FORCED CONVECTION ON ISOTHERMAL PLATES AND CHANNELS USING DIFFUSION VELOCITY

M. O. Petinrin[1], A. A. Dare[2],, S.A. Oke[3]

ABSTRACT

In many industrial applications, such as electronic systems, performance failure and breakdown usually occur due to poor thermal management, which could be adequately controlled through a proper understanding and management of the forced convection system and use of the vortex element method. The main contribution of this paper is that it shows how the vortex element method is capable of producing results similar to those reported in literature. The paper utilised vortex element method to model familiar problems in heat transfer, which is laminar flow over isothermal flat plate and isothermal two parallel-plate channels. Numerical models were developed using diffusion velocity method, a version of vortex element method, from vorticity transport equation and the energy equation for each of the cases. The velocity and temperature distributions, obtained for both plates and channels, were utilised to calculate Nusselt numbers with Reynolds numbers in the range of 20 to 120. The logarithmic plot of Nusselt number versus Reynolds number for forced convection on single horizontal plates yielded a slope of 0.46 and an intercept of -0.29 while that for forced convection in horizontal channels had a slope of 0.87 and an intercept of -0.88. The results obtained in this work show the diffusion velocity method to be a viable numerical tool for modelling fluid flow problems and also heat transfer problems.

Keywords: Diffusion velocity, forced convection, heat transfer, channels, plates.

1.0 NOMENCLATURE

u	velocity components in x direction
v	velocity components in y direction
T	fluid temperature
t	time
α	fluid thermal diffusivity
ν	kinematic viscosity
U	free-stream velocity
L	plate length
Δs	elemental length
Δt	time step
Re	Reynolds number
Γ	vortex strength or circulation
N	number of vortices on the plate surface and in space
u_s	slip velocity
d_{vj}	diffusion distance of each velocity vortex
d_{Tj}	diffusion distance of each temperature particle
v_e	velocity on the elemental surface
$\Delta\Gamma_q$	vortex strength
T_w	wall temperature
T_e	temperature on elemental surface
m, n	positions of the velocity along and normal to the plate
h	distance of the plates apart
Pr	Prandtl number
Nu	Nusselt number

[1]Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria
[2]Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria, Email: ademola_dare@yahoo.com
[3]Corresponding author, Department of Mechanical Engineering, University of Lagos, Lagos, Nigeria, E-mail: sa_oke@yahoo.com

2.0 INTRODUCTION

The Diffusion Velocity Method (DVM) is an existing mathematical model used to solve physical problems and was proposed by Ogami and Akamatsu (Ogami, 1999). In this method, diffusion velocity is expressed in such a way that vorticity is conserved as it would occur in the convection process. Unlike the other Vortex Element Methods (VEMs), this method handles the vorticity equation in a deterministic manner by calculating the diffusion velocity to account for diffusion in the flow. This suggests that simulating modelling results using this method, for the physical problem of forced convection on isothermal plates and on isothermal channels would be an important contribution to literature. The diffusion equation (Re = 0), boundary layer and two-dimensional flow around a circular cylinder (Re = 0.1 ~ 107), aerofoil, Burger equation and equations of incompressible fluid were successfully investigated (Ogami, 1999) using this method in order to prove the applicability of the method. Ogami (1999) also used the DVM to simulate diffusion of vorticity and temperature using the one-dimensional vorticity transport equation and compared the results obtained with the analytical solutions. Temperature and velocity distribution plots by Ogami (1991) showed close agreement with analytical results. The attempts of the foregoing authors were largely focused on the natural diffusion phenomena. Although the DVM is not a new methodology in the computational research field, it is however new in its application to forced convection on isothermal plates and channels.

Forced convection on isothermal plates and channels, which involves forced fluid flow over the surfaces of plates and channels, is a research area that has experienced a steadily growing number of investigations over the years. The main methods utilised to solve important problems in this area include the methods of; virtual flux, Rayleigh-Ritz, boundary element and finite element (Diaz, 1981; Tanno, 2006; Ramirez-Camacho and Barbosa, 2008). These methods are not vortex-based but alternatives, based on the Rayleigh-Ritz, virtual flux and boundary element methods. There is no evidence existing literature on the utilisation of the DVM, to solve the problem of forced convection on isothermal plates and channels. The recently documented investigations by Dare and Petinrin (2010), rather focused on natural convection. The next paragraph presents a review of the family of methods that the DVM evolves from, the Vortex Element Methods (VEMs).

The VEM, a numerical method in the field of computational sciences, has been widely used by different researchers as a modelling tool to study various flows due to its simplicity, grid-free attribute, lower computational overhead and better performance in the high Reynolds number *(Re)* regimes, than other numerical methods (Subramanian, 1996; Shirato and Matsumoto, 1997; Cottet and Koumosetsakos, 2000; Liu *et al*, 2005; Ogundare, 2006; Ogami and Fukumoto, 2010). Previously, when low speed computers were common and a good management of computer memory could not be compromised, VEMs' requirement for low computer memory gave them wide utility. A vast number of wind engineering and aerodynamic problems are solved using this method since flows in such problems do not follow the contour of the solid surface completely, making it difficult to investigate them using conventional numerical schemes, and which is readily overcome with VEM. Consequently, numerical methods such as VEMs are more appealing for flow regimes with only a small portion of the flow containing vorticity. Examples of applications of VEMs in their different versions include modelling of; air flows over cars, aeroplanes, winds blowing over bridges, and sea waves splashing against supporting columns of off-shore oil rigs.

A variety of computational tools such as finite element method, finite difference method and boundary element method have been used to solve complex, unsteady fluid flow and thermal engineering problems with the use of the DVM and other VEM methods. In the area of heat transfer, particularly lamina flow over isothermal flat plates and isothermal flow between two parallel plate channels, the demands of scientific research have created a need for simple methods of solving the complex problems encountered in this area, whose solutions would be useful to both researchers and practitioners in industry. The work presented here attempts to show how the DVM , which is a version of the VEM, is capable of producing similar results to the experimental results (Dare and Petinrin, 2010) that are to be found in literature, through application to the problem of forced convection heat transfer in fluid flow between isothermal plates and on isothermal channels . Dare and Petinrin (2010) employed the DVM to model natural convection along isothermal plates and channels. No attempt has been made so far to study forced convection on isothermal plates and channels using this method, which therefore forms the subject of the present work. Although the flow geometry of the present work and that of Dare and Petinrin (2010) are similar in that both focus on the convection processes, the two works differ in respect of applications; while the present work focuses on forced convection Dare and Petinrin (2010) considered natural convection. This paper focuses on the interaction of vortices in between horizontal plates based on forced convection while the work of Dare and Petinrin (2010) considers the interaction of vortices in between vertical plates, which has foundation in natural convection. Since the DVM is grid free, it has a high potential for computer memory saving.

3.0 LITERATURE REVIEW

Uchiyama and Naruse (2001) presented a numerical method for gas-solid two-phase free turbulent flow using the Lagrangian VEM with particles in simultaneous motion. In this method the change in the vorticity for the gas-phase is evaluated by an area weighing method, while the change of the viscous effect is simulated through a core spreading method. In a complimentary article, Uchiyama and Yagami (2001) developed a simulation approach for two-dimensional wake gas flows behind a flat plate, loaded with solid particles of Stokes numbers 0.15 and 1.4, using the Lagrangian VEM The solution of forced convection on isothermal plates and channels using diffusion velocity was not addressed anywhere in any of these two articles. Lakkis and Ghoniem (2003) presented a grid-free, Lagrangian method for the accurate simulation of low-Mach number, variable-density, diffusion-controlled reacting flow, by assuming a fast-chemistry model. In this method the conversion rate of reactants to products was limited by the local mixing rate in order to reduce the combustion problem to the solution of a convection-diffusion-generation equation with volumetric equation and vorticity generation at the reaction front. Cottet and Koumosetsakos (2000) presented a simple and efficient technique that uses a vortex method to predict the quantities of combustion products. Application of the DVM to the problem of forced convection on isothermal plates and channels was not tackled in either of these studies as well.

