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# Gravity Anomaly & Basis Symmetric Matrix Interpretation for Residuals

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***Abstract***– Gravity method has tremendously pervaded the field of geophysical exploration and thus extensively discussed and vividly illustrated in this work. The basis for symmetric matrix data gravity interpretation was earlier proposed, then extended to residuals and interpretation. A symmetric matrix representation is discussed further to residuals, spectral, interpretation, and optimistically forms a veritable tool for gravity data interpretation. The fundamental law of gravitational attraction is essential in delineating gravity data interpretation and the gravity effect or anomaly representation is evidently expressed by a finite difference matrix in the discussion.

A computational extension can be done based on the symmetric matrix representation with available gravity data and a vivid illustration for anomaly and residual computation has been made.

***Index Terms***– *Residuals, Symmetric Matrix, Gravity Data, Gravity Effect, Anomaly and Gravitational Attraction*

## I. INTRODUCTION

Gravity method has been a veritable and widely used tool in geophysical exploration for a span of several years over decades. The gravity method is a non-destructible versatile geophysical technique that measures differences in the earth's gravitational field at specific locations and employed for diverse geophysical and exploration activities.

In engineering and environmental sciences, it has gained huge prominence found numerous applications among locating voids and karst features, buried stream valleys, water table levels and the determination of soil layer thickness. The gravity method depends on variations produced in the measured gravitational field produced by different earth materials of different bulk densities i.e., mass (***Adewole O.O* et al**, 2011).

These variations can be interpreted by a variety of analytical and computer methods to determine the depth, geometry and density that causes the field variations.

The interpretation of Bourguer gravity anomalies ranges from just manually inspecting the grid or profiles for variations in the gravity field to more complex methods that involve separating the gravity anomaly due to an object of interest from some sort of regional gravity field. Several manual and computer techniques including graphical smoothing and polynomial surface fitting are applied in performing these techniques.

The gravity method can be a relatively easy technique to perform and interpreted. It requires only simple but precise data for processing. The technique has good depth penetration when compared to ground penetrating radar, high frequencies electromagnetic and D.C resistivity techniques and is not affected by the high conductivity values of near surface clay rich soils. Additionally, lateral boundaries of sub surface features can be easily obtained especially through the measurement of the derivatives of the gravitational field.

## II. THEORY

The gravity method involves variation in acceleration due to gravity and thus thye Newton's law of universal gravitation becomes extremely important. The acceleration due to gravity, g can be obtained from Newton's law of motion; where m is a test mass in equation (1)b, hence the Newton's law of universal gravitation is obtained where Me is the mass of the earth.

(1) a) – b):

$$g = \frac{GMm_e r'}{R_e{}^2} \qquad F = ma$$

The units for g are cm/s2 in c.g.s system and are commonly known as Gals, where the average acceleration due to gravity at the earth's surface is 980Gals. Variations in gravity from 10-1 to 10-3 are considered in most gravity studies and milliGal (mGal) is used by most workers. Even some detailed engineering and environmental work involve microGal (@Gal) variations.

## III. GRAVITY ANOMALIES AND RESIDUALS

### *A. Gravity Anomaly*

The difference between the observed gravity at a place and its theoretical value depending on its latitude is known as a gravity anomaly. It is also referred to spatially as the difference between the gravity fields observed at two points.

In geophysics, there are two kinds of gravity anomalies—free air and Bouguer anomalies—depending on type of correction applied to the observed gravity field at the point of observation.

### *B. Free Air and Bouguer Gravity Anomalies*

The observed gravity field after correction (corr) for latitude (theoretical gravity) and topography is known as the free air gravity anomaly, or simply the free air anomaly ($\Delta g_f$), which is given by:

(3) '$\Delta g_f$ = observed gravity – tidal corr – theoretical gravity + topographic corr + free air corr

If the attraction due to the effect of material between the plane of observation and the m.s.l. (known as the Bouguer correction ($b_C$)) is subtracted from the free air anomaly, the corrected gravity field is called the Bouguer gravity anomaly, or simply the Bouguer anomaly or gravity anomaly (Δg), and given by:

(4) $$\Delta g = \Delta g_f - b_C$$

The Bouguer anomaly is normally for a crustal density of 2.67g/cm3 . However, one can compute Bouguer anomaly for any other density, such as in the case of sedimentary basins where the Bouguer anomaly for a lower density corresponding to the sediments can be computed. The free air anomaly is found to be more suitable than the Bouguer anomaly in certain cases, such as in oceans, where the free air anomaly is mainly computed, as the plane of observation is same as the m.s.l.

