depth of slab (mm)            | 35    |
| Slab dead load $kN/m^2$                 | 1.344 |
| Imposed load on slab $kN/m^2$           | 3.2   |
| Finishes and Partition load             | 5.32  |
| Total load on slab ( $kN/m^2$ )         | 9.864 |

### ANALYSIS OF SLABS.

|  |          |
|--|----------|
| Fixity ratio for continuous supports on any side |          |
| Fixity ratio short span left side $i_1$          | 0        |
| Fixity ratio long span bottom $i_2$              | 0        |
| Fixity ratio short span right side $i_3$         | 0        |
| Fixity ratio long span top $i_4$                 | 0        |
| Reduced sides $a_r'$                             | 0.3      |
| Reduced sides $b_r'$                             | 0.9      |
| Bending moments in slab (kN.m)                   | 0.092425 |

### DESIGN

|   |  |
|---|--|
| Cover to reinforcement. (mm)                  | 5  |
| Concrete characteristic strength ( $N/mm^2$ ) | 20   |
| Steel Characteristic Strength ( $N/mm^2$ )    | 250  |
| Maximum lever arm factor                      | 0.95   |
| Maximum K- Value                              | 0.156  |
| Moment M                                      | 92425.38                                       |
| K- value                                      | 0.003136                                       |
| Lever arm factor                              | 0.996504                                       |
| Used Lever arm                                | 0.95   |
| Area of steel ( $mm^2$ )                      | 9.728581                                       |
| Provide                                       | <a href="#">R6@100mmc/c(283mm<sup>2</sup>)</a> |



### Appendix 9: Theoretical result of solid slab S3.

#### ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY

CODE B.S.8110 PART1 1997

Designed by Akinyele J.O

simply supported on two short sides

|   |        |
|---|--------|
| Slab Long span (ly) (m)                 | 1.085  |
| Slab Short span (lx) (m)                | 0.43   |
| Depth of Slab part (mm)                 | 40     |
| Distance between ribs on Long Span (m)  | 0      |
| Distance between ribs on Short Span (m) | 0      |
| Rib width (mm)                          | 0      |
| Depth of ribs (mm)                      | 0      |
| Total depth of slab (mm)                | 40     |
| Cover to reinforcement (mm)             | 5      |
| Effective depth of slab (mm)            | 35     |
| Slab dead load (kN/m <sup>2</sup> )     | 1.344  |
| Slab imposed load (kN/m <sup>2</sup> )  | 3.2    |
| Finishes and Partitions                 | 5.32   |
| Total load on slab (kN/m <sup>2</sup> ) | 9.864  |
| I <sub>1</sub> fixity at 1st support    | 0      |
| I <sub>2</sub> fixity at 2nd support    | 0      |
| Bending moment in Slab (kNm)            | 1.4515 |
|   | 1      |

#### DESIGN OF SLAB

|   |  |
|---|--|
| Concrete characteristic strength (N/mm <sup>2</sup> ) | 20   |
| Steel characteristic strength (N/mm <sup>2</sup> )    | 250  |
| Maximum Lever arm factor                              | 0.95   |
| Maximum K- value                                      | 0.156  |
| Moment M  | 145151   |
| K-value   | 0.0592   |
| Lever arm Factor                                      | 0.9291   |
| Used Lever arm  | 0.9291   |
| Area of Steel (mm <sup>2</sup> )                      | 187.93   |
| Provide   | <a href="#">R6@100mmc/c bottom (283mm<sup>2</sup>)</a> |

## Appendix 10: Theoretical result of solid slab S4.

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.

CODE B.S. 8110: PART 1: 1997.

Designed By : Engr. Akinyele J.O

Simply supported on all four sides.

|  |          |
|--|----------|
| Slab Length(Long span) (ly) (m)                    | 0.407    |
| Slab Width (Short span) (lx) (m)                   | 0.364    |
| Rib width (mm)                                     | 0        |
| Distance between ribs on Long span (m)             | 0        |
| Distance between ribs on Short span (m)            | 0        |
| Depth of slab portion (mm)                         | 40       |
| Depth of ribs (mm)                                 | 0        |
| Total depth of slab (mm)                           | 40       |
| Effective depth of slab (mm)                       | 35       |
| Slab dead load kN/m <sup>2</sup>                   | 1.344    |
| Imposed load on slab kN/m <sup>2</sup>             | 3.2      |
| Finishes and Partition load                        | 5.32     |
| Total load on slab (KN/m <sup>2</sup> )            | 9.864    |
| ANALYSIS OF SLABS.                                 |          |
| Fixity ratio for continuous supports on any side   |          |
| Fixity ratio short span left side i <sub>1</sub>   | 0        |
| Fixity ratio long span bottom i <sub>2</sub>       | 0        |
| Fixity ratio short span righth side i <sub>3</sub> | 0        |
| Fixity ratio long span top i <sub>4</sub>          | 0        |
| Reduced sides ar'                                  | 0.364    |
| Reduced sides br'                                  | 0.407    |
| Bending moments in slab (kN.m)                     | 0.071637 |

### DESIGN

|   |  |
|---|--|
| Cover to reinforcement. (mm)                          | 5  |
| Concrete characteristic strength (N/mm <sup>2</sup> ) | 20   |
| Steel Characteristic Strength (N/mm <sup>2</sup> )    | 250  |
| Maximum lever arm factor                              | 0.95   |
| Maximum K- Value                                      | 0.156  |
| Moment M  | 71637.56                                       |
| K- value  | 0.002475                                       |
| Lever arm factor                                      | 0.997242                                       |
| Used Lever arm  | 0.95   |
| Area of steel (mm <sup>2</sup> )                      | 7.678552                                       |
| Provide   | <a href="#">R6@100mmc/c(283mm<sup>2</sup>)</a> |

## Appendix 11: Theoretical result of solid slab S5.

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY

CODE: B.S. 8110: Part 1

1997

Designed By : Akinyele J.O

Simply supported on 3 sides

|  |  |
|--|--|
| Slab Long Span ( $l_y$ ) (m)                         | 1.312  |
| Slab Short Span ( $l_x$ ) (m)                        | 0.3  |
| Depth of Slab part (mm)                              | 40   |
| Distance between ribs on long span (m)               | 0  |
| Distance between ribs on short span (m)              | 0  |
| Rib width (mm)                                       | 0  |
| Rib depth (mm)                                       | 0  |
| Total depth of slab (mm)                             | 40   |
| Cover to reinforcement (mm)                          | 5  |
| Effective depth of slab (mm)                         | 35   |
| Slab dead load ( $\text{kN/m}^2$ )                   | 1.344  |
| Slab imposed load ( $\text{kN/m}^2$ )                | 3.2  |
| Finishes and Partition load                          | 5.32   |
| Total load on Slab $\text{kN/m}^2$                   | 9.864  |
| For case A, $= h_1+h_3 < b$                          |  |
| $l_1$ = fixity1 for left side of short span          | 0  |
| $l_2$ = fixity2 for supported side of long span      | 0  |
| $l_3$ = fixity3 for right side of short span         | 0  |
| Reduced span $b_r$ (m)                               | 1.312  |
| $K'$ = redistribution factor                         | 0.152439   |
| $h$ = factor to determine yield length               | 0.236725   |
| 1st Bending moment in slab (KNm)                     | 0.458949   |
| For case B, $= h_1+h_3 = b$                          |  |
| $b'$ = dimensions                                    | 0.6  |
| 2nd.Bending moment in slab (KNm)                     | 0.3196344  |
| DESIGN   |  |
| Concrete characteristic strength ( $\text{N/mm}^2$ ) | 20   |
| Steel Characteristic Strength ( $\text{N/mm}^2$ )    | 250  |
| Maximum Lever arm factor                             | 0.95   |
| Maximum K- value                                     | 0.156  |
| Used Bending moment (kNm)                            | 0.458949   |
| Moment $M$   | 458949.6   |
| K-value  | 0.015631   |
| Lever arm factor                                     | 0.98232  |
| Used Lever arm                                       | 0.95   |
| Area of Steel ( $\text{mm}^2$ )                      | 48.49369   |
| Provide  | <a href="#">R6@100mmc/c</a><br><a href="#">bottom(283mm<sup>2</sup>)</a> |

## Appendix 12: Theoretical result of solid slab S6

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY

CODE: B.S. 8110: Part 1

1997

Designed By : Akinyele J.O

Simply supported on 3 sides

|   |   |
|---|---|
| Slab Long Span ( $l_y$ ) (m)                    | 0.86  |
| Slab Short Span ( $l_x$ ) (m)                   | 0.36  |
| Depth of Slab part (mm)                         | 40  |
| Distance between ribs on long span (m)          | 0   |
| Distance between ribs on short span (m)         | 0   |
| Rib width (mm)                                  | 0   |
| Rib depth (mm)                                  | 0   |
| Total depth of slab (mm)                        | 40  |
| Cover to reinforcement (mm)                     | 5   |
| Effective depth of slab (mm)                    | 35  |
| Slab dead load ( $kN/m^2$ )                     | 1.344   |
| Slab imposed load ( $kN/m^2$ )                  | 3.2   |
| Finishes and Partition load                     | 5.32  |
| Total load on Slab $kN/m^2$                     | 9.864   |
| For case A, $= h_1+h_3 < b$                     |   |
| $l_1$ = fixity1 for left side of short span     | 0   |
| $l_2$ = fixity2 for supported side of long span | 0   |
| $l_3$ = fixity3 for right side of short span    | 0   |
| Reduced span $b_r$ (m)                          | 0.86  |
| $K'$ = redistribution factor                    | 0.27907   |
| $h$ = factor to determine yield length          | 0.255007  |
| 1st Bending moment in slab (kNm)                | 0.3240404   |
| For case B, $= h_1+h_3 = b$                     |   |
| $b'$ = dimensions                               | 0.72  |
| 2nd.Bending moment in slab (kNm)                | 0.3012834   |
| <b>DESIGN</b>                                   |   |
| Concrete characteristic strength ( $N/mm^2$ )   | 20  |
| Steel Characteristic Strength ( $N/mm^2$ )      | 250   |
| Maximum Lever arm factor                        | 0.95  |
| Maximum K- value                                | 0.156   |
| Used Bending moment (kNm)                       | 0.3240404   |
| Moment $M$                                      | 324040.1  |
| K-value   | 0.011037  |
| Lever arm factor                                | 0.987583  |
| Used Lever arm                                  | 0.95  |
| Area of Steel ( $mm^2$ )                        | 34.24191  |
| Provide   | <a href="#">R6@100mmc/c bottom(283mm<sup>2</sup>)</a> |

### Appendix 13: Experimental result of waffle slab W1

ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY  
CODE B.S.8110 PART1  
1997

Designed by Akinyele J.O

SUPPORTED ON TWO SIDE (simply supported)

|   |        |
|---|--------|
| Slab Long span (Ly) (m)                 | 1.353  |
| Slab Short span (Lx) (m)                | 0.43   |
| Depth of Slab part (mm)                 | 0      |
| Distance between ribs on Long Span (m)  | 0      |
| Distance between ribs on Short Span (m) | 0      |
| Rib width (mm)                          | 0      |
| Depth of ribs (mm)                      | 0      |
| Total depth of slab (mm)                | 0      |
| Cover to reinforcement (mm)             | 0      |
| Effective depth of slab (mm)            | 0      |
| Slab dead load (KN/m <sup>2</sup> )     | 0      |
| Slab imposed load (KN/m <sup>2</sup> )  | 288    |
| Total load on slab (KN/m <sup>2</sup> ) | 288    |
| Load distance from 1st Support          | 0.5515 |
| Load distance from 2nd support          | 0.5515 |
| Length of UDL                           | 0.25   |
| UDL obtained kN/m                       | 72     |
| Reactions at supports kN                | 9      |
| Bending moments in slab kNm             | 5.526  |

## Appendix 14: Experimental result of waffle slab W2

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.

CODE B.S. 8110: PART 1: 1997.

Designed By : Engr. Akinyele

J.O

Simply supported on all  
four sides.

|  |          |
|--|----------|
| Slab Length(Long span) ( $l_y$ ) (m)             | 0.9      |
| Slab Width (Short span) ( $l_x$ ) (m)            | 0.3      |
| Rib width (mm)                                   | 0        |
| Distance between ribs on Long span (m)           | 0        |
| Distance between ribs on Short span (m)          | 0        |
| Depth of slab portion (mm)                       | 0        |
| Depth of ribs (mm)                               | 0        |
| Total depth of slab (mm)                         | 0        |
| Effective depth of slab (mm)                     | 0        |
| Slab dead load $\text{kN/m}^2$                   | 0        |
| Imposed load on slab $\text{kN/m}^2$             | 144      |
| Finishes and Partition load                      | 0        |
| Total load on slab ( $\text{kN/m}^2$ )           | 1        |
| ANALYSIS OF SLABS.                               |          |
| Fixity ratio for continuous supports on any side |          |
| Fixity ratio short span left side $i_1$          | 0        |
| Fixity ratio long span bottom $i_2$              | 0        |
| Fixity ratio short span right side $i_3$         | 0        |
| Fixity ratio long span top $i_4$                 | 0        |
| Reduced sides                                    |          |
| $a_r'$   | 0.3      |
| Reduced sides                                    |          |
| $b_r'$   | 0.9      |
| Bending moments in slab (kN.m)                   | 1.121538 |

### Appendix 15: Experimental result of waffle slab W3

ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY  
CODE B.S.8110 PART1  
1997

Designed by Akinyele J.O

SUPPORTED ON TWO SIDE (simply supported)

|   |        |
|---|--------|
| Slab Long span (Ly) (m)                 | 1.085  |
| Slab Short span (Lx) (m)                | 0.43   |
| Depth of Slab part (mm)                 | 0      |
| Distance between ribs on Long Span (m)  | 0      |
| Distance between ribs on Short Span (m) | 0      |
| Rib width (mm)                          | 0      |
| Depth of ribs (mm)                      | 0      |
| Total depth of slab (mm)                | 0      |
| Cover to reinforcement (mm)             | 0      |
| Effective depth of slab (mm)            | 0      |
| Slab dead load (kN/m <sup>2</sup> )     | 0      |
| Slab imposed load (kN/m <sup>2</sup> )  | 192    |
| Total load on slab (kN/m <sup>2</sup> ) | 192    |
| Load distance from 1st Support          | 0.4175 |
| Load distance from 2nd support          | 0.4175 |
| Length of UDL                           | 0.25   |
| UDL obtained kN/m                       | 48     |
| Reactions at supports kN                | 6      |
| Bending moments in slab kNm             | 2.88   |

## Appendix 16 Experimental result of waffle slab W4

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.