Shirato and Matsumoto (1997) described the evaluation of unsteady pressure on an oscillating bluff body using Bernoulli's formula for unsteady flow by simulating flow around a body using a vortex method. Bin et al. (2009) proposed a Lagragian-Lagragian model to study gas-solid two-phase flow across a single cylinder and two tandem cylinders at high Reynold numbers by simulating the single-phase flow using a discrete vortex method and tracking the particle trajectories by the particle motion equation. None of these two separate papers by Bin *et al*. as well as Shirato and Matsumato addressed the problem of forced convection on isothermal plates and channels, particularly with reference to the DVM. Application of the VEM based on the Biot-Savart law has also been extended to numerical prediction of unsteady and complex characteristics of various flows in difficult engineering problems related to; flow-induced vibrations, off-design operation of fluid machinery, automobile aerodynamics, and biological fluid dynamics (Kamemoto, 2004). Barber and Fonty (2006) investigated the implementation of an alternative diffusion method that employed an exact analytical formulation which allows discrete vortices to diffuse smoothly in space and time. Gallati and Braschi (2002) carried out simulations of the flow around a circular cylinder in a uniform stream, obtained by the Lagrangian random VEM. They compared the simulations results obtained with the experimental results obtained from a hydrodynamic tunnel and found theirs to be closer to the experimental values. Huang et al. (2009) developed a fast multiple approach VEM to speed up the computation of velocities in order to simulate two-dimensional viscous incompressible flow based on a scheme of blob splitting and merging. The authors claimed good agreement of their results with those of previous experimental and numerical investigations, as evidenced by the data presented in their paper. Their summation is correct and their data useful for comparison with data generated using other methods. However, none of the foregoing studies dealt with the problem of forced convection on isothermal plates and channels using the DVM.

Khatir (2004) developed a three-dimensional exterior Neumann problem in terms of a source/sink boundary integral equation. The numerical scheme embedding this strategy was described well in the work and its accuracy and efficiency clearly outlined. The results arising from the work however, did not provide useful information for the problem of forced convection in isothermal plates and channels. Yokota et al. (2007) applied a decaying homogeneous isotropic turbulence VEM to the calculation of Reynolds number, and compared the results with those obtained from calculations based on the spectral method; the results indicated that the decaying homogeneous isotropic turbulence VEM requires more computation elements compared to spectral methods, although the two methods agreed on the dissipation wave number. He and Su (1998) presented some modifications of the viscous element method to better present the near-wall vorticity when obtaining numerical solutions for the Navier-Stokes equations. It was shown that the approach was more accurate in modelling the flow features and force coefficients without making different adhoc assumptions for different geometries. The method however, did not provide insight to the problem of forced convection in isothermal plates and channels. None of the foregoing studies however, tackled the application of the DVM to the problem of forced convection on isothermal plates and channels.

Smith and Stansby (1989) proposed an efficient algorithm for particle simulation of vorticity and heat transport. Further, Kamemoto and Miyasaka (2000) developed a vortex and heat element method for unsteady heat transfer around a circular cylinder in a uniform flow. Golia and Bernardo (2005) presented a vortex-thermal-blobs method for 3D-bouyancy driven flows. As in the other cases discussed in this section, none of these studies addressed the problem of forced convection on isothermal plates and channels, with particular reference to the DVM.

4.0 MATERIALS AND METHODS

4.1 Theoretical considerations

This section shows how the relevant theory for (a) velocity and temperature vortices and (b) forced convection on isothermal plates and channels was developed. However, it is first necessary to state the assumptions made in formulating the model here. The first important assumption relates to the energy equation, which is guided by the principle of conservation of mass.

The assumption states that no viscous dissipation and thermal generation exist. These equations are not quoted here since they are commonly available in the literature, such as the paper by Dare and Petinrin (2010). The second assumption is that the boundaries of the solids are non-deformable. The third assumption is that the fluid properties are invariant.

The velocity and temperature vortices with strengths Γ and Θ, respectively, on an elemental surface of a plate that is divided into m elements, are given by the expressions:

$$\Delta\Gamma_q = U\Delta s \tag{1}$$

and

and $$\Theta_q = Tu_{Tq}\Delta t = -\alpha\Delta t \left.\frac{\partial T}{\partial y}\right|_{y=\delta} \tag{2}$$

The term u_{Tq} in Equation 2 stands for thermal diffusion velocity. The symbols *U*, *m*, *δΔt* in the above two equations are defined by the expressions; $U = \nu \mathrm{Re}/L$, $m = L/\Delta s$, $\delta = \sqrt{\alpha\Delta t}$ and $\Delta t = 16L\ln 2/U\,\mathrm{Re}$, where *q* = 1, 2, 3,...*m*; *U* is the free-stream velocity; *L* is the plate length; Δs is the elemental length; Δt is the time step and *Re* is the Reynolds number. The vortices are initially distributed and separated at distances of Δ*s* before diffusing. The vorticity of each velocity vortex and the temperature of each temperature particle are respectively given by the expressions:

$$w(r,\Delta t) = \frac{\Gamma_i}{4\pi\nu\Delta t}\left[e^{\left(-r_1^2/4\nu\Delta t\right)} - e^{\left(-r_2^2/4\nu\Delta t\right)}\right] \tag{3}$$

and

$$T(r,\Delta t) = \frac{\Theta_i}{4\pi\alpha\Delta t}\left[e^{\left(-r_1^2/4\alpha\Delta t\right)} - e^{\left(-r_2^2/4\alpha\Delta t\right)}\right] \tag{4}$$

For flow on a flat plate, each velocity vortex or temperature particle has a corresponding negative image. The distances between a vortex, and the inducing vortex and its corresponding image can be deduced respectively by the expressions:

$$r_1(x_j, y_j) = \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \tag{5}$$

and

$$r_2(x_j, y_j) = \sqrt{(x_j - x_i)^2 + (y_j + y_i)^2} \tag{6}$$

where the symbols, *i* and *j* vary between the numbers 1, 2, 3,...*N*, and *N* is the number of vortices on the plate surface and in space. It is important to note here that most times *i* is not equal to *j*. Figure 1 shows how r_1 and r_2 are determined from a vortex of interest and an inducing vortex with its image.

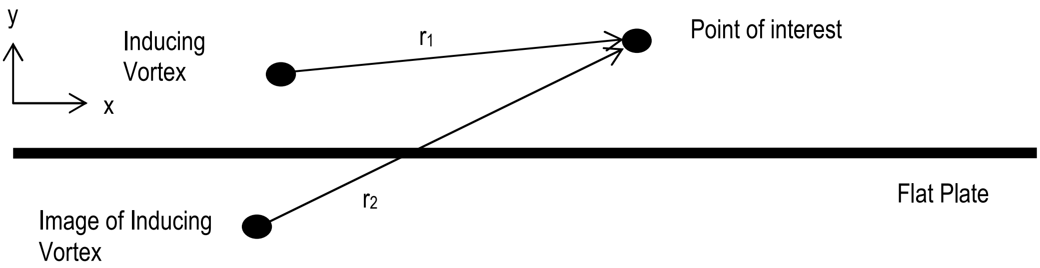

Figure 1 Interacting Vortices on a Horizontal Flat Plate

The image vortices have a negative strength to ensure no flow across the wall. For successive vortices the velocity vortex strength is given by the expression:

$$\Delta\Gamma_q = u_s \Delta s \quad (5)$$

In this equation, the slip velocity (u_s) is the sum of the free stream velocity (U) and the horizontal components of the inducing vortices and their images. Successive temperature particles are still created with Equation 2, as the trends are not the same in the creation of temperature vortices and velocity vortices.

The divergences of Equations 3 and 4 are, respectively, $\nabla.w$ and $\nabla.T$, and are written in the form:

$$\nabla.w = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} \quad (6)$$

and

$$\nabla.T = \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} \quad (7)$$

As described by Ogami (1999), the diffusion velocities of each velocity vortex and temperature particle are now, respectively, given by the expressions:-

$$u_{wj} = \sum_{i=1}^{N} [-\frac{\nu}{w} \nabla.w] \quad (8)$$

and

$$u_{Tj} = \sum_{i=1}^{N} [-\frac{\alpha}{T} \nabla.T] \quad (9)$$

The diffusion distances of each velocity vortex and temperature particle can be expressed, respectively, as:

$$d_{vj} = u_{wj} \times \Delta t \quad (10)$$

and

$$d_{Tj} = u_{Tj} \times \Delta t \quad (11)$$

These distances are added to the original positions to move the vortex and particle to other positions. For convection, each vortex is repositioned to a new location using the induced velocities (u,v) by the neighbouring velocity vortices. The slip flow condition is met when the velocity on each elemental surface is approximately zero and the numerically calculated temperature is approximately equal to the initial temperature of the plate. To check for the slip flow condition, the velocity on the elemental surface is determined from the expression:

$$v_e = U - \sum_{i=1}^{N} \left[\frac{\Gamma_i}{2\pi r_1} - \frac{\Gamma_i}{2\pi r_2} \right] \quad (12)$$

The corresponding temperature on the surface is determined from the expression:

$$T_e = \sum_{i=1}^{N} \frac{\Theta_i}{4\pi\alpha\Delta t} \left(e^{(-r_1^2/4\alpha\Delta t)} - e^{(-r_2^2/4\alpha\Delta t)} \right) + T_B \quad (13)$$

The practical substitution for the given boundary condition is given by the expression:

$$T_B = T_w erfc(\frac{y_j}{2\sqrt{\alpha\Delta t}}) \tag{14}$$

Equation 14 reflects the effect of the wall temperature (T_w) i.e., the boundary condition on the temperature domain. The parameters, v_e and T_e, are substituted for U and T in Equations 1 and 2 at each time step until the slip flow condition is met.