It is also found to be useful in assessing isostatic compensation as discussed in the section on isostasy in this article. Free air and Bouguer anomalies computed from the observed gravity field can be plotted as a profile with reference to distance from a fixed reference station, or in a two- dimensional map if data is recorded in a two-dimensional plane.

The contour interval is chosen depending on the accuracy of the gravity data: the contour interval should be approximately 2–3 times the latter. For example, in regional surveys, as the general accuracy of the gravity data is 1–2 mGal, the contour interval is generally 5 mGal.

In oil exploration, the contour interval is generally 1 mGal, while in mineral exploration it is 0.1 mGal or 0.01 mGal depending on the nature of surveys. In microgravity surveys, it is generally 0.001 mGal or 1 μGal.

A typical gravity profile and anomaly map adapted: ***New Jersey*** (USGS) is shown in Figure 1.

## IV. SEPARATION OF GRAVITY ANOMALIES

The gravity field recorded at Earth's surface is the cumulative effect of the sources at different levels. Mostly, the gravity effect of sources from the surface downward, at least up to the Moho, is recorded in these surveys. It is therefore essential to separate the observed field originating from different levels. In general, they are separated broadly in two groups, namely the regional and the residual fields, originating

The Bouguer anomaly is related to the observed gravity $g_B$ as follows:

(5) $g_B = g_{obs} - \delta_\lambda + \delta_{gF} - \delta_{gB} + \delta_{gT}$

Here,

$g_B$ is the Bouguer anomaly

$g_{obs}$ is the observed gravity

$\delta_\lambda$ is the lattitude correction, as the Earth is not a perfect sphere

$g_{gB}$ is the Bouguer correction which allows for the gravitational attraction of rocks between the measurement point and sea level

$g_T$ is a terrain correction compensating deviations from an infinite plain flat horizontal surface,

A Bouguer reduction is called *simple* or *incomplete* if the terrain is approximated by an infinite flat plate called the *Bouguer plate*. A *refined* or *complete* Bouguer reduction removes or eliminates the effects of terrain precisely.

The difference between the two, the differential gravitational effect of the unevenness of the terrain, is called the *terrain effect*, which is It is always negative (Hoffmann W et al, 2006).

### *B. Regional and Residual Gravity Fields*

In general, as the name indicates, the regional field is characteristic of the region as a whole and originates from deep-seated sources, while the residual field is confined to a localized area and or

Since the gravity method is concerned with determining sub-surface variations in mass distributions, most interpretation techniques involve the solution of the previous equation (1) due to some mass distribution. This can be accomplished by solving for the gravity field due to a generalized mass distribution using an integral equation.

In most gravity applications, recent work on the application of the gravity gradient tensor to exploration problem (Mickus et al, 2002, Mikhailov et al, 2007) could involve all three components of the gravity field.

Delineating the contour for the integral:

(6) $\Delta g = \int_S K(x, x_0, z_0)\rho(x_0, z_0)dx_0dz_0$

Where $\Delta g(x)$ g(x) is the gravity effect of a two dimensional mass distribution bounded by the surface, $s, x_0$ and $z_0$ are the source coordinates, $K(x, x_0, z_0)$ is the kernel giving the gravity effect of an elementary block of unit density, and $\rho(x_0, z_0)$ is the actual density of the block.

In equation (6), it is tacitly assumed that the density is a continuous function of space co-ordinates which renders the evaluation of the integral (6) very difficult. Thus, to perform numerical computations, a discrete variation of density with position is assumed. The integral (6) is then reduced to finite difference matrix:

(7) $\Delta g = \sum_{i=1}^{m} \sum_{i=1}^{n} k_{ii}\rho_{ii}$

Which in matrix notation could be expressed viz:

(8) $|\Delta| = |k||\rho|$

Where the body consists of m rows and n columns. The value of m is fixed through successive approximation to give a close fit between the calculated and observed gravity values. The limit on n is set by the memory space of the computer.