CODE B.S. 8110: PART 1: 1997.

Designed By : Engr. Akinyele J.O

Simple supported on all four sides.

|   |          |
|---|----------|
| Slab Length(Long span) (Ly) (m)                 | 0.407    |
| Slab Width (Short span) (Lx) (m)                | 0.364    |
| Rib width (mm)                                  | 20       |
| Distance between ribs on Long span (m)          | 0.4      |
| Distance between ribs on Short span (m)         | 0.4      |
| Depth of slab portion (mm)                      | 0        |
| Depth of ribs (mm)                              | 0        |
| Total depth of slab (mm)                        | 0        |
| Effective depth of slab (mm)                    | 0        |
| Slab dead load $\text{kN/m}^2$                  | 0        |
| Imposed load on slab $\text{kN/m}^2$            | 96       |
| Finishes and Partition load                     | 0        |
| Total load on slab ( $\text{kN/m}^2$ )          | 96       |
| ANALYSIS OF SLABS.                              |          |
| Fixity ratio for continuos supports on any side |          |
| Fixity ratio short span left side i1            | 0        |
| Fixity ratio long span bottom i2                | 0        |
| Fixity ratio short span rigth side i3           | 0        |
| Fixity ratio long span top i4                   | 0        |
| Reduced sides ar'                               | 0.364    |
| Reduced sides br'                               | 0.407    |
| Bending moments in slab (kN.m)                  | 0.590137 |



## Appendix 17: Experimental result of waffle slab W5

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY

CODE: B.S. 8110: Part 1 1997

Designed By : Akinyele J.O

Simply supported on 3 sides

|   |          |
|---|----------|
| Slab Long Span ( $l_y$ ) (m)                    | 1.312    |
| Slab Short Span ( $l_x$ ) (m)                   | 0.3      |
| Depth of Slab part (mm)                         | 0        |
| Distance between ribs on long span (m)          | 0        |
| Distance between ribs on short span (m)         | 0        |
| Rib width (mm)                                  | 0        |
| Rib depth (mm)                                  | 0        |
| Total depth of slab (mm)                        | 0        |
| Cover to reinforcement (mm)                     | 0        |
| Effective depth of slab (mm)                    | 0        |
| Slab dead load ( $\text{kN/m}^2$ )              | 0        |
| Slab imposed load ( $\text{kN/m}^2$ )           | 15.01    |
| Finishes and Partition load                     | 5.32     |
| Total load on Slab ( $\text{kN/m}^2$ )          | 20.33    |
| For case A, = $h_1+h_3 < b$                     |          |
| $l_1$ = fixity1 for left side of short span     | 0        |
| $l_2$ = fixity2 for supported side of long span | 0        |
| $l_3$ = fixity3 for right side of short span    | 0        |
| Reduced span $b_r$ (m)                          | 1.312    |
| $K'$ = redistribution factor                    | 0.152439 |
| $h$ = factor to determine yield length          | 0.236725 |
| 1st Bending moment in slab (kNm)                | 0.947269 |
| For case B, = $h_1+h_3 = b$                     |          |
| $b'$ = dimensions                               | 0.6      |
| 2nd.Bending moment in slab (kNm)                | 0.657679 |

## Appendix 18: Experimental result of waffle slab W6

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY

CODE: B.S. 8110: Part 1 1997

Designed By : Akinyele J.O

Simply supported on 3 sides

|   |          |
|---|----------|
| Slab Long Span ( $l_y$ ) (m)                    | 0.86     |
| Slab Short Span ( $l_x$ ) (m)                   | 0.36     |
| Depth of Slab part (mm)                         | 0        |
| Distance between ribs on long span (m)          | 0        |
| Distance between ribs on short span (m)         | 0        |
| Rib width (mm)                                  | 0        |
| Rib depth (mm)                                  | 0        |
| Total depth of slab (mm)                        | 0        |
| Cover to reinforcement (mm)                     | 5        |
| Effective depth of slab (mm)                    | 0        |
| Slab dead load ( $\text{kN/m}^2$ )              | 0        |
| Slab imposed load ( $\text{kN/m}^2$ )           | 33.44    |
| Finishes and Partition load                     | 5.32     |
| Total load on Slab ( $\text{kN/m}^2$ )          | 38.76    |
| For case A, = $h_1+h_3 < b$                     |          |
| $l_1$ = fixity1 for left side of short span     | 0        |
| $l_2$ = fixity2 for supported side of long span | 0        |
| $l_3$ = fixity3 for right side of short span    | 0        |
| Reduced span $b_r$ (m)                          | 0.86     |
| $K'$ = redistribution factor                    | 0.27907  |
| $h$ = factor to determine yield length          | 0.255007 |
| 1st Bending moment in slab (kNm)                | 1.27624  |
| For case B, = $h_1+h_3 = b$                     |          |
| $b'$ = dimensions                               | 0.72     |
| 2nd.Bending moment in slab (kNm)                | 1.1147   |

## Appendix 19: Experimental result of solid slab S1

ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY  
CODE B.S.8110 PART1  
1997

Designed by Akinyele J.O

SUPPORTED ON TWO SIDE (simply supported)

|   |        |
|---|--------|
| Slab Long span (Ly) (m)                 | 1.353  |
| Slab Short span (Lx) (m)                | 0.43   |
| Depth of Slab part (mm)                 | 0      |
| Distance between ribs on Long Span (m)  | 0      |
| Distance between ribs on Short Span (m) | 0      |
| Rib width (mm)                          | 0      |
| Depth of ribs (mm)                      | 0      |
| Total depth of slab (mm)                | 0      |
| Cover to reinforcement (mm)             | 0      |
| Effective depth of slab (mm)            | 0      |
| Slab dead load (kN/m <sup>2</sup> )     | 0      |
| Slab imposed load (kN/m <sup>2</sup> )  | 192    |
| Total load on slab (kN/m <sup>2</sup> ) | 192    |
| Load distance from 1st Support          | 0.5515 |
| Load distance from 2nd support          | 0.5515 |
| Length of UDL                           | 0.25   |
| UDL obtained kN/m                       | 48     |
| Reactions at supports kN                | 6      |
| Bending moments in slab kNm             | 3.684  |

## Appendix 20: Experimental result of solid slab S2

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.

CODE B.S. 8110: PART 1: 1997.

Designed By : Engr. Akinyele J.O

Simply supported on all four sides.

|  |          |
|--|----------|
| Slab Length(Long span) ( $l_y$ ) (m)             | 0.9      |
| Slab Width (Short span) ( $l_x$ ) (m)            | 0.3      |
| Rib width (mm)                                   | 0        |
| Distance between ribs on Long span (m)           | 0        |
| Distance between ribs on Short span (m)          | 0        |
| Depth of slab portion (mm)                       | 0        |
| Depth of ribs (mm)                               | 0        |
| Total depth of slab (mm)                         | 0        |
| Effective depth of slab (mm)                     | 0        |
| Slab dead load $\text{kN/m}^2$                   | 0        |
| Imposed load on slab $\text{kN/m}^2$             | 112      |
| Finishes and Partition load                      | 0        |
| Total load on slab ( $\text{kN/m}^2$ )           | 112      |
| ANALYSIS OF SLABS.                               |          |
| Fixity ratio for continuous supports on any side |          |
| Fixity ratio short span left side $i_1$          | 0        |
| Fixity ratio long span bottom $i_2$              | 0        |
| Fixity ratio short span right side $i_3$         | 0        |
| Fixity ratio long span top $i_4$                 | 0        |
| Reduced sides $ar'$                              | 0.3      |
| Reduced sides $br'$                              | 0.9      |
| Bending moments in slab (kN.m)                   | 0.872308 |

## Appendix 21: Experimental result of solid slab S3

ANALYSIS OF WAFFLE SLABS USING YIELD LINE THEORY  
CODE B.S.8110 PART1  
1997

Designed by Akinyele J.O

SUPPORTED ON TWO SIDE (simply supported)

|   |        |
|---|--------|
| Slab Long span (Ly) (m)                 | 1.085  |
| Slab Short span (Lx) (m)                | 0.43   |
| Depth of Slab part (mm)                 | 0      |
| Distance between ribs on Long Span (m)  | 0      |
| Distance between ribs on Short Span (m) | 0      |
| Rib width (mm)                          | 0      |
| Depth of ribs (mm)                      | 0      |
| Total depth of slab (mm)                | 0      |
| Cover to reinforcement (mm)             | 0      |
| Effective depth of slab (mm)            | 0      |
| Slab dead load (kN/m <sup>2</sup> )     | 0      |
| Slab imposed load (kN/m <sup>2</sup> )  | 244    |
| Total load on slab (kN/m <sup>2</sup> ) | 244    |
| Load distance from 1st Support          | 0.4175 |
| Load distance from 2nd support          | 0.4175 |
| Length of UDL                           | 0.25   |
| UDL obtained kN/m                       | 56     |
| Reactions at supports kN                | 7      |
| Bending moments in slab kNm             | 3.360  |

## Appendix 22: Experimental result of solid slab S4

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.

CODE B.S. 8110: PART 1: 1997.

Designed By : Engr. Akinyele J.O

Simply supported on all four sides.

|  |          |
|--|----------|
| Slab Length(Long span) ( $l_y$ ) (m)             | 0.407    |
| Slab Width (Short span) ( $l_x$ ) (m)            | 0.364    |
| Rib width (mm)                                   | 0        |
| Distance between ribs on Long span (m)           | 0        |
| Distance between ribs on Short span (m)          | 0        |
| Depth of slab portion (mm)                       | 0        |
| Depth of ribs (mm)                               | 0        |
| Total depth of slab (mm)                         | 0        |
| Effective depth of slab (mm)                     | 0        |
| Slab dead load $\text{kN/m}^2$                   | 0        |
| Imposed load on slab $\text{kN/m}^2$             | 96       |
| Finishes and Partition load                      | 0        |
| Total load on slab ( $\text{KN/m}^2$ )           | 96       |
| ANALYSIS OF SLABS.                               |          |
| Fixity ratio for continuous supports on any side |          |
| Fixity ratio short span left side $i_1$          | 0        |
| Fixity ratio long span bottom $i_2$              | 0        |
| Fixity ratio short span right side $i_3$         | 0        |
| Fixity ratio long span top $i_4$                 | 0        |
| Reduced sides $ar'$                              | 0.364    |
| Reduced sides $br'$                              | 0.407    |
| Bending moments in slab (kN.m)                   | 0.590152 |

## Appendix 23: Experimental result of solid slab S5

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY

CODE: B.S. 8110: Part 1 1997

Designed By : Akinyele J.O

Simply supported on 3 sides

|   |          |
|---|----------|
| Slab Long Span ( $l_y$ ) (m)                    | 1.312    |
| Slab Short Span ( $l_x$ ) (m)                   | 0.3      |
| Depth of Slab part (mm)                         | 0        |
| Distance between ribs on long span (m)          | 0        |
| Distance between ribs on short span (m)         | 0        |
| Rib width (mm)                                  | 0        |
| Rib depth (mm)                                  | 0        |
| Total depth of slab (mm)                        | 0        |
| Cover to reinforcement (mm)                     | 5        |
| Effective depth of slab (mm)                    | 0        |
| Slab dead load ( $\text{kN/m}^2$ )              | 0        |
| Slab imposed load ( $\text{kN/m}^2$ )           | 15.24    |
| Finishes and Partition load                     | 0        |
| Total load on Slab ( $\text{kN/m}^2$ )          | 15.24    |
| For case A, $= h_1+h_3 < b$                     |          |
| $l_1 =$ fixity1 for left side of short span     | 0        |
| $l_2 =$ fixity2 for supported side of long span | 0        |
| $l_3 =$ fixity3 for right side of short span    | 0        |
| Reduced span $b_r$ (m)                          | 1.312    |
| $K'$ = redistribution factor                    | 0.152439 |
| $H$ = factor to determine yield length          | 0.236725 |
| 1st Bending moment in slab (KNm)                | 0.71066  |
| For case B, $= h_1+h_3 = b$                     |          |
| $B'$ = dimensions                               | 0.6      |
| 2nd.Bending moment in slab (KNm)                | 0.4932   |

## Appendix 24: Experimental result of solid slab S6

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY

CODE: B.S. 8110: Part 1 1997

Designed By : Akinyele J.O

Simply supported on 3 sides

|   |          |
|---|----------|
| Slab Long Span ( $l_y$ ) (m)                    | 0.86     |
| Slab Short Span ( $l_x$ ) (m)                   | 0.36     |
| Depth of Slab part (mm)                         | 0        |
| Distance between ribs on long span (m)          | 0        |
| Distance between ribs on short span (m)         | 0        |
| Rib width (mm)                                  | 0        |
| Rib depth (mm)                                  | 0        |
| Total depth of slab (mm)                        | 0        |
| Cover to reinforcement (mm)                     | 5        |
| Effective depth of slab (mm)                    | 5        |
| Slab dead load ( $\text{kN/m}^2$ )              | 0        |
| Slab imposed load ( $\text{kN/m}^2$ )           | 32.3     |
| Finishes and Partition load                     | 0        |
| Total load on Slab ( $\text{kN/m}^2$ )          | 32.3     |
| For case A, = $h_1+h_3 < b$                     |          |
| $l_1$ = fixity1 for left side of short span     | 0        |
| $l_2$ = fixity2 for supported side of long span | 0        |
| $l_3$ = fixity3 for right side of short span    | 0        |
| Reduced span $b_r$ (m)                          | 0.86     |
| $K'$ = redistribution factor                    | 0.27907  |
| $H$ = factor to determine yield length          | 0.255007 |
| 1st Bending moment in slab (kNm)                | 1.062447 |
| For case B, = $h_1+h_3 = b$                     |          |
| $B'$ = dimensions                               | 0.72     |
| 2nd. Bending moment in slab (kNm)               | 0.988246 |



## Appendix 25: Result of waffle slab using the example in Mosley et al

ANALYSIS AND DESIGNED OF WAFFLE SLABS USING THE YIELD LINE THEORY.