When a vortex is very close to the wall or an inducing vortex occurs, i.e., when $r < \Delta s/\pi$ (r can be equal to r_1 or r_2) then $\Gamma_i r/2\pi r_0^2$ replaces $\Gamma_i/2\pi r$ where it is appropriate. In such circumstances, the minimal distance, r_0, is equal to, $\Delta s/\pi$ (Lewis, 1991). Likewise the temperature particles, $\Theta_i r/2\pi r_0^2$, replace $\Theta_i/2\pi r$.

The velocity and temperature distributions at specific locations in the hydrodynamic and thermal boundary layers are determined when the slip flow condition is met for the velocities and temperatures on the plate surfaces. The two components of the velocity distribution are obtained from the following two expressions:

$$v_{mnx}(r) = U - \sum_{i=1}^{N}\left[\frac{\Gamma_i}{2\pi r_1}\left(\frac{y_j - y_i}{r_1}\right) - \frac{\Gamma_i}{2\pi r_2}\left(\frac{y_j + y_i}{r_2}\right)\right] \tag{15}$$

and

$$v_{mny}(r) = U - \sum_{i=1}^{N}\left[\frac{\Gamma_i}{2\pi r_2}\left(\frac{x_j - x_i}{r_2}\right) - \frac{\Gamma_i}{2\pi r_1}\left(\frac{x_j - x_i}{r_1}\right)\right] \tag{16}$$

The temperature at any point can be obtained from the expression:

$$T_{mn}(r) = \sum_{i=1}^{N}\left[\frac{\Theta_i}{2\pi r_1} - \frac{\Theta_i}{2\pi r_2}\right] + T_B \tag{17}$$

In the foregoing three equations, the symbols m and n designate the positions of velocities or temperatures along and normal to the plate, respectively.

The type of channels described here are two parallel plates placed horizontally. The number of images of a vortex or a particle is infinite, which for ease of computation is reduced to eight. Therefore, there exist in the model developed here, one positive vortex with four positive images and four negative images (Ogundare, 2006). As discussed by Petinrin (2008), the vorticity of each velocity vortex and the temperature of each temperature particle can then be, respectively, given as:

$$\begin{aligned} w(r,\Delta t) = \frac{\Gamma_i}{4\pi\nu\Delta t}\Big[& e^{(-r_1^2/4\nu\Delta t)} - e^{(-r_2^2/4\nu\Delta t)} + e^{(-r_3^2/4\nu\Delta t)} + e^{(-r_4^2/4\nu\Delta t)} + e^{(-r_5^2/4\nu\Delta t)} \\ & + e^{(-r_6^2/4\nu\Delta t)} - e^{(-r_7^2/4\nu\Delta t)} - e^{(-r_8^2/4\nu\Delta t)} - e^{(-r_9^2/4\nu\Delta t)}\Big] \end{aligned} \tag{18}$$

and

$$\begin{aligned} T(r,\Delta t) = \frac{\Theta_i}{4\pi\alpha\Delta t}\Big[& e^{(-r_1^2/4\alpha\Delta t)} - e^{(-r_2^2/4\alpha\Delta t)} + e^{(-r_3^2/4\alpha\Delta t)} + e^{(-r_4^2/4\alpha\Delta t)} + e^{(-r_5^2/4\alpha\Delta t)} \\ & + e^{(-r_6^2/4\alpha\Delta t)} - e^{(-r_7^2/4\alpha\Delta t)} - e^{(-r_8^2/4\alpha\Delta t)} - e^{(-r_9^2/4\alpha\Delta t)}\Big] \end{aligned} \tag{19}$$

In addition, the distances of each vortex located at the points, (x_j y_j) from the inducing vortex located at the points, (x_i y_i) and its images can be determined from the distances, r_1 to r_9, and the distance, h, which is the distance between the plates, thus:

$$\begin{aligned}
r_1(x_j, y_j) &= \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2}\ , \\
r_2(x_j, y_j) &= \sqrt{(x_j - x_i)^2 + (y_j + y_i)^2}\ , \\
r_3(x_j, y_j) &= \sqrt{(x_j - x_i)^2 + (y_j - (y_i + 2h))^2}\ , \\
r_4(x_j, y_j) &= \sqrt{(x_j - x_i)^2 + (y_j - (y_i + 4h))^2}\ , \\
r_5(x_j, y_j) &= \sqrt{(x_j - x_i)^2 + (y_j - y_i + 2h)^2}\ , \\
r_6(x_j, y_j) &= \sqrt{(x_j - x_i)^2 + (y_j - y_i + 4h)^2}\ , \\
r_7(x_j, y_j) &= \sqrt{(x_j - x_i)^2 + (2h - y_j - y_i)^2}\ , \\
r_8(x_j, y_j) &= \sqrt{(x_j - x_i)^2 + (4h - y_j - y_i)^2} \text{ and} \\
r_9(x_j, y_j) &= \sqrt{(x_j - x_i)^2 + (2h + y_j + y_i)^2}
\end{aligned} \tag{20}$$

The distance, h, is also shown in Figure 2 to be the transverse distance between vortex points. This is intended since the walls will also be reflected as shown with imaginary lines in this figure. As such, the vortices, even at the image plane, will be separated by the distance h.

A representation of the foregoing is shown in Figure 2

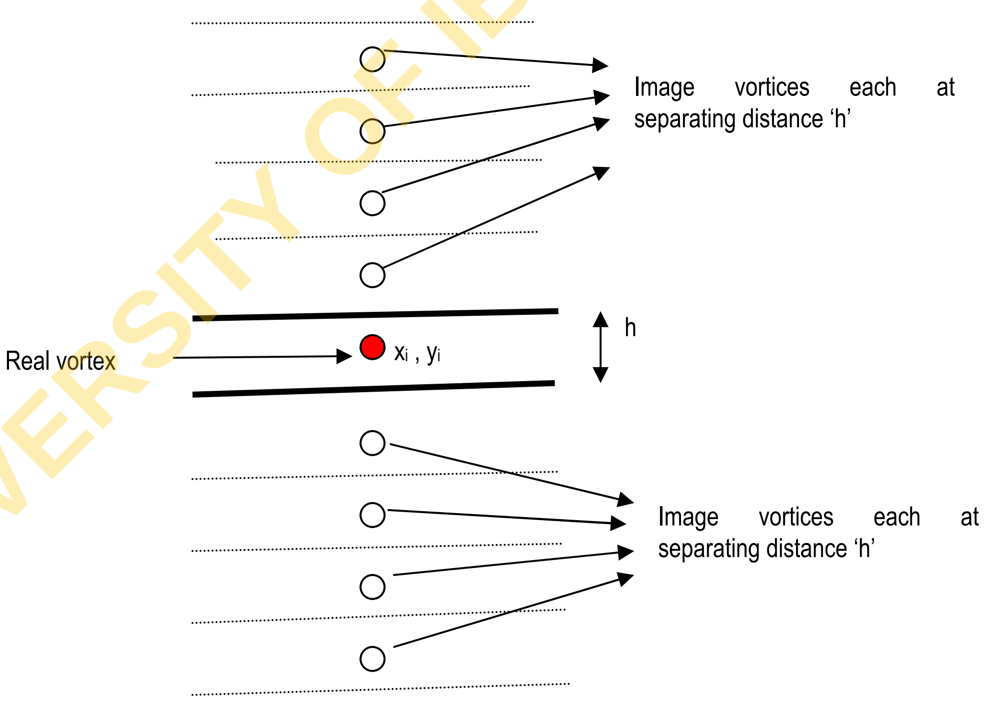

Figure 2 Representation of Vortices and Images in a Channel

4.2 Development of the Computer Simulation Model

4.2.1 Procedure and Analysis

The procedure used to develop the model used herein, and the methods used to analyse the data obtained in this work are described in this sub-section. Part of the data acquired and used in this work included values of the Nusselt number, details of the velocity and temperature distributions for the various flow conditions studied, while values of free stream velocity were specified from the relationship:

$$U = \frac{\nu \,\mathrm{Re}}{L} \tag{21}$$

The channels, consisting of two parallel horizontal plates were held at uniform wall temperatures. The simulation of forced convention was carried out first for the case when the two plates were held at the same temperature, thereafter for the case when the plates were kept at different temperatures. For the asymmetrical heating, one of the plates was kept at 40^{0}C while the other one was kept at 60^{0}C. The space between the plates was set to 0.1m, and a plate length of 0.5m maintained. This same procedure was also adopted for the simulation of forced convection over a single plate. For all cases, simulation was carried out for Reynolds numbers varying from 20 to 120 in steps of 20.