**Figure & schematics**:

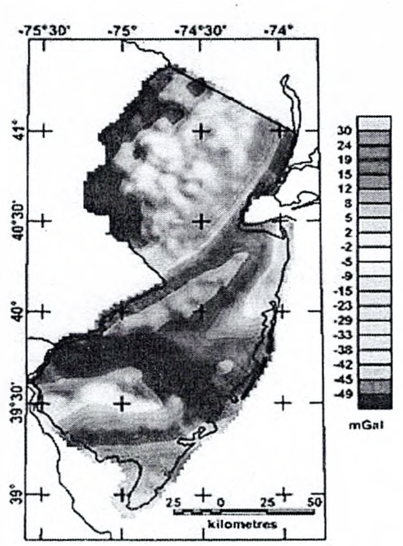


Figure 1: Bouguer anomaly map of the state of ***New Jersey*** (USGS)

## V. FURTHER DISCUSSION

Density is a very important parameter in gravity interpretation of data. The sub-surface variations of mass is very important and requires that the density of the material of interest or the density contrast between the material of interest and the surrounding material be known. The density can be determined by various techniques. The best technique is to acquire rock samples within the study area and determine their average density. The density logs obtained from drill logs can also be used but this is not always available.

Density can also be estimated from experimental relationships relating compression seismic velocities obtained from seismic refraction surveys and density (Nafe and Drake, 1957, Birch, 1961). Also, average density values comprises results from tables obtained from numerous measurements of rocks, soil and mineral samples (Jophnson and Olhoeft, 1984, Telford et al, 1990).

The regional-residual anomaly can be accomplished by many techniques. The simplest technique is the manual method such as the graphical smoothing where a simple smooth regional anomaly subtracted from the observed gravity anomaly to obtain a residual anomaly. The manual technique has the advantage of giving information on the lateral location of the source bodies which can be used select a correct regional anomaly.

Most other regional-residual anomaly separation techniques involve mathematical operations using a computer. The major limitation of the mathematical method is that they do not accurately represent the true residual gravity anomaly due to a specific body. Thus, they should not be used for quantitative interpretation of the sub-surface but only for qualitative interpretation (Ulyrch, 1968).

The most common mathematical techniques are surface fitting and weighted averaging. Surface fitting involves a least-square fitting of 2-D polynomial (Beltrac et al,19991, Kim et al, 2008) or 2-D Fourier series (James, 1968) of different orders to the original gridded Bouguer gravity data to represent a regional gravity anomaly map.

The higher the surface order, the greater the fit to the original data, however, high-order surfaces are usually not desired, as they contain part of the anomaly that is desired.

The kernel in equation (6) is the effect of a point due to an elementary two-dimensional rectangular block of unit density and is given by the formula (Heiland, 1946).

(9)

$$K_{ij} = 2y\left\{x'_{ij}\, ln\left(z^2_{ij} + {x^2}_{ij}/{z^2}_{ij} + x^2_{ij}\right)^{1/2} - (x_{ij} - WS)\, ln\left(z^2_{ij} + {(x_{ij} - WS)^2}/{z^2}_{ij-1} + (x_{ii} - WS)^2\right)^{1/2} + z_{ij}\left[tan^{-1}\, {x_{ii}}/{z_{ij}} - tan^{-1}\frac{(x_{ii} - WS)}{z_{ij-1}}\right] - z_{ij-1}\left[tan^{-1}(x_{ij} - WS)/{z_{ij-1}}\right]\right\}$$

Where K is the universal gravitation constant, $z_{ij}$ is the z coordinate of the far horizontal face, $x_{ij}$ is the x-co-ordinate of the far vertical face of j-th block reckoned from the observation point as origin, and WS is the width of the block.

### *B. Polynomial Approximation*

In this method, the regional and the residual fields are represented by low and high order surfaces, respectively. The observed gravity anomaly is approximated by a power series. The regional field gr along the x-axis can be represented by:

(10) $$g_r = a_0 + a_1x + a_2x^2 + \cdots \ldots \ldots \ldots . a_nx^n$$

where n is the order of the polynomial being used to approximate the regional field.

The coefficients are evaluated using the principles of least squares and the trends of different orders (n) are computed.