CODE B.S. 8110: PART 1: 1997.

Designed By : Engr. Akinyele J.O

Simple supported on all four sides.

|   |        |
|---|--------|
| Slab Length(Long span) (ly) (m)         | 6      |
| Slab Width (Short span) (lx) (m)        | 5      |
| Rib width (mm)                          | 125    |
| Distance between ribs on Long span (m)  | 0.4    |
| Distance between ribs on Short span (m) | 0.4    |
| Depth of slab portion (mm)              | 60     |
| Depth of ribs (mm)                      | 140    |
| Total depth of slab (mm)                | 200    |
| Effective depth of slab (mm)            | 40     |
| Slab dead load kN/m <sup>2</sup>        | 2.016  |
| Imposed load on slab kN/m <sup>2</sup>  | 10.384 |
| Finishes and Partition load             | 0      |
| Total load on slab (kN/m <sup>2</sup> ) | 12.4   |

### ANALYSIS OF SLABS.

|  |          |
|--|----------|
| Fixity ratio for continuous supports on any side |          |
| Fixity ratio short span left side i1             | 1        |
| Fixity ratio long span bottom i2                 | 1        |
| Fixity ratio short span right side i3            | 1        |
| Fixity ratio long span top i4                    | 1        |
| Reduced sides ar'                                | 3.535534 |
| Reduced sides br'                                | 4.242641 |
| Bending moments in slab (KN.m)                   | 7.664835 |

### DESIGN

|   |                                     |
|---|-------------------------------------|
| Cover to reinforcement. (mm)                        | 20                                  |
| Concrete characteristic stress (N/mm <sup>2</sup> ) | 30                                  |
| Steel Characteristic Stress (N/mm <sup>2</sup> )    | 460                                 |
| Maximum lever arm factor                            | 0.95                                |
| Maximum K- Value                                    | 0.156                               |
| Moment M  | 7664835                             |
| K- value  | 0.159684                            |
| Lever arm factor                                    | 0.769394                            |
| Used Lever arm                                      | 0.769394                            |
| Area of steel (mm <sup>2</sup> )                    | 461.5702                            |
| Provide   | <a href="#">T8@100mmc/c(503mm2)</a> |

### ANALYSIS AND DESIGN OF RIBS.

Adopting the Rankine Grashoff Theory.

|                                   |    |
|-----------------------------------|----|
| Number of Ribs on Long Span (ly)  | 10 |
| Number of Ribs on Short Span (lx) | 8  |

|   |   |
|---|---|
| Weight of Slabs (kN/m <sup>2</sup> )                  | 1.44  |
| Total Load on Slab (kN)                               | 43.2  |
| Weight of Ribs (kN/m)                                 | 0.42  |
| Total weight of Ribs on long span (ly), (kN)          | 25.2  |
| Total weight of Ribs on Short span (lx), (kN)         | 16.8  |
| Total weight of Finishes (kN)                         | 30  |
| Total Live Load (kN)                                  | 45  |
| Total Load on grid floor (KN)                         | 160.2   |
| Q=Load per Area (m2)                                  | 5.34  |
| Denominator for qx and qy                             | 1921  |
| Load sheared on short span (qx), (kN/m <sup>2</sup> ) | 3.602624  |
| Load sheared on Long span (qy), (kN/m <sup>2</sup> )  | 1.737376  |
| Moment on short span Mx (kN.m)                        | 4.50328   |
| Moment on long span My (kN.m)                         | 3.127277  |
| Shear force on short span, Vx, (kN)                   | 3.602624  |
| Shear force on long span, Vy, (kN)                    | 2.084852  |
| DESIGNED  |   |
| Steel characteristic stress (N/mm <sup>2</sup> )      | 460   |
| Short span Designed moment (Nmm)                      | 4503280   |
| Cover to ribs (mm)                                    | 40  |
| Effective depth of ribs                               | 100   |
| K.value   | 0.120087  |
| Lever arm Factor                                      | 0.841423  |
| Used Lever arm  | 0.841423  |
| Area of Steel (mm <sup>2</sup> )                      | 122.471   |
| Provide   | <a href="#">2T10@ bottom(157mm<sup>2</sup>)</a> |
| Long span Designed moment (Nmm)                       | 3127277   |
| K.value   | 0.083394  |
| Lever arm Factor                                      | 0.896661  |
| Used Lever arm  | 0.896661  |
| Area of Steel (mm <sup>2</sup> )                      | 79.80989  |
| Provide   | 2T10 Near bottom (157mm <sup>2</sup> )          |
| END DESIGNED  |   |

**Appendix 26: Waffle slab result of Mosley et al. Adopting BS 8110 coefficients.**

Length = 6m

Breadth = 5m

Total depth of waffle slab = 200mm

Depth of slab portion= 60mm

Depth of ribs= 140mm

Cover to reinforcements= 40mm

Effective depth = 160mm

Spacing between ribs in both axis= 400mm

Designed ultimate load = 12.4kN/m<sup>2</sup>

Slab is an internal panel of a floor system

Using the slab coefficient of Table 3.14 of BS 8110.

$L_y/l_x = 6/5 = 1.2$

$\beta_{sx} = +0.032$

$\beta_{sy} = + 0.024$

Moment at mid span along short span =  $0.032 \times 12.4 \times 5^2 = 9.92\text{kN/m/m}$

Moment carried by each rib=  $0.4 \times 9.92 = 3.97\text{kNm}$

$$\frac{m}{bd^2 f_{cu}} = \frac{3.97 \times 10^6}{400 \times 160^2 \times 30} = 0.013$$

$$A_s = \frac{m}{0.95 f_y z} = \frac{3.97 \times 10^6}{0.95 \times 460 \times 0.95 \times 160} = 60\text{mm}^2$$

Provide Two T10 bars in each rib along the small span.

Moment at Mid span along long span =  $0.024 \times 12.4 \times 5^2 = 7.44\text{kNm/m}$

Moment carried by each rib =  $0.4 \times 7.44 = 2.98\text{kNm}$

$$\frac{m}{bd^2 f_{cu}} = \frac{2.98 \times 10^6}{400 \times 160^2 \times 30} = 0.009$$

$$A_s = \frac{m}{0.95 f_y z} = \frac{2.98 \times 10^6}{0.95 \times 460 \times 0.95 \times 160} = 45\text{mm}$$

Provide Two T10 bars in each rib along the Long span.

Reinforcements at the top flange (slab portion)

$$A_s = 0.12 \times b \times h / 100 = 0.12 \times 1000 \times 60 / 100 = 72 \text{mm}^2 / \text{m}.$$

Provide D98 Mesh ( $98 \text{mm}^2$ ).

Shear

From table 3.15 of BS 8110, the shear force coefficient for a continuous edge supports is 0.33. Hence, for one rib, the shear at supports

$$V_{sx} = \beta_{vx} n l_x \times b = 0.33 \times 12.4 \times 5 \times 0.4 = 8.18 \text{kN}.$$

Maximum shear in the rib 0.6m from the centre line is

$$V = 8.18 - 0.6 \times 12.4 \times 0.4 = 5.20 \text{kN}.$$

## Appendix 27: Result of waffle W1 Theoretical (ETABS)

ETABS v9.0.0 File:WAFFLE W1THEORY Units:kN-m

### PROJECT INFORMATION

Company Name = eng

ETABS v9.0.0 File:WAFFLE W1THEORY Units:kN-m

### S T O R Y D A T A

| STORY  | SIMILAR TO | HEIGHT | ELEVATION |
|--------|------------|--------|-----------|
| STORY1 | None       | 0.750  | 0.750     |
| BASE   | None       |        | 0.000     |

ETABS v9.0.0 File:WAFFLE W1THEORY Units:kN-m

### S T A T I C L O A D C A S E S

| STATIC CASE | CASE TYPE   | AUTO LAT LOAD | SELF WT MULTIPLIER |
|-------------|-------------|---------------|--------------------|
| DEAD        | DEAD        | N/A           | 0.1000             |
| LIVE        | REDUCE LIVE | N/A           | 0.0000             |

ETABS v9.0.0 File:WAFFLE W1THEORY Units:kN-m

### M A S S S O U R C E D A T A

| MASS FROM | LATERAL MASS ONLY | LUMP MASS AT STORIES |
|-----------|-------------------|----------------------|
|-----------|-------------------|----------------------|

Masses Yes Yes

ETABS v9.0.0 File:WAFFLE W1THEORY Units:kN-m

### D I A P H R A G M M A S S D A T A

| STORY  | DIAPHRAGM | MASS-X | MASS-Y | MMI    | X-M   | Y-M   |
|--------|-----------|--------|--------|--------|-------|-------|
| STORY1 | D1        | 3.0170 | 3.0170 | 1.1743 | 0.677 | 0.215 |

ETABS v9.0.0 File:WAFFLE W1THEORY Units:kN-m

### A S S E M B L E D P O I N T M A S S E S

| STORY  | UX       | UY       | UZ       | RX     | RY       | RZ       |
|--------|----------|----------|----------|--------|----------|----------|
| STORY1 | 3.017041 | 3.017041 | 0.0000   | 0.0000 | 0.000000 | 1.174264 |
| BASE   | 0.752728 | 0.752728 | 0.000000 | 0.0000 | 0.000000 | 0.000000 |
| Totals | 3.769769 | 3.769769 | 0.000000 | 0.0000 | 0.000000 | 1.174264 |

ETABS v9.0.0 File:WAFFLE W1THEORY Units:kN-m May 9, 2009 4:47

CENTERS OF CUMULATIVE MASS & CENTER OF RIGIDITY

STORY DIAPHRAGM /-----CENTRE OF MASS-----//--CENTRE OF RIGIDITY--/

| LEVEL  | NAME  | MASS   | ORDINATE-X | ORDINATE-Y |
|--------|-------|--------|------------|------------|
| STORY1 | D1    | 3.0170 | 0.677      | 0.215      |
| 0.676  | 0.215 |        |            |            |

ETABS v9.0.0 File:WAFFLE W1THEORY Units:kN-m May 9, 2009 4:47

MODAL PERIODS AND FREQUENCIES

| MODE NUMBER | PERIOD (TIME) | FREQUENCY (CYCLES/TIME) | CIRCULAR FREQ (RADIANS/TIME) |
|-------------|---------------|-------------------------|------------------------------|
| Mode 1      | 0.01249       | 80.04585                | 502.94288                    |
| Mode 2      | 0.01096       | 91.26274                | 573.42070                    |
| Mode 3      | 0.00990       | 101.02017               | 634.72843                    |

ETABS v9.0.0 File:WAFFLE W1THEORY Units:KN-m

MODAL PARTICIPATING MASS RATIOS

| MODE ROTN NUMBER | RY-ROTN %MASS <SUM> | X-TRANS RZ-ROTN %MASS <SUM> | Y-TRANS %MASS <SUM> | Z-TRANS %MASS <SUM> | RX- %MASS |
|------------------|---------------------|-----------------------------|---------------------|---------------------|-----------|
| Mode 1           | 0.00 < 0>           | 100.00 <100>                | 0.00 < 0>           | 100.00              |           |
| Mode 2           | 100.00 <100>        | 0.00 < 0>                   | 0.00 <100>          | 0.00 < 0>           | 0.00      |
| Mode 3           | 0.00 <100>          | 0.00 <100>                  | 0.00 <100>          | 0.00 < 0>           | 0.00      |

ETABS v9.0.0 File:WAFFLE W1THEORY Units:kN-m

MODAL LOAD PARTICIPATION RATIOS (STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

| TYPE  | NAME | STATIC   | DYNAMIC  |
|-------|------|----------|----------|
| Load  | DEAD | 0.0000   | 0.0000   |
| Load  | LIVE | 0.0000   | 0.0000   |
| Accel | UX   | 100.0000 | 100.0000 |
| Accel | UY   | 100.0000 | 100.0000 |
| Accel | UZ   | 0.0000   | 0.0000   |
| Accel | RX   | 100.0000 | 100.0000 |

|       |    |          |          |
|-------|----|----------|----------|
| Accel | RY | 100.0000 | 100.0000 |
| Accel | RZ | 100.0000 | 100.0000 |

ETABS v9.0.0 File:WAFFLE W1THEORY Units:KN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

| LOAD | FX   | FY   | FZ   | MX    | MY     | MZ    |
|------|------|------|------|-------|--------|-------|
| DEAD | 0.00 | 0.00 | 7.10 | 1.525 | -4.800 | 0.00  |
| LIVE | 0.00 | 0.00 | 1.86 | 0.400 | -1.259 | 0.000 |

## Appendix 28: Result of waffle W2 Theoretical (ETABS)

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m  
PROJECT INFORMATION

Company Name = eng

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m  
S T O R Y D A T A

| STORY  | SIMILAR TO | HEIGHT | ELEVATION |
|--------|------------|--------|-----------|
| STORY1 | None       | 0.750  | 0.750     |
| BASE   | None       |        | 0.000     |

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

S T A T I C L O A D C A S E S

| STATIC CASE | CASE TYPE   | AUTO LAT LOAD | SELF WT MULTIPLIER |
|-------------|-------------|---------------|--------------------|
| DEAD        | DEAD        | N/A           | 0.1000             |
| LIVE        | REDUCE LIVE | N/A           | 0.0000             |

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

M A S S S O U R C E D A T A

| MASS FROM | LATERAL MASS ONLY | LUMP MASS AT STORIES |
|-----------|-------------------|----------------------|
|-----------|-------------------|----------------------|

Masses Yes Yes

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

D I A P H R A G M M A S S D A T A

| STORY  | DIAPHRAGM | MASS-X | MASS-Y | MMI    | X-M   | Y-M   |
|--------|-----------|--------|--------|--------|-------|-------|
| STORY1 | D1        | 2.2222 | 2.2222 | 0.4092 | 0.450 | 0.150 |