The input parameters, including the fluid's (air) thermophysical properties, which were recorded at the film temperature, that were used to simulate the forced convection on a horizontal plate are listed in Table 1. The input parameters for symmetrically heated, isothermal plates listed in Table 2 were used to simulate forced convection in channels.

Table 1 The Input Parameters for Forced Convection on a Single Plate [Lewis (1991); Petinrin (2008)]

Length of the plate	1 m
Reynolds number, Re	20-120
Fluid (air) temperature	10^{0}C
Plate wall temperature	70^{0}C
Kinematic viscosity of air at 35^{0}C	0.0000171 m^2/s
Thermal diffusivity of air at 35^{0}C	0.0000241 m^2/s

All the data contained in this table was occurs in both of the two references quoted here.

Table 2 The Input Parameters for Forced Convection in a Channel (Petinrin, 2008)

Length of the plates	0.5 m
Gap between plates	0.1m
Reynolds number	20-120
Fluid (air) temperature	10^{0}C
Lower plate wall temperature	60^{0}C
Upper plate wall temperature	60^{0}C
Kinematic viscosity of air at 350C	0.0000171 m^2/s
Thermal diffusivity of air at 350C	0.0000241 m^2/s

Logical flowcharts for implementation by computer of the scheme of the theory described herein are shown in Figures 3 and 4. Figure 3 consists of four primary steps representing the creation of temperature and velocity vortices, subjection of these to the diffusion equation, imposition of convection on both streams of particles and finally estimation of temperature and velocity on the conditions of no slip of temperature and velocity. Figure 4 is an expanded flow chart of the process illustrated in Figure 3, giving more details.

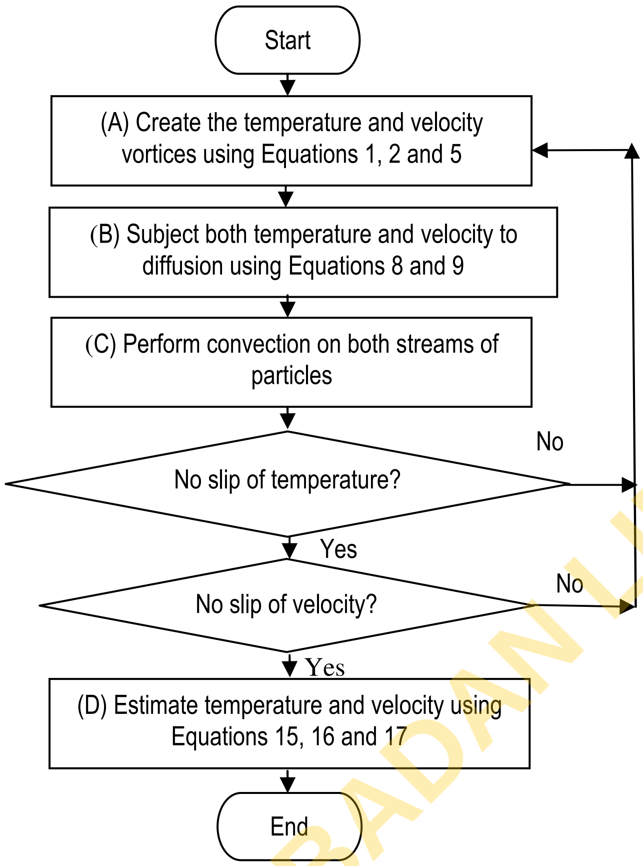

Figure 3 Flow Chart for Implementation of the Diffusion Velocity Technique in Forced Convection

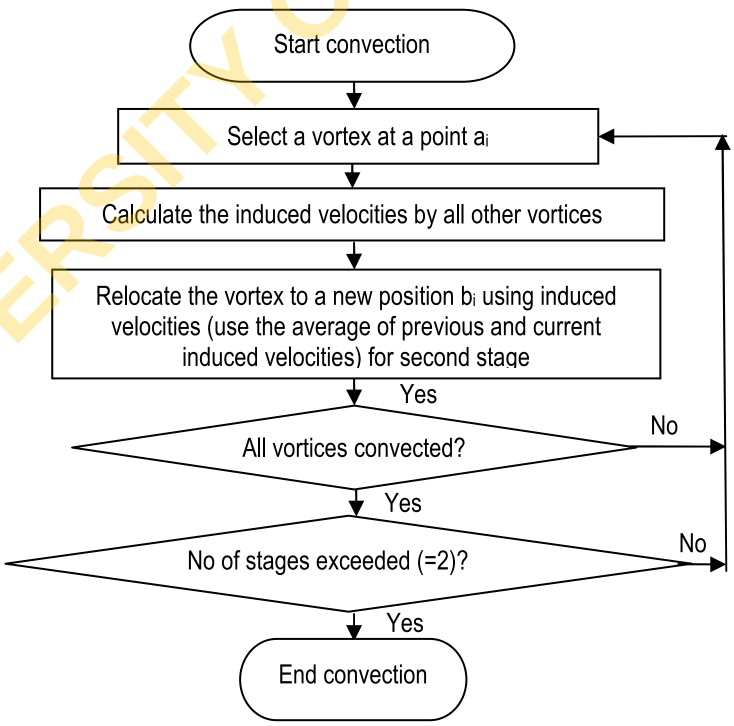

Figure 4 Implementation Scheme for the Convective Process

Implementation of the DVM was carried out for forced convection on a horizontal flat plate and on the inside of a horizontal flat channel, such as the ones shown in Figures 5 and 6, respectively.

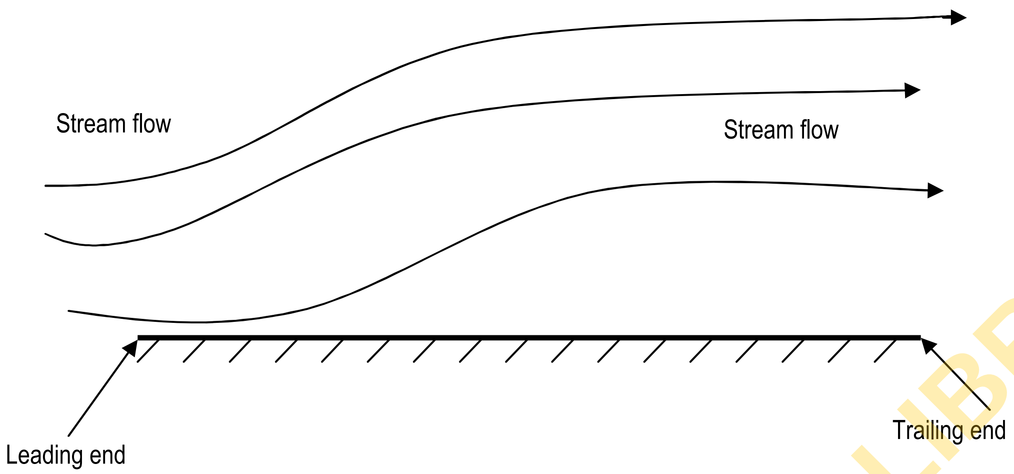

Figure 5 Forced Convection Over a Horizontal Flat Plate (Geometry 1: Plate)

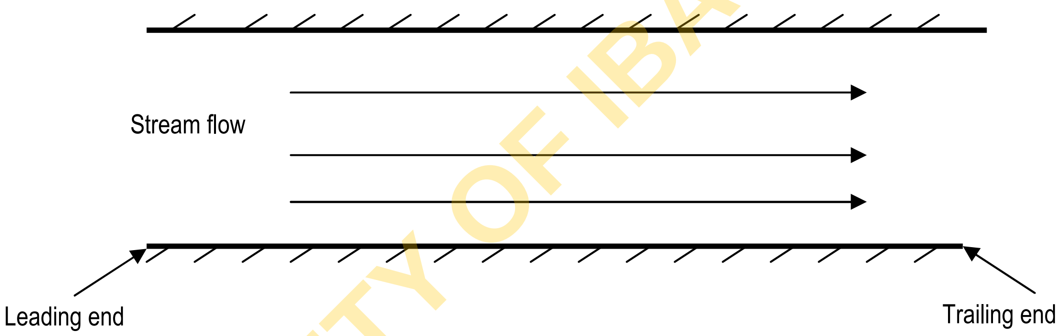

Figure 6 Forced Convection over a Horizontal Flat Plate (Geometry 2: Channel)

4.2.2 Model Performance Evaluation

Correlation was used as a performance measure, through comparison of the output of the simulation undertaken herein with results sourced from existing literature. The model utilized consisted of multiple parameters, with each of the parameters taking on new values at every stage of assessment. Comparison of the results arising from the application of the model on the basis of the values of individual parameters of the model with results obtained from literature was found challenging. The aggregation of parameters with the resulting overall values obtain was used as an alternative basis for comparison. Several runs of each simulation were made before their results were used for correlation analysis. The purpose of the several runs is as follows. Nusselt number depends on Reynolds number and Prandtl number. Since however the model's equation relating *Nu* to *Re* and Pr are not the same with the cited literature, a correlation factor R2 between Nu and Re was determined and used in making comparison with values obtained from literature. The literature on correlation as a performance criterion (Miller and Freud, 1987) is well established and has in the past permitted correlation coefficients values of 0.5 and above as acceptable for concluding on a strong relationship between a set of parameters. Otherwise, values less than 0.5 suggest a poor relationship between entities.