One of the low order trend (say 2, 3, ...) is selected as the regional field, and its difference from the observed field is the residual field. However, in some practical applications or interpretations, the selection of the order of polynomial (n) to represent the regional field is quite arbitrary, and depends considerably on the experience of the interpreter.

If the depth to the shallow sources (such as basement) is known in certain sections, from seismic profiles or borehole information, some constraints on the order of polynomial can be imposed that will provide the right magnitude of the residual field at these points.

For two-dimensional data, surfaces of different order are approximated in the (x, y) direction over the data grid can be represented for various degrees of the $n - th$ order polynomial as the regional field. Some literatures show almost coinciding smoothly varying field drawn based on visual inspection as observed and reported.

The regional field shows can show different variations of gravity behavior or residual anomalies revealed from seismic information or confined to an investigation region attributable to various sources in, some instances could be deep seated or shallow exposures as likely examples or illustrations.

### *C. Digital Filtering*

Digital filtering of geophysical data is based on the principles of Fourier transform and signal processing. The observed gravity field g(x) can be represented by discrete Fourier transform in frequency domain as:

(11)

$$g(x) = \sum_{f=-n/2}^{n/2} g(f)\exp(^{2\pi i.f.z}/_{\lambda})$$

where f is the frequency, n is the number of observations, z is the depth to the causative sources, λ is the wavelength, and g(f) is the transform of the gravity field given by:

(12)

$$g(f) = \frac{1}{n}\sum_{x=1}^{n} g(x)\exp(^{-2\pi i.f.x}/_{n})$$

g(f) is known as the amplitude spectrum corresponding to frequencies f, and the corresponding power or energy spectrum is

(13) $$g^2(f) = g(f)\, g^*(f)$$

where * denotes complex conjugate.

In practice, equally spaced digital data is generated from the recorded data by interpolation, or alternatively the Bouguer anomaly profile is digitized at equal intervals depending on stations' spacing and depth of investigation. However, the sampling interval should be such that it represents the highest frequency present in the data set, known as the Nyquist frequency, which is equal to $1/(2\Delta x)$, where $\Delta x$ is the sampling interval. The digital data is transformed into the frequency domain providing the amplitudes of various frequencies (wavelengths) present in the data set.

In the case of maps, the two-dimensional version of the discrete Fourier transform is used, and the computed amplitude spectrum can be averaged in concentric circles for similar frequencies to provide the variation of g(f) with f, known as the radial spectrum. In these cases, $f = (k^2 + m^2)^{1/2}$ where k and m are frequencies along the (x, y) axis.

'*

The plot of log of amplitude spectrum versus frequencies (Eqs. 11 and 12) '* provides straight-line segments corresponding to sources distributed at different levels.

A typical plot of amplitude spectrum has been shown in some extant literatures (Mishra, 1990), which shows three linear segments corresponding to sources distributed at three different levels.

The first segment corresponding to low frequencies represents deeper sources compared to other segments corresponding to higher frequencies. In practice, two to three layers can be effectively separated in a spectral plot. Depending on these frequency bands, low and high pass filters can be designed to separate the regional and the residual fields.

The selection of cut-off frequency for low and high pass filters is arbitrary and depends largely on the experience of the interpreter. It is, therefore advisable to design three or four alternatives of these filters, and through trial and error based on other available information and experience, decide the appropriate cut-off frequency/wavelength.

As is apparent from literature revelations (Mishra, 1990, Tiwari and Mishra, 1999), most of the energy is concentrated in the first segment, and therefore it is not possible to assign frequency bands for the regional and the residual fields based on spectral plots as suggested by several workers. It is advisable to assign the first few frequencies (say 1–2 frequencies) to the regional field, and the remaining field represents the residual field.

## VI. ILLUSTRATIONS

Having extensively delineated the salient concepts in this scope and field of an explicitly treated and elaborate concept based on a detailed qualitative and extensive treatment of gravity field, anomaly and derivations from the basis symmetric matrix and the contour integral or interpretation, we presented some very interesting and nice graphical illustrations.