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

A S S E M B L E D P O I N T M A S S E S

| STORY  | UX       | UY       | UZ       | RX       | RY       | RZ       |
|--------|----------|----------|----------|----------|----------|----------|
| STORY1 | 2.222178 | 2.222178 | 0.000000 | 0.000000 | 0.000000 | 0.409217 |
| BASE   | 0.752728 | 0.752728 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| Totals | 2.974906 | 2.974906 | 0.000000 | 0.000000 | 0.000000 | 0.409217 |



ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

C E N T E R S O F C U M U L A T I V E M A S S & C E N T E R  
S O F R I G I D I T Y

STORY DIAPHRAGM /-----CENTRE OF MASS-----//--CENTRE  
OF RIGIDITY--/

| LEVEL  | NAME | MASS   | ORDINATE-X | ORDINATE-Y | ORDINATE-X | ORDINATE-Y |
|--------|------|--------|------------|------------|------------|------------|
| STORY1 | D1   | 2.2222 | 0.450      | 0.150      | 0.450      | 0.150      |

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

M O D A L P E R I O D S A N D F R E Q U E N C I E S

| MODE NUMBER | PERIOD (TIME) | FREQUENCY (CYCLES/TIME) | CIRCULAR FREQ (RADIANS/TIME) |
|-------------|---------------|-------------------------|------------------------------|
| Mode 1      | 0.01249       | 80.04847                | 502.95938                    |
| Mode 2      | 0.00917       | 109.05392               | 685.20598                    |
| Mode 3      | 0.00882       | 113.41486               | 712.60661                    |

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

M O D A L P A R T I C I P A T I N G M A S S R A T I O S

| MODE NUMBER | X-TRANS %MASS <SUM> | Y-TRANS %MASS <SUM> | Z-TRANS %MASS <SUM> | RX-ROTN %MASS <SUM> | RY-ROTN %MASS <SUM> | RZ-ROTN %MASS |
|-------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------|
| Mode 1      | 0.00 < 0>           | 0.00 < 0>           | 100.00 <100>        | 0.00 < 0>           | 0.00 < 0>           | 100.00        |
| Mode 2      | 100.00 <100>        | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           | 0.00          |
| Mode 3      | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           | 0.00          |

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

M O D A L L O A D P A R T I C I P A T I O N R A T I O S  
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

| TYPE  | NAME | STATIC   | DYNAMIC  |
|-------|------|----------|----------|
| Load  | DEAD | 0.0000   | 0.0000   |
| Load  | LIVE | 0.0000   | 0.0000   |
| Accel | UX   | 100.0000 | 100.0000 |
| Accel | UY   | 100.0000 | 100.0000 |
| Accel | UZ   | 0.0000   | 0.0000   |
| Accel | RX   | 100.0000 | 100.0000 |
| Accel | RY   | 100.0000 | 100.0000 |
| Accel | RZ   | 100.0000 | 100.0000 |

ETABS v9.0.0 File:WAFFLE W2THEORY Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

| LOAD | FX   | FY   | FZ   | MX    | MY     | MZ    |
|------|------|------|------|-------|--------|-------|
| DEAD | 0.00 | 0.00 | 3.11 | 0.467 | -1.401 | 0.000 |
| LIVE | 0.00 | 0.00 | 0.23 | 0.035 | -0.105 | 0.000 |

## Appendix 29: Result of waffle W3 Theoretical (ETABS)

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m

### PROJECT INFORMATION

Company Name = eng

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m  
S T O R Y D A T A

| STORY  | SIMILAR TO | HEIGHT | ELEVATION |
|--------|------------|--------|-----------|
| STORY1 | None       | 0.750  | 0.750     |
| BASE   | None       |        | 0.000     |

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m  
S T A T I C L O A D C A S E S

| STATIC CASE | CASE TYPE   | AUTO LAT LOAD | SELF WT MULTIPLIER |
|-------------|-------------|---------------|--------------------|
| DEAD        | DEAD        | N/A           | 0.1000             |
| LIVE        | REDUCE LIVE | N/A           | 0.0000             |

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m  
M A S S S O U R C E D A T A

| MASS FROM | LATERAL MASS ONLY | LUMP MASS AT STORIES |
|-----------|-------------------|----------------------|
|-----------|-------------------|----------------------|

Masses Yes Yes

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m  
D I A P H R A G M M A S S D A T A

| STORY  | DIAPHRAGM | MASS-X | MASS-Y | MMI    | X-M   | Y-M   |
|--------|-----------|--------|--------|--------|-------|-------|
| STORY1 | D1        | 2.5350 | 2.5350 | 0.6810 | 0.543 | 0.215 |

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m

### A S S E M B L E D P O I N T M A S S E S

| STORY  | UX       | UY       | UZ       | RX       | RY     | RZ       |
|--------|----------|----------|----------|----------|--------|----------|
| STORY1 | 2.535048 | 2.535048 | 0.000000 | 0.000000 | 0.000  | 0.680962 |
| BASE   | 0.752728 | 0.752728 | 0.000000 | 0.000000 | 0.0000 | 0.000000 |
| Totals | 3.287776 | 3.287776 | 0.000000 | 0.000000 | 0.0000 | 0.680962 |

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m

C E N T E R S O F C U M U L A T I V E M A S S & C E N T E R  
S O F R I G I D I T Y

STORY DIAPHRAGM /-----CENTRE OF MASS-----//--CENTRE  
OF RIGIDITY--/

| LEVEL  | NAME | MASS   | ORDINATE-X | ORDINATE-Y | ORDINATE-X | ORDINATE-Y |
|--------|------|--------|------------|------------|------------|------------|
| STORY1 | D1   | 2.5350 | 0.543      | 0.215      | 0.542      | 0.215      |

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m

M O D A L P E R I O D S A N D F R E Q U E N C I E S

| MODE NUMBER | PERIOD (TIME) | FREQUENCY (CYCLES/TIME) | CIRCULAR FREQ (RADIAN/TIME) |
|-------------|---------------|-------------------------|-----------------------------|
| Mode 1      | 0.01145       | 87.32430                | 548.67473                   |
| Mode 2      | 0.00993       | 100.66678               | 632.50804                   |
| Mode 3      | 0.00892       | 112.09162               | 704.29241                   |

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m

M O D A L P A R T I C I P A T I N G M A S S R A T I O S

| MODE NUMBER | X-TRANS %MASS <SUM> | Y-TRANS %MASS <SUM> | Z-TRANS %MASS <SUM> | RX-ROTN %MASS <SUM> | RY-ROTN %MASS <SUM> | RZ-ROTN %MASS <SUM> |
|-------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| Mode 1      | 0.00 < 0>           | 0.00 < 0>           | 100.00 <100>        | 0.00 < 0>           | 0.00 < 0>           | 100.00 <100>        |
| Mode 2      | 100.00 <100>        | 0.00 < 0>           | 0.00 < 0>           | 0.00 <100>          | 0.00 < 0>           | 0.00 < 0>           |
| Mode 3      | 0.00 <100>          | 0.00 <100>          | 0.00 <100>          | 0.00 <100>          | 0.00 < 0>           | 0.00 < 0>           |

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m

M O D A L L O A D P A R T I C I P A T I O N R A T I O S  
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

| TYPE  | NAME | STATIC   | DYNAMIC  |
|-------|------|----------|----------|
| Load  | DEAD | 0.0000   | 0.0000   |
| Load  | LIVE | 0.0000   | 0.0000   |
| Accel | UX   | 100.0000 | 100.0000 |
| Accel | UY   | 100.0000 | 100.0000 |
| Accel | UZ   | 0.0000   | 0.0000   |
| Accel | RX   | 100.0000 | 100.0000 |
| Accel | RY   | 100.0000 | 100.0000 |
| Accel | RZ   | 100.0000 | 100.0000 |

ETABS v9.0.0 File:WAFFLE W3 THEORY Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

| LOAD | FX   | FY   | FZ   | MX    | MY     | MZ    |
|------|------|------|------|-------|--------|-------|
| DEAD | 0.00 | 0.00 | 6.01 | 1.292 | -3.260 | 0.000 |
| LIVE | 0.00 | 0.00 | 1.49 | 0.321 | -0.810 | 0.000 |

### Appendix 30: Result of waffle W4 Theoretical (ETABS)

ETABS v9.0.0 File:WAFFLE W4THEORY Units:kN-m  
PROJECT INFORMATION

Company Name = eng

ETABS v9.0.0 File:WAFFLE W4THEORY Units:kN-m

#### S T O R Y D A T A

| STORY  | SIMILAR TO | HEIGHT | ELEVATION |
|--------|------------|--------|-----------|
| STORY1 | None       | 0.750  | 0.750     |
| BASE   | None       |        | 0.000     |

ETABS v9.0.0 File:WAFFLE W4THEORY Units:kN-m

#### S T A T I C L O A D C A S E S

| STATIC CASE | CASE TYPE   | AUTO LAT LOAD | SELF WT MULTIPLIER |
|-------------|-------------|---------------|--------------------|
| DEAD        | DEAD        | N/A           | 0.1000             |
| LIVE        | REDUCE LIVE | N/A           | 0.0000             |

ETABS v9.0.0 File:WAFFLE W4THEORY Units:kN-m

#### M A S S S O U R C E D A T A

| MASS FROM | LATERAL MASS ONLY | LUMP MASS AT STORIES |
|-----------|-------------------|----------------------|
|-----------|-------------------|----------------------|

Masses Yes Yes

ETABS v9.0.0 File:WAFFLE W4THEORY Units:kN-m

#### D I A P H R A G M M A S S D A T A

| STORY  | DIAPHRAGM | MASS-X | MASS-Y | MMI    | X-M   | Y-M   |
|--------|-----------|--------|--------|--------|-------|-------|
| STORY1 | D1        | 1.6142 | 1.6142 | 0.1072 | 0.204 | 0.182 |

ETABS v9.0.0 File:WAFFLE W4THEORY Units:kN-m

#### A S S E M B L E D P O I N T M A S S E S

| STORY  | UX       | UY       | UZ       | RX       | RY       | RZ       |
|--------|----------|----------|----------|----------|----------|----------|
| STORY1 | 1.614198 | 1.614198 | 0.000000 | 0.000000 | 0.000000 | 0.107212 |
| BASE   | 0.752728 | 0.752728 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| Totals | 2.366926 | 2.366926 | 0.000000 | 0.000000 | 0.000000 | 0.107212 |

ETABS v9.0.0 File:WAFFLE W4THEORY Units:kN-m

C E N T E R S O F C U M U L A T I V E M A S S & C E N T E R  
S O F R I G I D I T Y

STORY DIAPHRAGM /-----CENTRE OF MASS-----//--CENTRE OF  
RIGIDITY--/

| LEVEL  | NAME | MASS   | ORDINATE-X | ORDINATE-Y | ORDINATE-X | ORDINATE-Y |
|--------|------|--------|------------|------------|------------|------------|
| STORY1 | D1   | 1.6142 | 0.204      | 0.182      | 0.203      | 0.182      |

ETABS v9.0.0 File:WAFFLE W4THEORY Units:kN-m

M O D A L P E R I O D S A N D F R E Q U E N C I E S

| MODE NUMBER | PERIOD (TIME) | FREQUENCY (CYCLES/TIME) | CIRCULAR FREQ (RADIAN/TIME) |
|-------------|---------------|-------------------------|-----------------------------|
| Mode 1      | 0.00974       | 102.65408               | 644.99462                   |
| Mode 2      | 0.00934       | 107.10545               | 672.96340                   |
| Mode 3      | 0.00686       | 145.86851               | 916.51886                   |

ETABS v9.0.0 File:WAFFLE W4THEORY Units:kN-m

M O D A L P A R T I C I P A T I N G M A S S R A T I O S

| MODE NUMBER | X-TRANS %MASS <SUM> | Y-TRANS %MASS <SUM> | Z-TRANS %MASS <SUM> | RX-ROTN %MASS <SUM> | RY-ROTN %MASS <SUM> | RZ-ROTN %MASS |
|-------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------|
| Mode 1      | 0.00 < 0>           | 0.00 < 0>           | 100.00 <100>        | 0.00 < 0>           | 100.00              | 100.00        |
| Mode 2      | 100.00 <100>        | 0.00 < 0>           | 0.00 < 0>           | 0.00 <100>          | 0.00 < 0>           | 0.00          |
| Mode 3      | 0.00 <100>          | 0.00 <100>          | 0.00 <100>          | 0.00 < 0>           | 0.00 < 0>           | 0.00          |

ETABS v9.0.0 File:WAFFLE W4THEORY Units:kN-m

M O D A L L O A D P A R T I C I P A T I O N R A T I O S  
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

| TYPE  | NAME | STATIC   | DYNAMIC  |
|-------|------|----------|----------|
| Load  | DEAD | 0.0000   | 0.0000   |
| Load  | LIVE | 0.0000   | 0.0000   |
| Accel | UX   | 100.0000 | 100.0000 |
| Accel | UY   | 100.0000 | 100.0000 |
| Accel | UZ   | 0.0000   | 0.0000   |
| Accel | RX   | 100.0000 | 100.0000 |
| Accel | RY   | 100.0000 | 100.0000 |

Accel            RZ                    100.0000    100.0000

ETABS v9.0.0    File:WAFFLE W4THEORY    Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

| LOAD | FX   | FY   | FZ   | MX    | MY     | MZ    |
|------|------|------|------|-------|--------|-------|
| DEAD | 0.00 | 0.00 | 2.32 | 0.422 | -0.473 | 0.000 |
| LIVE | 0.00 | 0.00 | 0.03 | 0.005 | -0.005 | 0.000 |



## Appendix 31: Result of waffle W5 Theoretical (ETABS)

ETABS v9.0.0 File:WAFFLE W5THEORY Units:kN-m

### PROJECT INFORMATION

Company Name = eng

ETABS v9.0.0 File:WAFFLE W5THEORY Units:kN-m

### S T O R Y D A T A

| STORY  | SIMILAR TO | HEIGHT | ELEVATION |
|--------|------------|--------|-----------|
| STORY1 | None       | 0.750  | 0.750     |
| BASE   | None       |        | 0.000     |

ETABS v9.0.0 File:WAFFLE W5THEORY Units:kN-m

### S T A T I C L O A D C A S E S

| STATIC CASE | CASE TYPE   | AUTO LAT LOAD | SELF WT MULTIPLIER |
|-------------|-------------|---------------|--------------------|
| DEAD        | DEAD        | N/A           | 0.1000             |
| LIVE        | REDUCE LIVE | N/A           | 0.0000             |