5.0 RESULTS AND DISCUSSION

A Visual Basic 6.0 programming language was used to simulate the vortex numerical model that was developed here. The results of the simulation were automatically displayed on a Microsoft Excel Workbook.

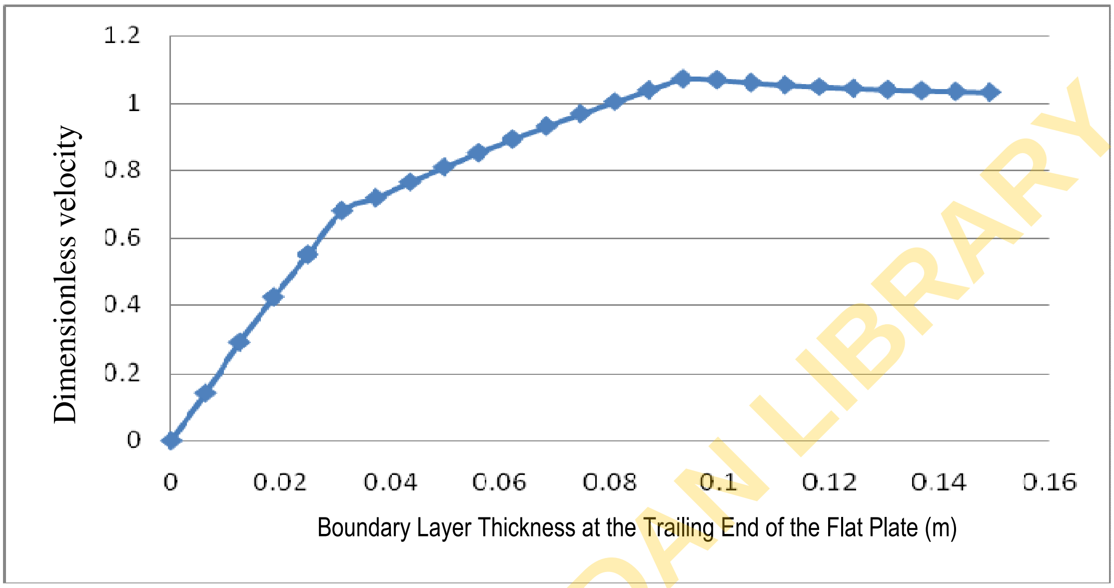

Figure 7 Velocity Distribution at the Trailing End of a Plate at Re = 80

Figure 7 is a representative plot of velocity distribution at the trailing end of the plate. The value of free stream velocity was recorded at a distance of 0.125m normal to the plate at the plate's trailing edge as 0.001376 m/s It can be seen from this figure that the velocity increased with increasing distance from the plate wall, within the boundary layer thickness from a value of zero (which represents the no slip condition at the wall) to a value close to the free-stream velocity in view of the asymptotic nature of the velocity profile. The steep gradient of the velocity curve near the plate wall is a sign of a more pronounced fluid friction there.

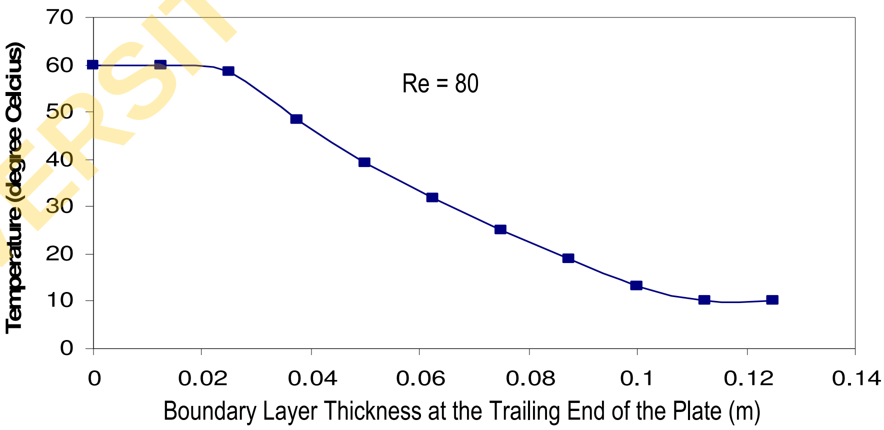

Figure 8 Temperature Distribution at the Trailing End of the Flat Plate

A plot of temperature distribution at *Re* = 80 at the trailing end of the plate is presented in Figure 8. The figure shows that temperature decreases asymptotically from a value equal to 60°C, which is the wall temperature, to a value equal to the free-stream temperature of 10°C, with increasing distance away from the wall, through the boundary layer thickness. However, there is a slow decay in temperature within the boundary layer region of 0-0.03 m, which is the region close to the plate. The curve exhibits a zero gradient for values of distance between 0.1 – 0.12m.

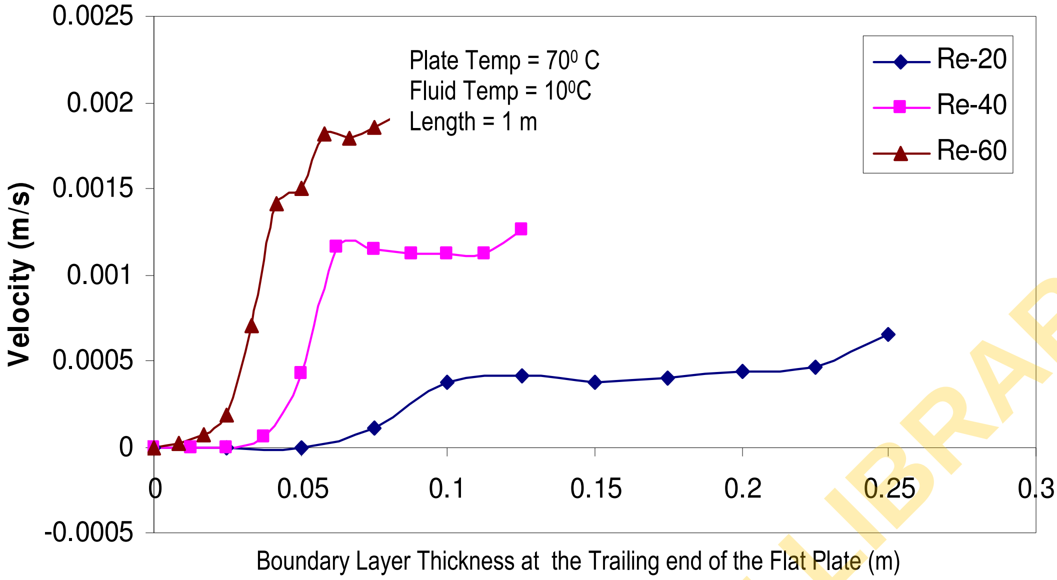

Figure 9 Velocity Distributions for Different Reynolds Numbers

The curves of boundary layer velocity distribution for *Re* values of 20, 40 and 60, and for a plate length of 0.5m are shown in Figure 9. It can be deduced from the curves that the boundary layer thickness is inversely proportional to the Reynolds number, which from Equation 21 is known to be directly proportional to the fluid velocity. Similar observations were also reported by Ogundare (2006) using the random vortex technique. The plot at the trailing edge was considered more representative since the overall heat transfer irrespective of the length can be estimated using the temperature and velocity values at the trailing edge. The free stream velocities for Re values of 20, 40 and 60 are seen from this figure to be 0.00052, 0.0012 and 0.002 m/s, respectively. It is also observed in these figures that as *Re* increases as the boundary layer thickness decreases in both cases of velocity (Figure 9) and temperature (Figure 10).