**Gravity Anomaly Variations with Distance Based on Experimental Observation & Model**

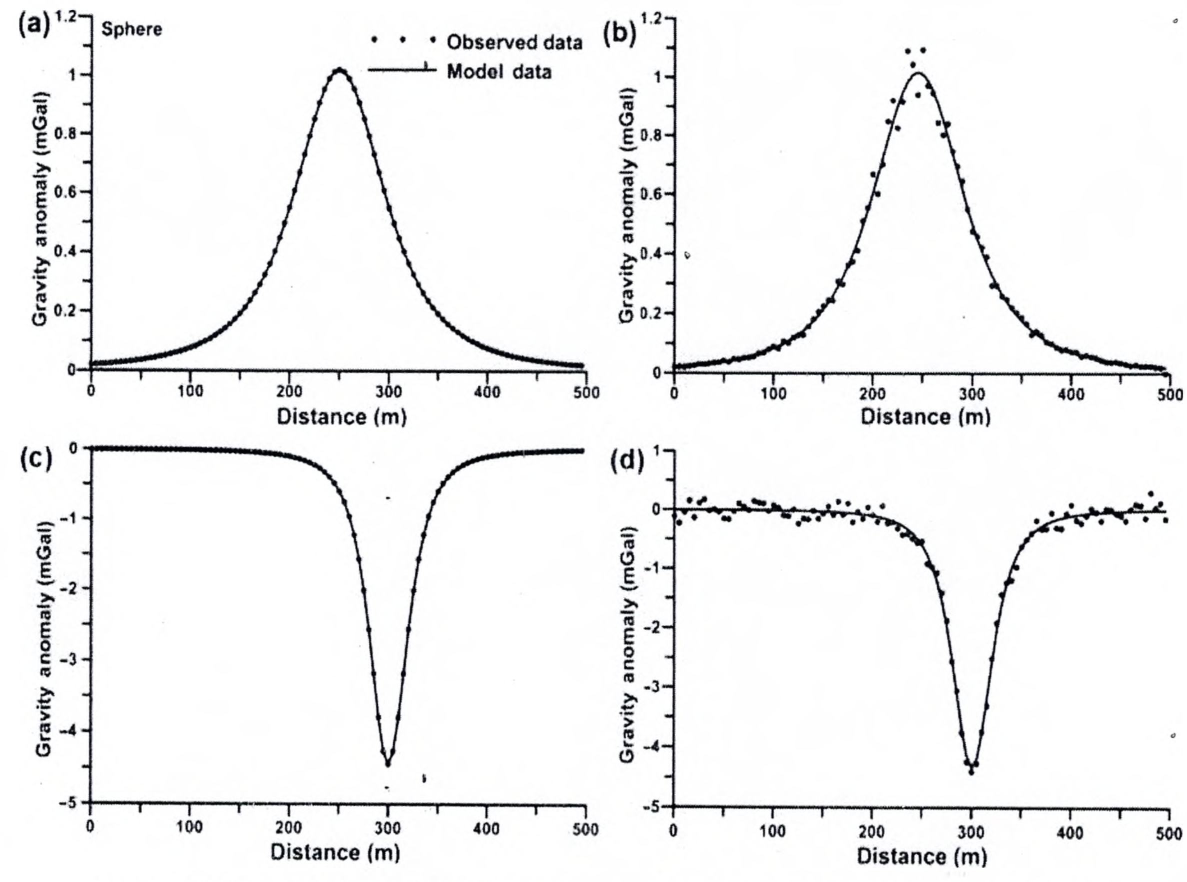


Fig 2., a-d): Schematics illustrations of the gravity anomaly variation with distance (computational fit)

The schematics in fig 2., (a – d) as displayed show the gravity anomaly variations with distance based on experimental observations and model fitted to data from experimentally observed data. The fits appear quite interestingly nice and represented almost a perfect fits of the experimental data.

## CONCLUSION

Interpretation of gravity data is very pertinent, crucial and found extremely important in geophysical exploration, prospecting and related applications, thus it is expedient to have a robust tool for gravity data interpretation.

The fundamental basis of symmetric matrix has been elaborated and hopefully would be a veritable analytical tool in gravity data interpretation. A numerical extension can be made and appropriate data can be thoroughly treated.

We can then complete the treatment following the polynomial approximation fit and our contour delineation with the computer based digital filtering.

Finally, having thoroughly delineated the salient concepts, we have presented some interesting and nice schematics or graphs to illustrate this extensive treatment and discussion so far.

### *Comments*

This research investigation has been on for a considerable length of time, investigation is still going on and more results would still be presented in the nearer future. Thank you all for your attentions.

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