ETABS v9.0.0 File:WAFFLE W5THEORY Units:kN-m

### M A S S S O U R C E D A T A

| MASS FROM | LATERAL MASS ONLY | LUMP MASS AT STORIES |
|-----------|-------------------|----------------------|
| Masses    | Yes               | Yes                  |

ETABS v9.0.0 File:WAFFLE W5THEORY Units:kN-m

### D I A P H R A G M M A S S D A T A

| STORY  | DIAPHRAGM | MASS-X | MASS-Y | MMI    | X-M   | Y-M   |
|--------|-----------|--------|--------|--------|-------|-------|
| STORY1 | D1        | 2.0627 | 2.0627 | 0.7605 | 0.656 | 0.150 |

ETABS v9.0.0 File:WAFFLE W5THEORY Units:kN-m

### A S S E M B L E D P O I N T M A S S E S

| STORY  | UX       | UY       | UZ       | RX       | RY       | RZ       |
|--------|----------|----------|----------|----------|----------|----------|
| STORY1 | 2.062712 | 2.062712 | 0.000000 | 0.000000 | 0.000000 | 0.760534 |
| BASE   | 0.752728 | 0.752728 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| Totals | 2.815440 | 2.815440 | 0.000000 | 0.000000 | 0.000000 | 0.760534 |

ETABS v9.0.0 File:WAFFLE W5THEORY Units:kN-m

CENTERS OF CUMULATIVE MASS & CENTER OF RIGIDITY

| STORY OF RIGIDITY--/ | DIAPHRAGM | /-----CENTRE OF MASS-----//--CENTRE OF RIGIDITY--/ |            |            |            |            |
|----------------------|-----------|--|------------|------------|------------|------------|
| LEVEL                | NAME      | MASS   | ORDINATE-X | ORDINATE-Y | ORDINATE-X | ORDINATE-Y |
| STORY1               | D1        | 2.0627   | 0.656      | 0.150      | 0.656      | 0.150      |

ETABS v9.0.0 File:WAFFLE W5THEORY Units:kN-m

MODAL PERIODS AND FREQUENCIES

| MODE NUMBER | PERIOD (TIME) | FREQUENCY (CYCLES/TIME) | CIRCULAR FREQ (RADIAN/TIME) |
|-------------|---------------|-------------------------|-----------------------------|
| Mode 1      | 0.01247       | 80.18465                | 503.81500                   |
| Mode 2      | 0.00954       | 104.83219               | 658.68006                   |
| Mode 3      | 0.00954       | 104.87250               | 658.93333                   |

ETABS v9.0.0 File:WAFFLE W5THEORY Units:kN-m

MODAL PARTICIPATING MASS RATIOS

| MODE NUMBER | X-TRANS %MASS <SUM> | Y-TRANS %MASS <SUM> | Z-TRANS %MASS <SUM> | RX-ROTN %MASS <SUM> | RY-ROTN %MASS <SUM> | RZ-ROTN %MASS |
|-------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------|
| Mode 1      | 0.00 < 0>           | 0.00 < 0>           | 100.00 <100>        | 0.00 < 0>           | 100.00              | 100.00        |
| Mode 2      | 100.00 <100>        | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           | 0.00                | 0.00          |
| Mode 3      | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           | 0.00                | 0.00          |

ETABS v9.0.0 File:WAFFLE W5THEORY Units:kN-m

MODAL LOAD PARTICIPATION RATIOS (STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

| TYPE  | NAME | STATIC   | DYNAMIC  |
|-------|------|----------|----------|
| Load  | DEAD | 0.0000   | 0.0000   |
| Load  | LIVE | 0.0000   | 0.0000   |
| Accel | UX   | 100.0000 | 100.0000 |
| Accel | UY   | 100.0000 | 100.0000 |
| Accel | UZ   | 0.0000   | 0.0000   |
| Accel | RX   | 100.0000 | 100.0000 |
| Accel | RY   | 100.0000 | 100.0000 |
| Accel | RZ   | 100.0000 | 100.0000 |

ETABS v9.0.0 File:WAFFLE W5THEORY Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

| LOAD | FX   | FY   | FZ   | MX    | MY     | MZ    |
|------|------|------|------|-------|--------|-------|
| DEAD | 0.00 | 0.00 | 3.06 | 0.460 | -2.010 | 0.000 |
| LIVE | 0.00 | 0.00 | 0.72 | 0.108 | -0.473 | 0.000 |

## Appendix 32: Result of waffle W6 Theoretical (ETABS)

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

### PROJECT INFORMATION

Company Name = eng

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

### S T O R Y D A T A

| STORY  | SIMILAR TO | HEIGHT | ELEVATION |
|--------|------------|--------|-----------|
| STORY1 | None       | 0.750  | 0.750     |
| BASE   | None       |        | 0.000     |

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

### S T A T I C L O A D C A S E S

| STATIC CASE | CASE TYPE   | AUTO LAT LOAD | SELF WT MULTIPLIER |
|-------------|-------------|---------------|--------------------|
| DEAD        | DEAD        | N/A           | 0.1000             |
| LIVE        | REDUCE LIVE | N/A           | 0.0000             |

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

### M A S S S O U R C E D A T A

| MASS FROM | LATERAL MASS ONLY | LUMP MASS AT STORIES |
|-----------|-------------------|----------------------|
|-----------|-------------------|----------------------|

Masses Yes Yes

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

### D I A P H R A G M M A S S D A T A

| STORY  | DIAPHRAGM | MASS-X | MASS-Y | MMI    | X-M   | Y-M   |
|--------|-----------|--------|--------|--------|-------|-------|
| STORY1 | D1        | 1.6898 | 1.6898 | 0.3085 | 0.430 | 0.180 |

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

### A S S E M B L E D P O I N T M A S S E S

| STORY  | UX       | UY       | UZ       | RX       | RY       | RZ       |
|--------|----------|----------|----------|----------|----------|----------|
| STORY1 | 1.689796 | 1.689796 | 0.000000 | 0.000000 | 0.000000 | 0.308515 |
| BASE   | 0.752728 | 0.752728 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| Totals | 2.442524 | 2.442524 | 0.000000 | 0.000000 | 0.000000 | 0.308515 |

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

CENTERS OF CUMULATIVE MASS & CENTER OF RIGIDITY

STORY DIAPHRAGM /-----CENTRE OF MASS-----//--CENTRE OF RIGIDITY--/

| LEVEL  | NAME | MASS   | ORDINATE-X | ORDINATE-Y | ORDINATE-X | ORDINATE-Y |
|--------|------|--------|------------|------------|------------|------------|
| STORY1 | D1   | 1.6898 | 0.430      | 0.180      | 0.430      | 0.180      |

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

MODAL PERIODS AND FREQUENCIES

| MODE NUMBER | PERIOD (TIME) | FREQUENCY (CYCLES/TIME) | CIRCULAR FREQ (RADIANS/TIME) |
|-------------|---------------|-------------------------|------------------------------|
| Mode 1      | 0.01037       | 96.47473                | 606.16863                    |
| Mode 2      | 0.00834       | 119.94595               | 753.64261                    |
| Mode 3      | 0.00780       | 128.27638               | 805.98424                    |

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

MODAL PARTICIPATING MASS RATIOS

| MODE NUMBER | X-TRANS<br><SUM> | Y-TRANS<br>%MASS <SUM> | Z-TRANS<br>%MASS <SUM> | RX-ROTN<br>%MASS <SUM> | RY-ROTN<br>%MASS <SUM> | RZ-ROTN<br>%MASS |
|-------------|------------------|------------------------|------------------------|------------------------|------------------------|------------------|
| Mode 1      | 0.00 < 0>        | 0.00 < 0>              | 100.00 <100>           | 0.00 < 0>              | 100.00                 | 100.00           |
| Mode 2      | 100.00 <100>     | 0.00 < 0>              | 0.00 <100>             | 0.00 < 0>              | 0.00                   | 0.00             |
| Mode 3      | 0.00 <100>       | 0.00 <100>             | 0.00 <100>             | 0.00 < 0>              | 0.00                   | 0.00             |

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

MODAL LOAD PARTICIPATION RATIOS  
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

| TYPE  | NAME | STATIC   | DYNAMIC  |
|-------|------|----------|----------|
| Load  | DEAD | 0.0000   | 0.0000   |
| Load  | LIVE | 0.0000   | 0.0000   |
| Accel | UX   | 100.0000 | 100.0000 |
| Accel | UY   | 100.0000 | 100.0000 |
| Accel | UZ   | 0.0000   | 0.0000   |
| Accel | RX   | 100.0000 | 100.0000 |
| Accel | RY   | 100.0000 | 100.0000 |
| Accel | RZ   | 100.0000 | 100.0000 |

ETABS v9.0.0 File:WAFFLE W6THEORY Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

| LOAD | FX   | FY   | FZ   | MX    | MY     | MZ    |
|------|------|------|------|-------|--------|-------|
| DEAD | 0.00 | 0.00 | 2.70 | 0.486 | -1.160 | 0.000 |
| LIVE | 0.00 | 0.00 | 0.24 | 0.044 | -0.105 | 0.000 |

### Appendix 33: Result of waffle W1 Experimental (ETABS)

ETABS v9.0.0 File:WAFFLE W1EXPERIMENT Units:kN-m

#### PROJECT INFORMATION

Company Name = eng

ETABS v9.0.0 File:WAFFLE W1EXPERIMENT Units:kN-m

#### S T O R Y D A T A

| STORY  | SIMILAR TO | HEIGHT | ELEVATION |
|--------|------------|--------|-----------|
| STORY1 | None       | 0.750  | 0.750     |
| BASE   | None       |        | 0.000     |

ETABS v9.0.0 File:WAFFLE W1EXPERIMENT Units:kN-m

#### S T A T I C L O A D C A S E S

| STATIC CASE | CASE TYPE   | AUTO LAT LOAD | SELF WT MULTIPLIER |
|-------------|-------------|---------------|--------------------|
| DEAD        | DEAD        | N/A           | 0.1000             |
| LIVE        | REDUCE LIVE | N/A           | 0.0000             |

ETABS v9.0.0 File:WAFFLE W1EXPERIMENT Units:kN-m

#### M A S S S O U R C E D A T A

| MASS FROM | LATERAL MASS ONLY | LUMP MASS AT STORIES |
|-----------|-------------------|----------------------|
|-----------|-------------------|----------------------|

Masses Yes Yes

ETABS v9.0.0 File:WAFFLE W1EXPERIMENT Units:kN-m

#### D I A P H R A G M M A S S D A T A

| STORY  | DIAPHRAGM | MASS-X | MASS-Y | MMI    | X-M   | Y-M   |
|--------|-----------|--------|--------|--------|-------|-------|
| STORY1 | D1        | 3.0170 | 3.0170 | 1.1743 | 0.677 | 0.215 |

ETABS v9.0.0 File:WAFFLE W1EXPERIMENT Units:kN-m

A S S E M B L E D P O I N T M A S S E S

| STORY  | UX       | UY       | UZ       | RX       | RY       | RZ       |
|--------|----------|----------|----------|----------|----------|----------|
| STORY1 | 3.017041 | 3.017041 | 0.000000 | 0.000000 | 0.000000 | 1.174264 |
| BASE   | 0.752728 | 0.752728 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| Totals | 3.769769 | 3.769769 | 0.000000 | 0.000000 | 0.000000 | 1.174264 |

ETABS v9.0.0 File:WAFFLE W1EXPERIMENT Units:kN-m

C E N T E R S O F C U M U L A T I V E M A S S & C E N T E R S O F R I G I D I T Y

| STORY  | DIAPHRAGM | /-----CENTRE OF MASS-----//--CENTRE OF RIGIDITY--/ |            |            |            |            |
|--------|-----------|--|------------|------------|------------|------------|
| LEVEL  | NAME      | MASS   | ORDINATE-X | ORDINATE-Y | ORDINATE-X | ORDINATE-Y |
| STORY1 | D1        | 3.0170   | 0.677      | 0.215      | 0.676      | 0.215      |

ETABS v9.0.0 File:WAFFLE W1EXPERIMENT Units:kN-m

M O D A L P E R I O D S A N D F R E Q U E N C I E S

| MODE NUMBER | PERIOD (TIME) | FREQUENCY (CYCLES/TIME) | CIRCULAR FREQ (RADIAN/TIME) |
|-------------|---------------|-------------------------|-----------------------------|
| Mode 1      | 0.01249       | 80.04585                | 502.94288                   |
| Mode 2      | 0.01096       | 91.26274                | 573.42070                   |
| Mode 3      | 0.00990       | 101.02017               | 634.72843                   |

ETABS v9.0.0 File:WAFFLE W1EXPERIMENT Units:kN-m

M O D A L P A R T I C I P A T I N G M A S S R A T I O S

| MODE NUMBER | X-TRANS %MASS <SUM> | Y-TRANS %MASS <SUM> | Z-TRANS %MASS <SUM> | RX-ROTN %MASS <SUM> | RY-ROTN %MASS <SUM> | RZ-ROTN %MASS <SUM> |
|-------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| Mode 1      | 0.00 < 0>           | 0.00 < 0>           | 100.00 <100>        | 0.00 < 0>           | 0.00 < 0>           | 100.00 <100>        |
| Mode 2      | 100.00 <100>        | 0.00 < 0>           | 0.00 < 0>           | 0.00 <100>          | 0.00 < 0>           | 0.00 < 0>           |
| Mode 3      | 0.00 <100>          | 0.00 <100>          | 0.00 <100>          | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           |



ETABS v9.0.0 File:WAFFLE W1EXPERIMENT Units:kN-m

MODAL LOAD PARTICIPATION RATIOS  
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

| TYPE  | NAME | STATIC   | DYNAMIC  |
|-------|------|----------|----------|
| Load  | DEAD | 0.0000   | 0.0000   |
| Load  | LIVE | 0.0000   | 0.0000   |
| Accel | UX   | 100.0000 | 100.0000 |
| Accel | UY   | 100.0000 | 100.0000 |
| Accel | UZ   | 0.0000   | 0.0000   |
| Accel | RX   | 100.0000 | 100.0000 |
| Accel | RY   | 100.0000 | 100.0000 |
| Accel | RZ   | 100.0000 | 100.0000 |