All along the boundary layer, excepting below a value of about 0.01m from the plate wall, the fluid velocity is seen in Figure 9 to increase non-linearly with increasing values of Re. This from Equation 21, points to a non-linearly increasing value of fluid coefficient of friction. However, it is necessary that further testing be conducted with more data collected for different values of *Re* and different velocities in order to support this inference.

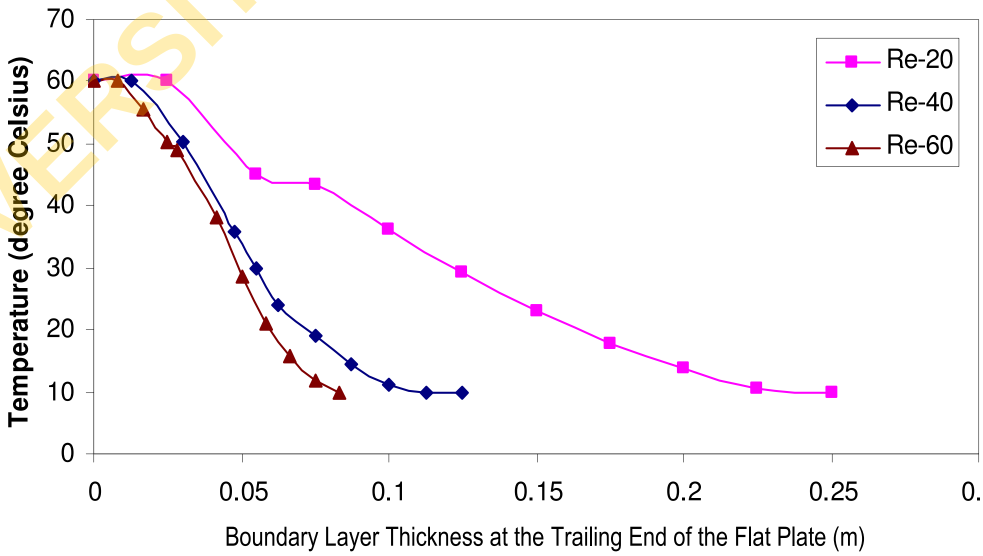

Figure 10 Temperature Distribution for Varying Reynolds Number

The curves of boundary layer temperature distribution for Re values of 20, 40 and 60, and for a plate length of 0.5m are shown in Figure 10. It can be deduced from the curves that the boundary layer thickness is inversely proportional to the Reynolds number. Similar observations were also reported by Ogundare (2006) using the random vortex technique. It is also observed in these figures that as *Re* increases the boundary layer thickness decreases in both cases of velocity (Figure 9) and temperature (Figure 10).

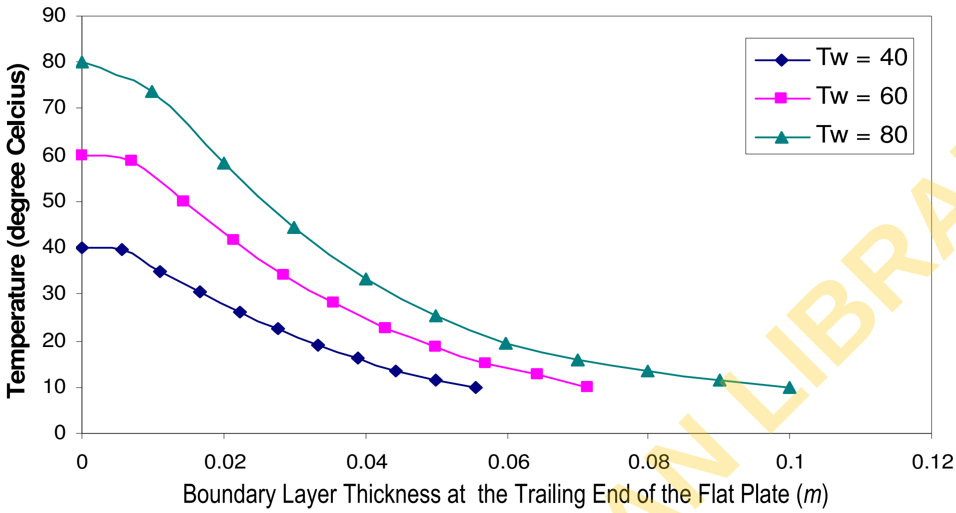

Figure 11 Effect of Wall Temperature for Forced Convection on a Single Plate

Figure 11 shows plots of temperature distribution for a free-stream temperature, T_f, of 10⁰C and for wall temperatures, T_w, equal to 40, 60 and 80⁰C. The plate length was 1m, and the *Re* was set to 150. The plots show that an increase in the value of the wall temperature at a constant fluid temperature, leads to an increase in the thickness of the thermal boundary layer. As was noted in the case of the velocity boundary layer thickness, this trend is consistent with the work of Ogundare (2006), who established similar trends using a random vortex numerical technique for heat and fluid flow over a flat plate maintained at a constant temperature. Worth noting at this stage is the fact that at the wall, the effect of friction is well pronounced and non linear, and further that as the flow moves away from the wall, the free stream condition is approached. Moreover, it is known that the thickness of thermal boundary layer increases with increasing temperature.

Values of the Nusselt number, *Nu*, for corresponding values of Reynolds numbers ranging from 20 to 120 in steps of 20 are presented in Table 3. A *log-log* plot of the Nusselt number against the Reynolds number, for values of *Re* ranging from 20 to 120 in steps of 20, gave rise to the following relationship:

$$Nu = 0.4592Re - 0.2905 \quad (22)$$

with a correlation coefficient (R) defined as $R^2 = 0.9904$.

Table 3 Variations of Reynolds Number with Nusselt Number for Forced Convection on a Single Plate (Petinrin, 2008)

Re	*Nu*	*Log(Re)*	*Log(Nu)*
20	2.16	1.30	0.33
40	2.80	1.60	0.44
60	3.30	1.79	0.52
80	3.87	1.90	0.59
100	4.25	2.00	0.63
120	4.32	2.08	0.64

The second term at the right hand side of Equation 22 can be expressed as:

$$-0.2905 = 0.33x\ ln(Pr) \tag{23}$$

At 35⁰C, which is the average of the free stream temperature and wall temperature, the Prandtl number, *Pr* is 0.71. This gives a value of *x* that is equal to 0.57 and a relationship between the Nusselt number, Reynolds number and Prandtl number that is given by the expression, $Nu = 0.57Re^{0.}46Pr0.33$ (Cottet and Koumosetsakos, 2000). This expression generates a value that is more representative of the fluid condition which varies in temperature from 10^0C at the wall to 70^0C at free stream, and therefore its use here. The relationship obtained by Incropera and Dewitt (2005) is given as $Nu = 0.332Re^{0.5}Pr^{0.33}$.

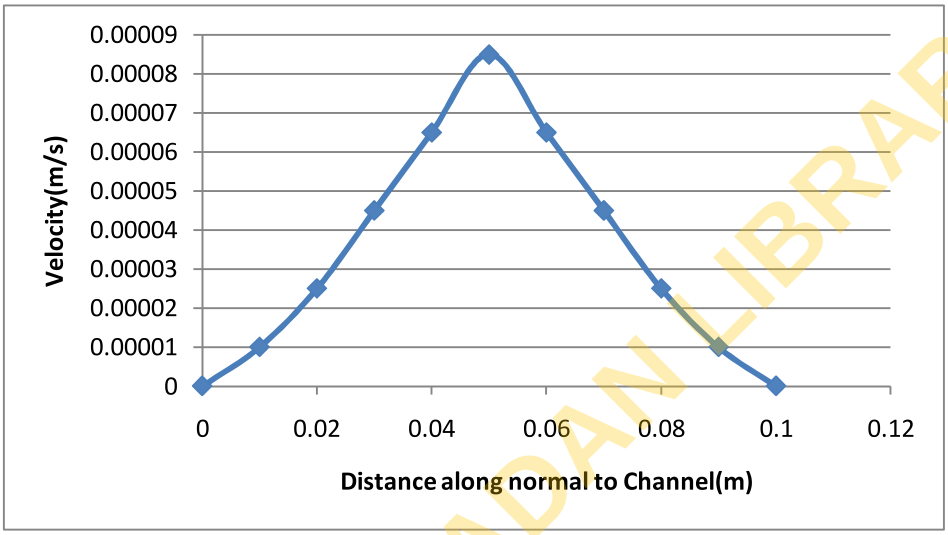

Figure 12 Velocity Distribution for Forced Convection in a Channel

Figure 12 is a plot of fluid velocity against distance measured normal to the channel wall at the trailing end of the channel, for a flow with a mainstream velocity U_o of 0.000088m/s. The values of velocity *u* are given in the dimensional form. It is evident in this figure that the velocity increases from the two plate walls towards the axis of the channel. From the symmetry of the plot, it is expected that the maximum velocity will occur at the axis of the channel, which from a close look at the figure is the case. The curve profile in Figure 12 has a distorted Vee-shaped but could have been parabolic if the channel width had been small, in consistence with the work of Ofi and Heterington (1977) who obtained a near parabolic shape for natural convection in a channel with width of 0.02m

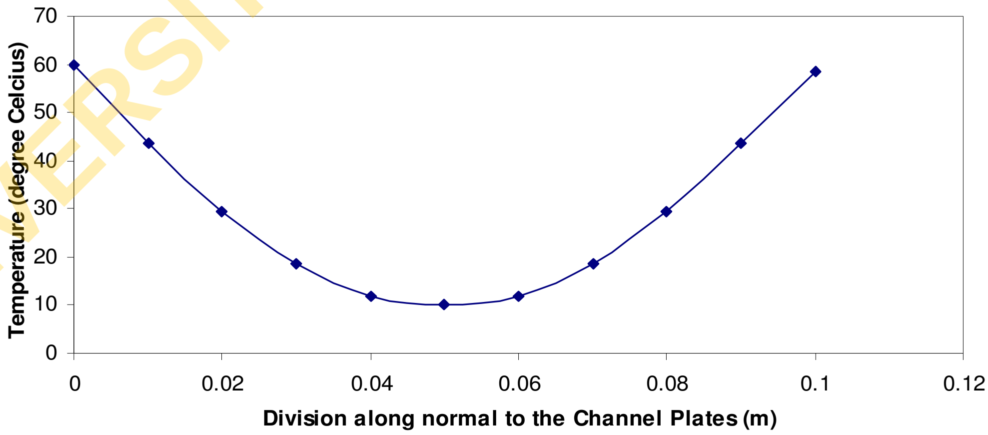

Figure 13 Temperature Distribution for Forced Convection in a Channel

Figure 13 is a plot of fluid temperature against distance measured normal to the channel wall at the trailing end. The figure shows the temperature to decrease parabolically from the two plate walls towards the axis of the channel. This trend is expected in steady state fluid flow, because the temperature of the walls, which is 60^0C is greater than the fluid temperature, referred to as free stream temperature, at 10^0C.

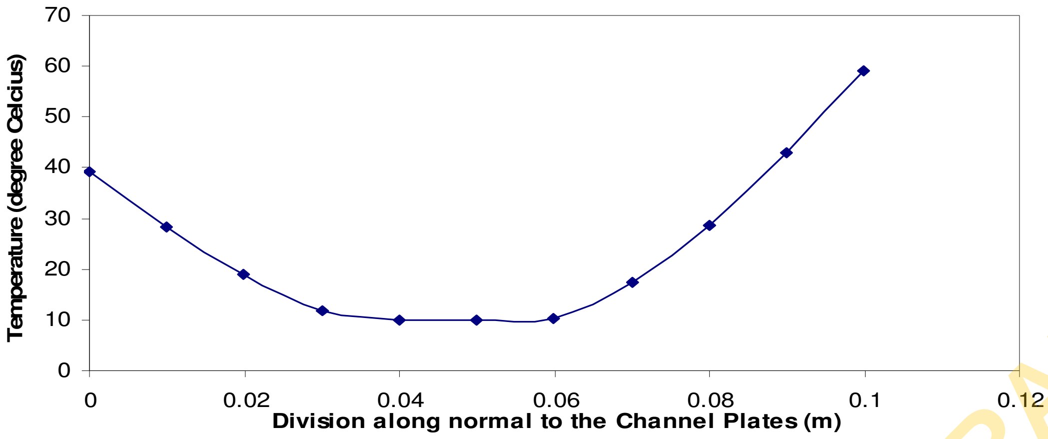

Figure 14 Temperature Distribution for Asymmetrically Heated Isothermal Plates

Figure 14 shows the temperature distribution of forced convection in asymmetrically heated isothermal parallel plates. One of the plates was kept at 40⁰C while the other was kept at 60⁰C. As expected, it can be seen from Figure 14 that the lowest fluid temperature at free stream is skewed towards the plate with a temperature of 40⁰C away from the one with a temperature of 60⁰C.

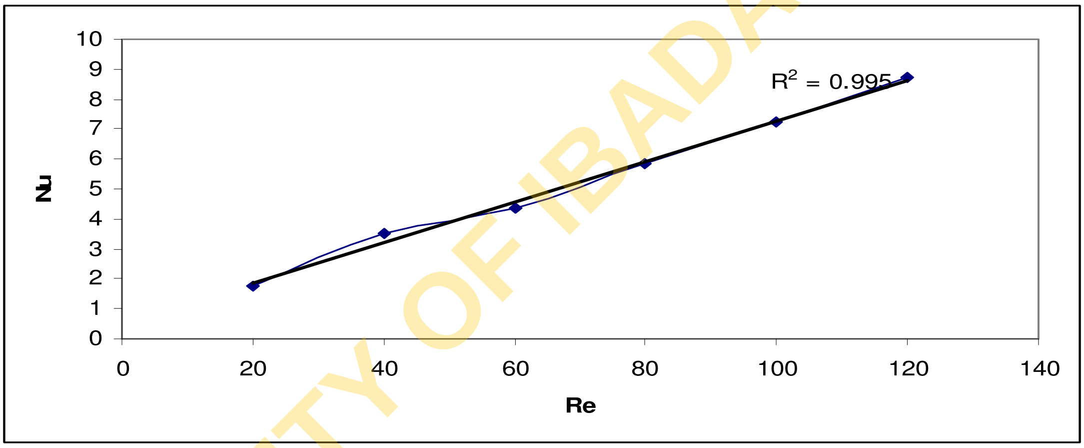

Figure 15 Nusselt Number against the Reynolds Number for a Horizontal Channel

The graph of *Nu* versus *Re* for the horizontal channel used in this work is presented in Figure 15 for values of *Re* varying from 20 to 120 in steps of 20.The results imply direct proportionality between the *Nu* and *Re* numbers for this case.

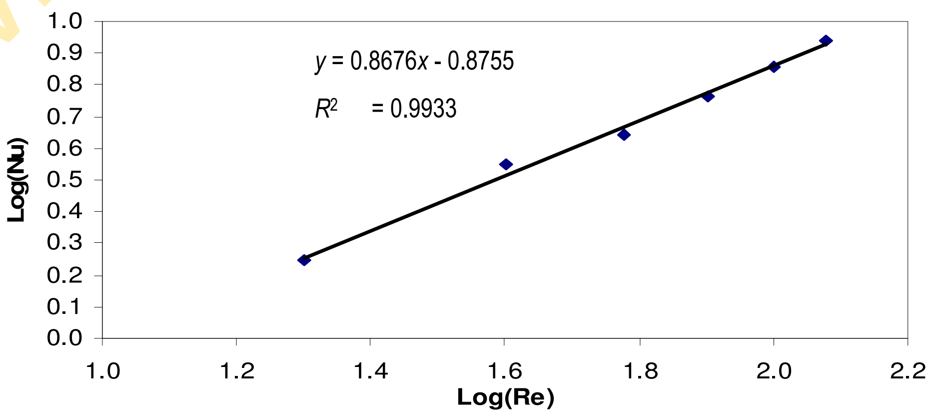

Figure 16 Logarithmic Plot of Nusselt Number Against that for Reynolds Number for Horizontal Channels

Figure 16 shows the corresponding log-log plot of the Nusselt number against the Reynolds number. The slope of the plot is 0.87 while the intercept is -0.88. At 35^{0}C, the Prandtl number, *Pr* is 0.71, therefore, using the same approach as was adopted for a single plate the relationship between Nusselt number, Reynolds number and Prandtl number for empirical correlation of the form $Nu = CRe^mPr^n$, for n = 0.33 is $Nu = 0.15Re^{0.87}Pr^{0.33}$. The two expressions for Nusselt number are useful; the earlier derived expression establishes the relationship between Nusselt number and the Reynolds number, while the later derivation gives a comprehensive relationship between the Nusselt number, Reynolds number and Prandtl number.