ETABS v9.0.0 File:WAFFLE W1EXPERIMENT Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

| LOAD | FX   | FY   | FZ    | MX    | MY      | MZ    |
|------|------|------|-------|-------|---------|-------|
| DEAD | 0.00 | 0.00 | 15.24 | 3.277 | -10.310 | 0.000 |
| LIVE | 0.00 | 0.00 | 6.75  | 1.452 | -4.569  | 0.000 |

### Appendix 34: Result of waffle W2 Experimental (ETABS)

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

#### PROJECT INFORMATION

Company Name = eng

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

#### S T O R Y D A T A

| STORY  | SIMILAR TO | HEIGHT | ELEVATION |
|--------|------------|--------|-----------|
| STORY1 | None       | 0.750  | 0.750     |
| BASE   | None       |        | 0.000     |

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

#### S T A T I C L O A D C A S E S

| STATIC CASE | CASE TYPE   | AUTO LAT LOAD | SELF WT MULTIPLIER |
|-------------|-------------|---------------|--------------------|
| DEAD        | DEAD        | N/A           | 0.1000             |
| LIVE        | REDUCE LIVE | N/A           | 0.0000             |

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

#### M A S S S O U R C E D A T A

| MASS FROM | LATERAL MASS ONLY | LUMP MASS AT STORIES |
|-----------|-------------------|----------------------|
|-----------|-------------------|----------------------|

Masses Yes Yes

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

#### D I A P H R A G M M A S S D A T A

| STORY  | DIAPHRAGM | MASS-X | MASS-Y | MMI    | X-M   | Y-M   |
|--------|-----------|--------|--------|--------|-------|-------|
| STORY1 | D1        | 2.1975 | 2.1975 | 0.4053 | 0.454 | 0.150 |

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

A S S E M B L E D P O I N T M A S S E S

| STORY  | UX       | UY       | UZ       | RX       | RY       | RZ       |
|--------|----------|----------|----------|----------|----------|----------|
| STORY1 | 2.197480 | 2.197480 | 0.000000 | 0.000000 | 0.000000 | 0.405288 |
| BASE   | 0.752728 | 0.752728 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| Totals | 2.950208 | 2.950208 | 0.000000 | 0.000000 | 0.000000 | 0.405288 |

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

C E N T E R S O F C U M U L A T I V E M A S S & C E N T E R S O F R I G I D I T Y

| STORY  | DIAPHRAGM | /-----CENTRE OF MASS-----//--CENTRE OF RIGIDITY--/ |            |            |            |            |
|--------|-----------|--|------------|------------|------------|------------|
| LEVEL  | NAME      | MASS   | ORDINATE-X | ORDINATE-Y | ORDINATE-X | ORDINATE-Y |
| STORY1 | D1        | 2.1975   | 0.454      | 0.150      | 0.474      | 0.150      |

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

M O D A L P E R I O D S A N D F R E Q U E N C I E S

| MODE NUMBER | PERIOD (TIME) | FREQUENCY (CYCLES/TIME) | CIRCULAR FREQ (RADIAN/TIME) |
|-------------|---------------|-------------------------|-----------------------------|
| Mode 1      | 0.01283       | 77.94835                | 489.76390                   |
| Mode 2      | 0.00922       | 108.49457               | 681.69150                   |
| Mode 3      | 0.00892       | 112.12289               | 704.48889                   |

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

M O D A L P A R T I C I P A T I N G M A S S R A T I O S

| MODE NUMBER | X-TRANS %MASS <SUM> | Y-TRANS %MASS <SUM> | Z-TRANS %MASS <SUM> | RX-ROTN %MASS <SUM> | RY-ROTN %MASS <SUM> | RZ-ROTN %MASS <SUM> |
|-------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| Mode 1      | 0.00 < 0>           | 99.82 <100>         | 0.00 < 0>           | 99.82 <100>         | 0.00 < 0>           | 0.16 < 0>           |
| Mode 2      | 100.00 <100>        | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           |
| Mode 3      | 0.00 < 0>           | 0.18 <100>          | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           | 0.18 < 0>           |

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

MODAL LOAD PARTICIPATION RATIOS  
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

| TYPE  | NAME | STATIC   | DYNAMIC  |
|-------|------|----------|----------|
| Load  | DEAD | 0.0683   | 0.0000   |
| Load  | LIVE | 0.0017   | 0.0000   |
| Accel | UX   | 100.0000 | 100.0000 |
| Accel | UY   | 100.0000 | 100.0000 |
| Accel | UZ   | 0.0000   | 0.0000   |
| Accel | RX   | 100.0000 | 100.0000 |
| Accel | RY   | 100.0000 | 100.0000 |
| Accel | RZ   | 104.3987 | 100.0000 |

ETABS v9.0.0 File:WAFFLE W2EXPERIMENT Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

| LOAD | FX   | FY   | FZ   | MX    | MY     | MZ    |
|------|------|------|------|-------|--------|-------|
| DEAD | 0.00 | 0.00 | 3.11 | 0.467 | -1.401 | 0.000 |
| LIVE | 0.00 | 0.00 | 0.76 | 0.114 | -0.399 | 0.000 |

## Appendix 35: Result of waffle W3 Experimental (ETABS)

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

### PROJECT INFORMATION

Company Name = eng

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

### S T O R Y D A T A

| STORY  | SIMILAR TO | HEIGHT | ELEVATION |
|--------|------------|--------|-----------|
| STORY1 | None       | 0.750  | 0.750     |
| BASE   | None       |        | 0.000     |

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

### S T A T I C L O A D C A S E S

| STATIC CASE | CASE TYPE   | AUTO LAT LOAD | SELF WT MULTIPLIER |
|-------------|-------------|---------------|--------------------|
| DEAD        | DEAD        | N/A           | 0.1000             |
| LIVE        | REDUCE LIVE | N/A           | 0.0000             |

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

### M A S S S O U R C E D A T A

| MASS FROM | LATERAL MASS ONLY | LUMP MASS AT STORIES |
|-----------|-------------------|----------------------|
|-----------|-------------------|----------------------|

Masses Yes Yes

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

### D I A P H R A G M M A S S D A T A

| STORY  | DIAPHRAGM | MASS-X | MASS-Y | MMI    | X-M   | Y-M   |
|--------|-----------|--------|--------|--------|-------|-------|
| STORY1 | D1        | 2.5350 | 2.5350 | 0.6810 | 0.543 | 0.215 |

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

A S S E M B L E D P O I N T M A S S E S

| STORY  | UX       | UY       | UZ       | RX       | RY       | RZ       |
|--------|----------|----------|----------|----------|----------|----------|
| STORY1 | 2.535048 | 2.535048 | 0.000000 | 0.000000 | 0.000000 | 0.680962 |
| BASE   | 0.752728 | 0.752728 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| Totals | 3.287776 | 3.287776 | 0.000000 | 0.000000 | 0.000000 | 0.680962 |

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

C E N T E R S O F C U M U L A T I V E M A S S & C E N T E R  
S O F R I G I D I T Y

| STORY  | DIAPHRAGM | /-----CENTRE OF MASS-----//--CENTRE OF RIGIDITY--/ |            |            |            |            |
|--------|-----------|--|------------|------------|------------|------------|
| LEVEL  | NAME      | MASS   | ORDINATE-X | ORDINATE-Y | ORDINATE-X | ORDINATE-Y |
| STORY1 | D1        | 2.5350   | 0.543      | 0.215      | 0.542      | 0.215      |

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

M O D A L P E R I O D S A N D F R E Q U E N C I E S

| MODE NUMBER | PERIOD (TIME) | FREQUENCY (CYCLES/TIME) | CIRCULAR FREQ (RADIAN/TIME) |
|-------------|---------------|-------------------------|-----------------------------|
| Mode 1      | 0.01145       | 87.32430                | 548.67473                   |
| Mode 2      | 0.00993       | 100.66678               | 632.50804                   |
| Mode 3      | 0.00892       | 112.09162               | 704.29241                   |

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

M O D A L P A R T I C I P A T I N G M A S S R A T I O S

| MODE NUMBER | X-TRANS %MASS <SUM> | Y-TRANS %MASS <SUM> | Z-TRANS %MASS <SUM> | RX-ROTN %MASS <SUM> | RY-ROTN %MASS <SUM> | RZ-ROTN %MASS |
|-------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------|
| Mode 1      | 0.00 < 0>           | 0.00 < 0>           | 100.00 <100>        | 0.00 < 0>           | 100.00 <100>        |               |
| Mode 2      | 100.00 <100>        | 0.00 < 0>           | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>          |               |
| Mode 3      | 0.00 <100>          | 0.00 <100>          | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>          |               |

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

MODAL LOAD PARTICIPATION RATIOS  
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

| TYPE  | NAME | STATIC   | DYNAMIC  |
|-------|------|----------|----------|
| Load  | DEAD | 0.0000   | 0.0000   |
| Load  | LIVE | 0.0000   | 0.0000   |
| Accel | UX   | 100.0000 | 100.0000 |
| Accel | UY   | 100.0000 | 100.0000 |
| Accel | UZ   | 0.0000   | 0.0000   |
| Accel | RX   | 100.0000 | 100.0000 |
| Accel | RY   | 100.0000 | 100.0000 |
| Accel | RZ   | 100.0000 | 100.0000 |

ETABS v9.0.0 File:WAFFLE W3EXPERIMENT Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

| LOAD | FX   | FY   | FZ    | MX    | MY     | MZ    |
|------|------|------|-------|-------|--------|-------|
| DEAD | 0.00 | 0.00 | 11.02 | 2.369 | -5.978 | 0.000 |
| LIVE | 0.00 | 0.00 | 4.51  | 0.969 | -2.445 | 0.000 |

### Appendix 36: Result of waffle W4 Experimental (ETABS)

ETABS v9.0.0 File:WAFFLE W4EXPERIMENT Units:kN-m

PROJECT INFORMATION

Company Name = eng

ETABS v9.0.0 File:WAFFLE W4EXPERIMENT Units:kN-m

S T O R Y D A T A

| STORY  | SIMILAR TO | HEIGHT | ELEVATION |
|--------|------------|--------|-----------|
| STORY1 | None       | 0.750  | 0.750     |
| BASE   | None       |        | 0.000     |

ETABS v9.0.0 File:WAFFLE W4EXPERIMENT Units:kN-m

S T A T I C L O A D C A S E S

| STATIC CASE | CASE TYPE   | AUTO LAT LOAD | SELF WT MULTIPLIER |
|-------------|-------------|---------------|--------------------|
| DEAD        | DEAD        | N/A           | 0.1000             |
| LIVE        | REDUCE LIVE | N/A           | 0.0000             |

ETABS v9.0.0 File:WAFFLE W4EXPERIMENT Units:kN-m

M A S S S O U R C E D A T A

| MASS FROM | LATERAL MASS ONLY | LUMP MASS AT STORIES |
|-----------|-------------------|----------------------|
| Masses    | Yes               | Yes                  |

ETABS v9.0.0 File:WAFFLE W4EXPERIMENT Units:kN-m

D I A P H R A G M M A S S D A T A

| STORY  | DIAPHRAGM | MASS-X | MASS-Y | MMI    | X-M   | Y-M   |
|--------|-----------|--------|--------|--------|-------|-------|
| STORY1 | D1        | 1.6154 | 1.6154 | 0.1075 | 0.204 | 0.182 |



ETABS v9.0.0 File:WAFFLE W4EXPERIMENT Units:kN-m

A S S E M B L E D P O I N T M A S S E S

| STORY  | UX       | UY       | UZ       | RX       | RY       | RZ       |
|--------|----------|----------|----------|----------|----------|----------|
| STORY1 | 1.615446 | 1.615446 | 0.000000 | 0.000000 | 0.000000 | 0.107542 |
| BASE   | 0.752728 | 0.752728 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| Totals | 2.368174 | 2.368174 | 0.000000 | 0.000000 | 0.000000 | 0.107542 |

ETABS v9.0.0 File:WAFFLE W4EXPERIMENT Units:kN-m

C E N T E R S O F C U M U L A T I V E M A S S & C E N T E R S O F R I G I D I T Y

| STORY  | DIAPHRAGM | /-----CENTRE OF MASS-----//--CENTRE OF RIGIDITY--/ |            |            |            |            |
|--------|-----------|--|------------|------------|------------|------------|
| LEVEL  | NAME      | MASS   | ORDINATE-X | ORDINATE-Y | ORDINATE-X | ORDINATE-Y |
| STORY1 | D1        | 1.6154   | 0.204      | 0.182      | 0.204      | 0.182      |

ETABS v9.0.0 File:WAFFLE W4EXPERIMENT Units:kN-m

M O D A L P E R I O D S A N D F R E Q U E N C I E S

| MODE NUMBER | PERIOD (TIME) | FREQUENCY (CYCLES/TIME) | CIRCULAR FREQ (RADIAN/TIME) |
|-------------|---------------|-------------------------|-----------------------------|
| Mode 1      | 0.00973       | 102.72929               | 645.46719                   |
| Mode 2      | 0.00934       | 107.06637               | 672.71782                   |
| Mode 3      | 0.00686       | 145.82088               | 916.21964                   |

ETABS v9.0.0 File:WAFFLE W4EXPERIMENT Units:kN-m

M O D A L P A R T I C I P A T I N G M A S S R A T I O S

| MODE NUMBER | X-TRANS %MASS <SUM> | Y-TRANS %MASS <SUM> | Z-TRANS %MASS <SUM> | RX-ROTN %MASS <SUM> | RY-ROTN %MASS <SUM> | RZ-ROTN %MASS <SUM> |
|-------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| Mode 1      | 0.00 < 0>           | 0.00 < 0>           | 100.00 <100>        | 0.00 < 0>           | 0.00 < 0>           | 100.00 <100>        |
| Mode 2      | 100.00 <100>        | 0.00 < 0>           | 0.00 < 0>           | 0.00 <100>          | 0.00 < 0>           | 0.00 < 0>           |
| Mode 3      | 0.00 <100>          | 0.00 <100>          | 0.00 <100>          | 0.00 < 0>           | 0.00 < 0>           | 0.00 < 0>           |