6.0 CONCLUSION

In this study, use of the diffusion velocity method, a version of the Vortex Element Method, has been successfully done and demonstrated as an effective way of modelling forced convection problems on isothermal plates and channels. In this work, trends relating the velocity and temperature boundary layer thicknesses and profiles to other parameters such as Reynolds number and viscosity were clearly identified using convectional flow along a flat plate and a flat channel. For the case of a flat plate, the velocity exhibited asymptotic growth from a value of zero at the plate wall to its free stream velocity. The temperature distribution for the flat plate also showed an asymptotic decrease from the plate wall temperature to the free stream temperature. The temperature distributions for the channel had parabolic profiles for free stream temperatures that were lower than the channel wall temperatures, for both the symmetric and asymmetric cases considered here. However, the pattern of velocity distributions did not have well defined parabolic profiles. This could only be attributed to slight overestimation of the velocities. Both the asymptotic growth and decay exhibited by the velocity and temperature for the flat plate and parabolic profiles of the temperature distributions for the channels have a strong support in literature support (Incropera and Dewitt, 2005). However, despite the fact that the model's relationship between *Nu* and *Re*, as well as *Pr* was not the same as that of Incropera and Dewitt (2005), this study has established the fact that the diffusion velocity method is a viable numerical tool capable of modelling fluid and heat transfer problems.

7.0 RECOMMENDATIONS

It is recommended that future work undertakes a comparative analysis of the various methods available for developing solutions of problems on forced convection on isothermal plates and channels, such as the methods of Virtual Flux, Rayleigh-Ritz, Boundary Element and Finite Element. Their performance and limitations should also be compared with the proposed approach in this work. Possibilities of forming hybrids using some of these methods with the diffusion velocity method should be looked into. Further possibilities of incorporating other simulation approaches into the method proposed in this work should also be explored. Further work which focuses on experimentation is desirable. Experimental investigations are necessary in the determination of the logarithmic plot of Nusselt number versus Reynolds number for forced convection on single horizontal plates and for forced convection in horizontal channels. This would serve to validate the models developed here.

8.0 ACKNOWLEDGEMENT

The authors are grateful to the following people who helped in the significant improvement of the earlier draft of the article: Shashank B Thakre, Kyoji Kamemoto and Yechun Wang.

REFERENCES

1. Barber R. W. and Fonty A. "Numerical Simulation of Low Reynolds Number Flows Using a Lagrangian Discrete Vortex Method", 2006, Retrieved 15th Jan; 2008 from www.cse.scitech.ac.uk/ceg/misc/vortex/vortex.shtml
2. Bin C., Cong W., Zhiwei W., Liejin G. "Investigation of Gas-Solid Two Phase Flow Across Circular Cylinders with Discrete Vortex Method", Applied Thermal Engineering, Vol. 29, 2009, pp. 1457-1466.
3. Cottet G.H., Koumosetsakos P., "Vortex Methods- Theory and Practice, Cambridge University Press, 2000.
4. Dare A.A., Petinrin M.O. "Modelling of Natural Convection along Isothermal Plates and in Channels Using Diffusion Velocity Method", Maejo International Journal of Science and Technology, Vol. 4, No. 1, 2010, pp. 43-52.
5. Diaz L.A. and Suryanarayana N.V. "Application of Rayleigh-Ritz Method to Forced Convection Turbulent Heat Transfer", Heat and Mass Transfer, Vol. 15, No. 2, 1981, pp. 67-77.
6. Gallati, M. and Braschi, G. "Vortex Method Simulation of the Flow Past a Cylinder", Applied Simulation and Modelling (ASM 2002), Track 363-216, June 25-28, Crete, Greece, 2002.

7. Golia, C. and Bernardo, B. "A Vortex-Thermal Blobs Method for 3D-Buoyancy Driven Flows", Proc. of the 3rd International Conference on Vortex Flows and Vortex Models (ICVFM2005), Yokohama, Japan, Nov. 21-23, 2005, pp. 275-280.
8. Incropera, F. P. and Dewitt, D. P.: Fundamentals of Heat and Mass Tranfer, 5th edition, John Wiley & Sons, New York, 2005.
9. He F., Su T.-C. "A Numerical Study of Bluff Body Aerodynamics in High Reynolds Number Flows", Journal of Wind Engineering and Industrial Aerodynamics, Vol. 77/78, 1998, pp. 393-407.
10. Huang M.-J., Su H.-X., Chen L.-C.: A fast resurrected core-spreading vortex method with no-slip boundary conditions, Journal of Computational Physics, 228, 1916-1931, (2009).
11. Kamemoto, K. and Miyasaka, T. "Development of a Vortex and Heat Elements Method and its Application to Analysis of Unsteady Heat Transfer Around a Circular Cylinder in a Uniform Flow". Vortex Methods, World Scientific, 2000, pp. 135-144.
12. Kamemoto, K. "On Contribution of Advanced Vortex Element Methods Toward Virtual Reality of Unsteady Vortical Flows in the New Generation of CFD", J. Braz. Soc. Mech. Sci. & Eng, Rio de Janeiro, 2004, Vol. 26, No. 4, Retrieved 3rd Sept 2007; from www.scielo.br/scielo.php?script_arttext&pid=s1678-5878200300020 0005.
13. Khatir Z. "A Boundary Element Method for the Numerical Investigation of Near-Wall Fluid Flow with Vortex Method Simulation", Engineering Analysis with Boundary Elements, Vol. 28, 2004, pp. 1405-1416.
14. Kreith, F. and Bohn, M. S.: Principles of heat transfer, 5th edition, West Publishing Company, St. Paul, (1993).
15. Lakkis I., Ghoniem A.F.: Axisymmetric Vortex Method for Low-Mach Number, Diffusion-Controlled Combustion, Journal of Computational Physics, 184, pp. 435-475, (2003).
16. Lewis, R. I.: Vortex Element Methods for Fluid Dynamic Analysis of Engineering systems, University Press, Cambridge, (1991).
17. Liu, L; Feng, J; Fan, J. and Cen, K. "Recent Development of Vortex Method in Incompressible Viscous Bluff Body Flows", J. of Zhejiang Univ. Sci., Vol. 6A, No. 4, 2005, pp. 283-288, Retrieved 20th Feb; 2008 from http://www.zju.edu.cn/jzus/2005/A0504/A050406.pdf
18. Miller I., Freud J.E., Probability and Statistics for Engineers, Prentice Hall of India Private Limited, New Delhi, 1987.
19. Ofi, O. and Hetherington, A.J. "Application of the Finite Element Method to Natural Convection Heat Transfer from The Open Vertical Channel", International Journal of Heat and Mass Transfer, Vol. 20, 1977, pp. 1195-1204.
20. Ogundare, M. A., "Vortex Element Modelling of Fluid and Heat Flow in Vertical Channels and Ducts, Unpublished Ph.D Thesis, Mechanical Engineering, University of Ibadan, Ibadan, 2006.
21. Ogami, Y. "Simulation of Heat-Vortex Interaction by the Diffusion Velocity Method", ESAIM Proceedings, Third International Workshop on Vortex Flows and Related Numerical Methods, Vol. 7, 1999, pp. 314-324.
22. Ogami Y., Fukumoto K. "Simulation of Combustion by Vortex Method", Computers and Fluids, Vol. 39, No. 4, 2010, pp. 592-603.
23. Petinrin, M. O. "Diffusion Velocity Modelling Of Forced and Natural Convection in Isothermal Plates and Channels", Unpublished M.Sc Thesis, Mechanical Engineering, University of Ibadan, Ibadan, (2008).
24. Ramirez-Camacho R.G. and Barbosa J.R. "The Boundary Element Method Applied to Forced Convection Heat Problems", International Communications in Heat and Mass Transfer, Vol. 35, No. 1, 2008, pp. 1-11.
25. Subramanian, S. "A New Mesh-Free Vortex Method", FAMU-FSU College of Engineering, 1996, Retrieved 20th Feb; 2008 from http://www.eng.fsu.edu/~dommelen/papers/subram/style_a/node78.html
26. Shirato H., Matsumoto M. "Unsteady Pressure Evaluation on Oscillating Bluff Body by Vortex Method", Journal of Wind Engineering, Vol. 67/68, 1997, pp. 349-359.
27. Smith, P.A. and Stansby, P.K. "An Efficient Surface Algorithm for Random Particle Simulation of Vorticity and Heat Transport, J. Comp. Phys., Vol. 81, 1989, pp. 349-371.
28. Tanno I., Morinishi K., Matsuno K., Nishida H. "Validation of Virtual Flux Method for Forced Convection Flows", Transactions of the Japan Society of Mechanical Engineers B, Vol. 72, No. 714, 2006, pp. 217-224.
29. Uchiyama T., Naruse M. "A Numerical Method for Gas-Solid Two-Phase Free Turbulent Flow Using a Vortex Method", Powder Technology, Vol. 119, 2001, pp. 206-214.
30. Uchiyama T., Yagami H. "Numerical Analysis of Gas-Particle Two-Phase Wake Flow by Vortex Method", Powder Technology, Vol. 149, 2001, pp. 112-120.
31. Yokota R., Sheel T.K., Obi S. "Calculation of Isotropic Turbulence Using a Pure Lagrangian Vortex Method", Journal of Computational Physics, Vol. 226, 2007, pp. 1589-1606.