ETABS v9.0.0 File:WAFFLE W4EXPERIMENT Units:kN-m

MODAL LOAD PARTICIPATION RATIOS  
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

| TYPE  | NAME | STATIC   | DYNAMIC  |
|-------|------|----------|----------|
| Load  | DEAD | 0.0000   | 0.0000   |
| Load  | LIVE | 0.0000   | 0.0000   |
| Accel | UX   | 100.0000 | 100.0000 |
| Accel | UY   | 100.0000 | 100.0000 |
| Accel | UZ   | 0.0000   | 0.0000   |
| Accel | RX   | 100.0000 | 100.0000 |
| Accel | RY   | 100.0000 | 100.0000 |
| Accel | RZ   | 100.0000 | 100.0000 |

ETABS v9.0.0 File:WAFFLE W4EXPERIMENT Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

| LOAD | FX   | FY   | FZ   | MX    | MY     | MZ    |
|------|------|------|------|-------|--------|-------|
| DEAD | 0.00 | 0.00 | 2.45 | 0.446 | -0.498 | 0.000 |
| LIVE | 0.00 | 0.00 | 1.58 | 0.287 | -0.321 | 0.000 |

## Appendix 37: Result of waffle W5 Experimental (ETABS)

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m  
PROJECT INFORMATION

Company Name = eng

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

### S T O R Y D A T A

| STORY  | SIMILAR TO | HEIGHT | ELEVATION |
|--------|------------|--------|-----------|
| STORY1 | None       | 0.750  | 0.750     |
| BASE   | None       |        | 0.000     |

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

### S T A T I C L O A D C A S E S

| STATIC CASE | CASE TYPE   | AUTO LAT LOAD | SELF WT MULTIPLIER |
|-------------|-------------|---------------|--------------------|
| DEAD        | DEAD        | N/A           | 0.1000             |
| LIVE        | REDUCE LIVE | N/A           | 0.0000             |

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

### M A S S S O U R C E D A T A

| MASS FROM | LATERAL MASS ONLY | LUMP MASS AT STORIES |
|-----------|-------------------|----------------------|
|-----------|-------------------|----------------------|

Masses Yes Yes

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

### D I A P H R A G M M A S S D A T A

| STORY  | DIAPHRAGM | MASS-X | MASS-Y | MMI    | X-M   | Y-M   |
|--------|-----------|--------|--------|--------|-------|-------|
| STORY1 | D1        | 2.2570 | 2.2570 | 0.8492 | 0.668 | 0.147 |

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

### A S S E M B L E D P O I N T M A S S E S

| STORY  | UX       | UY       | UZ       | RX       | RY       | RZ       |
|--------|----------|----------|----------|----------|----------|----------|
| STORY1 | 2.749433 | 2.749433 | 0.000000 | 0.000000 | 0.000000 | 0.849182 |
| BASE   | 0.752728 | 0.752728 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| Totals | 3.502160 | 3.502160 | 0.000000 | 0.000000 | 0.000000 | 0.849182 |

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

C E N T E R S O F C U M U L A T I V E M A S S & C E N T E R S  
O F R I G I D I T Y

STORY DIAPHRAGM /-----CENTRE OF MASS-----//--CENTRE OF  
RIGIDITY--/

| LEVEL  | NAME | MASS   | ORDINATE-X | ORDINATE-Y | ORDINATE-X | ORDINATE-Y |
|--------|------|--------|------------|------------|------------|------------|
| STORY1 | D1   | 2.2570 | 0.668      | 0.147      | 0.655      | 0.169      |

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

M O D A L P E R I O D S A N D F R E Q U E N C I E S

| MODE NUMBER | PERIOD (TIME) | FREQUENCY (CYCLES/TIME) | CIRCULAR FREQ (RADIANS/TIME) |
|-------------|---------------|-------------------------|------------------------------|
| Mode 1      | 0.01392       | 71.85817                | 451.49820                    |
| Mode 2      | 0.01073       | 93.21213                | 585.66912                    |
| Mode 3      | 0.01045       | 95.73303                | 601.50835                    |
| Mode 4      | 0.00074       | 1342.60301              | 8435.82349                   |
| Mode 5      | 0.00043       | 2303.89344              | 14475.78944                  |
| Mode 6      | 0.00015       | 6552.63176              | 41171.39961                  |
| Mode 7      | 0.00000       | 11613.02352             | 72966.77872                  |
| Mode 8      | 0.00000       | 13258.95988             | 83308.50193                  |
| Mode 9      | 0.00000       | 22731.23830             | 142824.58252                 |

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

M O D A L P A R T I C I P A T I N G M A S S R A T I O S

| MODE NUMBER | X-TRANS %MASS <SUM> | Y-TRANS %MASS <SUM> | Z-TRANS %MASS <SUM> | RX-ROTN %MASS <SUM> | RY-ROTN %MASS <SUM> | RZ-ROTN %MASS |
|-------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------|
| Mode 1      | 0.00 < 0>           | 100.00 <100>        | 0.00 < 0>           | 100.00 <100>        | 0.00 < 0>           | 0.00 <100>    |
| Mode 2      | 0.07 < 0>           | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>    |
| Mode 3      | 99.93 <100>         | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>    |
| Mode 4      | 0.00 <100>          | 0.00 <100>          | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>          | 0.00 <100>    |
| Mode 5      | 0.00 <100>          | 0.00 <100>          | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>          | 0.00 <100>    |
| Mode 6      | 0.00 <100>          | 0.00 <100>          | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>          | 0.00 <100>    |
| Mode 7      | 0.00 <100>          | 0.00 <100>          | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>          | 0.00 <100>    |
| Mode 8      | 0.00 <100>          | 0.00 <100>          | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>          | 0.00 <100>    |
| Mode 9      | 0.00 <100>          | 0.00 <100>          | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>          | 0.00 <100>    |

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

MODAL LOAD PARTICIPATION RATIOS  
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

| TYPE  | NAME | STATIC   | DYNAMIC  |
|-------|------|----------|----------|
| Load  | DEAD | 0.5812   | 0.0000   |
| Load  | LIVE | 0.0041   | 0.0000   |
| Accel | UX   | 100.0000 | 100.0000 |
| Accel | UY   | 100.0000 | 100.0000 |
| Accel | UZ   | 0.0000   | 0.0000   |
| Accel | RX   | 100.0000 | 100.0000 |
| Accel | RY   | 100.0000 | 100.0000 |
| Accel | RZ   | 99.7702  | 100.0000 |

ETABS v9.0.0 File:WAFFLE W5EXPERIMENT Units:kN-m

TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

| LOAD | FX   | FY   | FZ   | MX    | MY     | MZ    |
|------|------|------|------|-------|--------|-------|
| DEAD | 0.00 | 0.00 | 3.59 | 0.593 | -2.353 | 0.000 |
| LIVE | 0.00 | 0.00 | 1.72 | 0.258 | -1.129 | 0.000 |

## Appendix 38: Result of waffle W6 Experimental (ETABS)

ETABS v9.0.0 File:WAFFLE W6EXPERIMENT Units:kN-m

### PROJECT INFORMATION

Company Name = eng

ETABS v9.0.0 File:WAFFLE W6EXPERIMENT Units:kN-m

### S T O R Y D A T A

| STORY  | SIMILAR TO | HEIGHT | ELEVATION |
|--------|------------|--------|-----------|
| STORY1 | None       | 0.750  | 0.750     |
| BASE   | None       |        | 0.000     |

ETABS v9.0.0 File:WAFFLE W6EXPERIMENT Units:kN-m

### S T A T I C L O A D C A S E S

| STATIC CASE | CASE TYPE   | AUTO LAT LOAD | SELF WT MULTIPLIER |
|-------------|-------------|---------------|--------------------|
| DEAD        | DEAD        | N/A           | 0.1000             |
| LIVE        | REDUCE LIVE | N/A           | 0.0000             |

ETABS v9.0.0 File:WAFFLE W6EXPERIMENT Units:kN-m

### M A S S S O U R C E D A T A

| MASS FROM | LATERAL MASS ONLY | LUMP MASS AT STORIES |
|-----------|-------------------|----------------------|
|-----------|-------------------|----------------------|

Masses Yes Yes

ETABS v9.0.0 File:WAFFLE W6EXPERIMENT Units:kN-m

### D I A P H R A G M M A S S D A T A

| STORY  | DIAPHRAGM | MASS-X | MASS-Y | MMI    | X-M   | Y-M   |
|--------|-----------|--------|--------|--------|-------|-------|
| STORY1 | D1        | 1.6898 | 1.6898 | 0.3085 | 0.430 | 0.180 |

ETABS v9.0.0 File:WAFFLE W6EXPERIMENT Units:kN-m

### A S S E M B L E D P O I N T M A S S E S

| STORY  | UX       | UY       | UZ       | RX       | RY       | RZ       |
|--------|----------|----------|----------|----------|----------|----------|
| STORY1 | 1.689796 | 1.689796 | 0.000000 | 0.000000 | 0.000000 | 0.308515 |
| BASE   | 0.752728 | 0.752728 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |

Totals 2.442524 2.442524 0.000000 0.000000 0.000000 0.308515

ETABS v9.0.0 File:WAFFLE W6EXPERIMENT Units:kN-m

C E N T E R S O F C U M U L A T I V E M A S S & C E N T E R S  
O F R I G I D I T Y

STORY DIAPHRAGM /-----CENTRE OF MASS-----//--CENTRE OF  
RIGIDITY--/

| LEVEL  | NAME | MASS   | ORDINATE-X | ORDINATE-Y | ORDINATE-X | ORDINATE-Y |
|--------|------|--------|------------|------------|------------|------------|
| STORY1 | D1   | 1.6898 | 0.430      | 0.180      | 0.430      | 0.180      |

ETABS v9.0.0 File:WAFFLE W6EXPERIMENT Units:kN-m

M O D A L P E R I O D S A N D F R E Q U E N C I E S

| MODE NUMBER | PERIOD (TIME) | FREQUENCY (CYCLES/TIME) | CIRCULAR FREQ (RADIANS/TIME) |
|-------------|---------------|-------------------------|------------------------------|
| Mode 1      | 0.01037       | 96.47473                | 606.16863                    |
| Mode 2      | 0.00834       | 119.94595               | 753.64261                    |
| Mode 3      | 0.00780       | 128.27638               | 805.98424                    |

ETABS v9.0.0 File:WAFFLE W6EXPERIMENT Units:kN-m

M O D A L P A R T I C I P A T I N G M A S S R A T I O S

| MODE NUMBER | X-TRANS %MASS <SUM> | Y-TRANS %MASS <SUM> | Z-TRANS %MASS <SUM> | RX-ROTN %MASS <SUM> | RY-ROTN %MASS <SUM> | RZ-ROTN %MASS |
|-------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------|
| Mode 1      | 0.00 < 0>           | 100.00 <100>        | 0.00 < 0>           | 100.00 <100>        | 0.00 < 0>           | 0.00 <100>    |
| Mode 2      | 100.00 <100>        | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>    |
| Mode 3      | 0.00 <100>          | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>          | 0.00 < 0>           | 0.00 <100>    |

ETABS v9.0.0 File:WAFFLE W6EXPERIMENT Units:kN-m

M O D A L L O A D P A R T I C I P A T I O N R A T I O S  
(STATIC AND DYNAMIC RATIOS ARE IN PERCENT)

| TYPE  | NAME | STATIC   | DYNAMIC  |
|-------|------|----------|----------|
| Load  | DEAD | 0.0000   | 0.0000   |
| Load  | LIVE | 0.0000   | 0.0000   |
| Accel | UX   | 100.0000 | 100.0000 |
| Accel | UY   | 100.0000 | 100.0000 |
| Accel | UZ   | 0.0000   | 0.0000   |
| Accel | RX   | 100.0000 | 100.0000 |
| Accel | RY   | 100.0000 | 100.0000 |

Accel            RZ                    100.0000    100.0000

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TOTAL REACTIVE FORCES (RECOVERED LOADS) AT ORIGIN

| LOAD | FX   | FY   | FZ   | MX    | MY     | MZ    |
|------|------|------|------|-------|--------|-------|
| DEAD | 0.00 | 0.00 | 2.70 | 0.486 | -1.160 | 0.000 |
| LIVE | 0.00 | 0.00 | 1.11 | 0.200 | -0.477 | 0.000 |



**Appendix 39: Test results for panel W1**

| <b>Load</b> | <b>(kN)</b> | <b>Deflections (mm)</b> | <b>Crack width</b> |
|-------------|-------------|-------------------------|--------------------|
| 2           |             | 0.10                    | -                  |
| 4           |             | 0.15                    | -                  |
| 6           |             | 0.23                    | -                  |
| 8           |             | 0.31                    | -                  |
| 10          |             | 0.40                    | -                  |
| 12          |             | 0.49                    | -                  |
| 14          |             | 0.80                    | -                  |
| 16          |             | 0.99                    | -                  |
| 18          |             | 1.19                    | 0.60               |

**Appendix 40 : Test results for panel W2**

| <b>Load</b> | <b>(kN)</b> | <b>Deflections (mm)</b> | <b>Crack width (mm)</b> |
|-------------|-------------|-------------------------|-------------------------|
| 1           |             | 0.34                    | -                       |
| 2           |             | 0.71                    | -                       |
| 3           |             | 0.89                    | -                       |
| 4           |             | 1.34                    | -                       |
| 5           |             | 1.51                    | -                       |
| 6           |             | 1.70                    | -                       |
| 7           |             | 1.90                    | -                       |
| 8           |             | 2.70                    | -                       |
| 9           |             | 3.64                    | 0.35                    |

**Appendix 41 : Test results for panel W3**

| <b>Load (kN)</b> | <b>Deflection (mm)</b> | <b>Crack width (mm)</b> |
|------------------|------------------------|-------------------------|
| 2                | 0                      | -                       |
| 4                | 1.11                   | -                       |
| 6                | 1.79                   | -                       |
| 8                | 2.45                   | -                       |
| 10               | 2.99                   | -                       |
| 12               | 3.90                   | 0.34                    |

**Appendix 42: Test results for panel W4**

| <b>Load (kN)</b> | <b>Deflection (mm)</b> | <b>Crack width (mm)</b> |
|------------------|------------------------|-------------------------|
| 1                | 1.45                   | -                       |
| 2                | 3.31                   | -                       |
| 3                | 4.69                   | -                       |
| 4                | 5.90                   | -                       |
| 5                | 6.16                   | -                       |
| 6                | 6.60                   | 0.25                    |

**Appendix 43: Test results for panel W5**

| <b>Load (mm)</b> | <b>Deflection (mm)</b> | <b>Crack width (mm)</b> |
|------------------|------------------------|-------------------------|
| 1                | 1.44                   | -                       |
| 2                | 3.77                   | -                       |
| 3                | 4.69                   | -                       |
| 4                | 6.05                   | -                       |
| 5                | 7.34                   | -                       |
| 6                | 7.35                   | -                       |
| 7                | 7.72                   | -                       |
| 8                | 8.17                   | 0.35                    |

**Appendix 44: Test results for panel W6**

| <b>Load (kN)</b> | <b>Deflection (mm)</b> | <b>Crack width (mm)</b> |
|------------------|------------------------|-------------------------|
| 2                | 0.43                   | -                       |
| 4                | 0.78                   | -                       |
| 6                | 1.22                   | -                       |
| 8                | 1.69                   | -                       |
| 10               | 2.40                   | -                       |
| 12               | 3.29                   | 0.62                    |

**Appendix 45: Test results for panel S1**

| <b>Load (kN)</b> | <b>Deflection (mm)</b> | <b>Crack width (mm)</b> |
|------------------|------------------------|-------------------------|
| 2                | 0.73                   | -                       |
| 4                | 1.47                   | -                       |
| 6                | 2.32                   | -                       |
| 8                | 2.86                   | -                       |
| 10               | 3.49                   | -                       |
| 12               | 3.56                   | 1.20                    |

**Appendix 46: Test results of panel S2**

| <b>Load (kN)</b> | <b>Deflection (mm)</b> | <b>Crack width (mm)</b> |
|------------------|------------------------|-------------------------|
| 1                | 1.10                   | -                       |
| 2                | 1.58                   | -                       |
| 3                | 2.38                   | -                       |
| 4                | 4.01                   | -                       |
| 5                | 5.49                   | -                       |
| 6                | 7.38                   | -                       |
| 7                | 9.28                   | 0.75                    |

**Appendix 47: Test results for panel S3**

| <b>Load (kN)</b> | <b>Deflection (mm)</b> | <b>Crack width (mm)</b> |
|------------------|------------------------|-------------------------|
| 2                | 0.71                   | -                       |
| 4                | 1.89                   | -                       |
| 6                | 3.33                   | -                       |
| 8                | 4.29                   | -                       |
| 10               | 5.08                   | -                       |
| 12               | 6.34                   | -                       |
| 14               | 7.44                   | 0.90                    |

**Appendix 48: Test results for panel S4**

| <b>Load (kN)</b> | <b>Deflection (mm)</b> | <b>Crack width (mm)</b> |
|------------------|------------------------|-------------------------|
| 1                | 1.19                   | -                       |
| 2                | 1.99                   | -                       |
| 3                | 2.62                   | -                       |
| 4                | 3.57                   | -                       |
| 5                | 4.82                   | -                       |
| 6                | 6.44                   | 0.70                    |

**Appendix 49: Test results for panel S5**

| <b>Load (kN)</b> | <b>Deflection (mm)</b> | <b>Crack width (mm)</b> |
|------------------|------------------------|-------------------------|
| 1                | 2.14                   | -                       |
| 2                | 6.18                   | -                       |
| 3                | 7.52                   | -                       |
| 4                | 9.30                   | -                       |
| 5                | 11.75                  | -                       |
| 6                | 12.18                  | 1.10                    |

**Appendix 50: Test results for panel S6**

| <b>Load (kN)</b> | <b>Deflection (mm)</b> | <b>Crack width (mm)</b> |
|------------------|------------------------|-------------------------|
| 2                | 0.14                   | -                       |
| 4                | 0.91                   | -                       |
| 6                | 1.79                   | -                       |
| 8                | 2.82                   | -                       |
| 10               | 3.89                   | 1.30                    |

**Appendix 51: Analysis of variance table for W1 and S1**

| <b>Source of variation</b> | <b>Degree of freedom</b> | <b>Sum of square</b> | <b>Mean square</b> | <b>Variance ratio F</b> |
|----------------------------|--------------------------|----------------------|--------------------|-------------------------|
| Specimens                  | 1                        | 12.83                | 12.83              | 21.93**                 |
| Error                      | 13                       | 7.61                 | 0.585              |                         |
| <i>Total</i>               | <i>14</i>                | <i>20.44</i>         |                    |                         |

F\*\* > P<sub>0.05</sub> significant

P<sub>0.05</sub> = 4.67 (F tabulated)

**Appendix 52: Analysis of variance table for W2 and S2**

| <b>Source of variation</b> | <b>Degree of freedom</b> | <b>Sum of square</b> | <b>Mean square</b> | <b>Variance ratio F</b> |
|----------------------------|--------------------------|----------------------|--------------------|-------------------------|
| Specimens                  | 1                        | 34.83                | 34.83              | 7.01**                  |
| Error                      | 14                       | 69.60                | 4.97               |                         |
| <i>Total</i>               | <i>15</i>                | <i>104.43</i>        |                    |                         |

F\*\* > P<sub>0.05</sub> significant

P<sub>0.05</sub> = 4.60 (F tabulated)

**Appendix 53: Analysis of variance table for W3 and S3**

| <b>Source of variation</b> | <b>Degree of freedom</b> | <b>Sum of square</b> | <b>Mean square</b> | <b>Variance ratio F</b> |
|----------------------------|--------------------------|----------------------|--------------------|-------------------------|
| Specimens                  | 1                        | 14.46                | 14.46              | 3.637*                  |
| Error                      | 11                       | 43.74                | 3.98               |                         |
| <i>Total</i>               | <i>12</i>                | <i>58.20</i>         |                    |                         |

F\* < P<sub>0.05</sub> no significance

P<sub>0.05</sub> = 4.96 (F tabulated)

**Appendix 54: Analysis of variance table for W4 and S4**

| <b>Source of variation</b> | <b>Degree of freedom</b> | <b>Sum of square</b> | <b>Mean square</b> | <b>Variance ratio F</b> |
|----------------------------|--------------------------|----------------------|--------------------|-------------------------|
| Specimens                  | 1                        | 4.30                 | 4.30               | 0.591*                  |
| Error                      | 14                       | 101.96               | 7.28               |                         |
| <i>Total</i>               | <i>15</i>                | <i>106.26</i>        |                    |                         |

F\* < P<sub>0.05</sub> no significance

P<sub>0.05</sub>= 4.60 (F tabulated)

**Appendix 55: Analysis of variance table for W5 and S5**

| <b>Source of variation</b> | <b>Degree of freedom</b> | <b>Sum of square</b> | <b>Mean square</b> | <b>Variance ratio F</b> |
|----------------------------|--------------------------|----------------------|--------------------|-------------------------|
| Specimens                  | 1                        | 18.94                | 18.94              | 2.076*                  |
| Error                      | 12                       | 109.42               | 9.12               |                         |
| <i>Total</i>               | <i>13</i>                | <i>128.36</i>        |                    |                         |

F\* < P<sub>0.05</sub> no significance

P<sub>0.05</sub>= 4.75 (F tabulated)

**Appendix 56: Analysis of variance table for W6 and S6**

| <b>Source of variation</b> | <b>Degree of freedom</b> | <b>Sum of square</b> | <b>Mean square</b> | <b>Variance ratio F</b> |
|----------------------------|--------------------------|----------------------|--------------------|-------------------------|
| Specimens                  | 1                        | 0.00                 | 0.00               | 0.000*                  |
| Error                      | 10                       | 61.532               | 6.153              |                         |
| Total                      | 11                       | 61.532               |                    |                         |

F\* < P<sub>0.05</sub> no significance

P<sub>0.05</sub>= 4.96 (F tabulated)



**Appendix 57: Correlation analysis for W1 and W3**

|    |                     | <b>W1</b> | <b>W3</b> |
|----|---------------------|-----------|-----------|
| W1 | Pearson Correlation | 1         | .984**    |
|    | Sig. (2-tailed)     | .         | .000      |
|    | N                   | 9         | 6         |
| W3 | Pearson Correlation | .984**    | 1         |
|    | Sig. (2-tailed)     | .000      | .         |
|    | N                   | 6         | 6         |

\*\* Correlation is significant at 0.01 levels

**Appendix 58: Correlation analysis for W2 and W4**

|    |                     | <b>W2</b> | <b>W4</b> |
|----|---------------------|-----------|-----------|
| W2 | Pearson Correlation | 1         | .988**    |
|    | Sig. (2-tailed)     | .         | .000      |
|    | N                   | 9         | 8         |
| W4 | Pearson Correlation | .988**    | 1         |
|    | Sig. (2-tailed)     | .000      | .         |
|    | N                   | 8         | 8         |

\*\* Correlation is significant at 0.01 levels

**Appendix 59: Correlation analysis for W5 and W6**

|    |                     | <b>W5</b> | <b>W6</b> |
|----|---------------------|-----------|-----------|
| W1 | Pearson Correlation | 1         | .919**    |
|    | Sig. (2-tailed)     | .         | .003      |
|    | N                   | 8         | 7         |
| W3 | Pearson Correlation | .919**    | 1         |
|    | Sig. (2-tailed)     | .003      | .         |
|    | N                   | 7         | 7         |

\*\* Correlation is significant at 0.01 levels

**Appendix 60: Correlation analysis for S1 and S3**

|    |                     | S1     | S3     |
|----|---------------------|--------|--------|
| W1 | Pearson Correlation | 1      | .983** |
|    | Sig. (2-tailed)     | .      | .000   |
|    | N                   | 6      | 6      |
| W3 | Pearson Correlation | .983** | 1      |
|    | Sig. (2-tailed)     | .000   | .      |
|    | N                   | 6      | 6      |

\*\* Correlation is significant at 0.01 levels

**Appendix 61: Correlation analysis for S2 and S4**

|    |                     | S2     | S4     |
|----|---------------------|--------|--------|
| W1 | Pearson Correlation | 1      | .997** |
|    | Sig. (2-tailed)     | .      | .000   |
|    | N                   | 7      | 7      |
| W3 | Pearson Correlation | .997** | 1      |
|    | Sig. (2-tailed)     | .000   | .000   |
|    | N                   | 7      | 8      |

\*\* Correlation is significant at 0.01 levels

**Appendix 62: Correlation analysis for S5 and S6**

|    |                     | S5     | S6     |
|----|---------------------|--------|--------|
| W1 | Pearson Correlation | 1      | .974** |
|    | Sig. (2-tailed)     | .      | .005   |
|    | N                   | 6      | 5      |
| W3 | Pearson Correlation | .974** | 1      |
|    | Sig. (2-tailed)     | .005   | .      |
|    | N                   | 5      | 5      |

\*\* Correlation is significant at 0.01 levels

### Appendix 63: Estimation of flexural crack width at service for Slab S1

Modulus of elasticity of steel  $E_s = 200 \text{ kN/mm}^2$ .

Calculate neutral axis position and steel stress.

Instantaneous modulus of elasticity =  $20 \text{ kN/mm}^2$

$$E_c = 20/2 = 10 \text{ kN/mm}^2$$

The neutral axis position is giving by

$$\frac{1}{2}bx^2 + \frac{E_s}{E_c}A_sx - \frac{E_s}{E_s}A_s d = 0$$

In this case  $A_s$  = area of distribution bars in the slab =  $377 \text{ mm}^2$

Effective depth,  $d = 40 - (5+3) = 32 \text{ mm}$

$$\text{Thus} \quad \frac{1}{2} \times 1000x^2 + \frac{200}{10} \times 377 \times x - \frac{200}{10} \times 377 \times 32 = 0$$

$$x = -\frac{15 \pm \sqrt{15^2 + 4 \times 483}}{2} = 15.71$$

The neutral axis  $X = 15.71 \text{ mm}$

The stress in the reinforcement at the crack section,

$$f_s = \frac{M}{(d - x/3)A_s} = \frac{3.684 \times 10^6}{(32 - 4.8)377} = 359 \text{ N/mm}^2$$

$$\epsilon_1 = \frac{y}{(d - x)} \cdot \frac{f_s}{E_s} = \frac{y}{17.7} \cdot \frac{359}{200 \times 10^3} = y101 \times 10^{-6}$$

$y$  is the distance from the neutral axis to the bottom of the slab =  $25.7 \text{ mm}$

$$\epsilon_1 = 25.7 \times 101 \times 10^{-6} = 2.58 \times 10^{-3}$$

$\epsilon_m$  = the average concrete strain,

$$\varepsilon_m = \varepsilon_1 - \frac{b_t(h-x)(a'-x)}{3E_sA_s(d-x)} = \left(3.31 - \left(\frac{1000(40-25.7)(40-25.7)}{3 \times 200 \times 377 \times 17.7}\right)\right) \times 10^{-3} = 2.529 \times 10^{-3}$$

$$w_{\max} = \frac{3a_{cr}\varepsilon_m}{1 + 2\left(\frac{a_{cr} - c_{\min}}{h-x}\right)} = \frac{3 \times 4.07 \times 2.529 \times 10^{-3}}{1 + 2\left(\frac{4.07 - 5}{14.3}\right)} = 0.035 \text{ mm}$$

The estimated maximum crack width at service = 0.035 mm.

#### Appendix 64: Analysis of variance table of crack width for Waffle and Solid slabs

| Source of variation | Degree of freedom | Sum of square | Mean square | Variance ratio F |
|---------------------|-------------------|---------------|-------------|------------------|
| Specimens           | 1                 | 0.770         | 0.770       | 15.46*           |
| Error               | 10                | 0.498         | 0.049       |                  |
| <i>Total</i>        | <i>11</i>         | <i>1.268</i>  |             |                  |

F\* > P<sub>0.05</sub> there is significance difference

P<sub>0.05</sub> = 4.96 (F tabulated)